

2x2 BG Factorial Designs

- Definition and advantage of factorial research designs
- 5 terms necessary to understand factorial designs
- 5 patterns of factorial results for a 2x2 factorial designs
- Descriptive & misleading main effects
- The F-tests of a Factorial ANOVA
- Using LSD to describe the pattern of an interaction

Introduction to factorial designs

Factorial designs have 2 (or more) Independent Variables

An Example...

Forty clients at a local clinic volunteered to participate in a research project designed to examine the individual and combined effects of the client's Initial Diagnosis (either general anxiety or social anxiety) and the Type of Therapy they receive (either group or individual). Twenty of the participants had been diagnosed with general anxiety and 20 had been diagnosed as having social anxiety. One-half of the clients with each diagnosis were assigned to receive group therapy and one-half received individual therapy. All clients underwent 6 months of the prescribed treatment, and then completed a battery of assessments which were combined into a DV score of "wellness from anxiety", for which larger scores indicate better outcome.

Here is a depiction of this design.

Showing this design is a 2x2 Factorial

Initial Diagnosis	Type of Therapy	
	Group	Individual
General Anxiety	clients diagnosed w/ general anxiety who received group therapy	clients diagnosed w/ general anxiety who received individual therapy
Social Anxiety	clients diagnosed w/ social anxiety who received group therapy	clients diagnosed w/ social anxiety who received individual therapy

Participants in each "cell" of this design have a unique combination of IV conditions.

What's involved in a 2x2 factorial design ?

There are 3 variables examined ...

- 1-- the DV (dependent, outcome, response, measured, etc. variable)
- 2 -- one IV (independent, treatment, manipulated, grouping, etc. variable)
- 3 – second IV (independent, treatment, manipulated, grouping, etc. variable)

There are 3 effects examined ...

- 1 -- the interaction of the two IVs -- how they jointly relate to DV
- 2 -- the main effect of the one IV -- how it relates to the DV independently of the interaction and the other main effect
- 3 -- the main effect of the other IV -- how it relates to the DV independently of the interaction and the other main effect

For the example...

- 1 -- the "interaction" of Initial Diagnosis & Type of Therapy
- 2 -- the "main effect" of Initial Diagnosis
- 3 -- the "main effect" of Type of Therapy

The difficult part of learning about factorial designs is the large set of new terms that must be acquired. Here's a summary;;

cell means -- the mean DV score of all the folks with a particular combination of IV treatments

marginal means -- the mean DV score of all the folks in a particular condition of the specified IV (aggregated across conditions of the other IV)

Main effects involve the comparison of marginal means.

Simple effects involve the comparison of cell means.

Interactions involve the comparison of simple effects.

Identifying Cell Means and Marginal Means

Initial Diagnosis	Type of Therapy		
	Group	Individual	
General Anxiety	50	50	50
Social Anxiety	90	10	50
	70	30	

Cell means → mean DV of subjects in a design cell

Marginal means → average mean DV of all subjects in one condition of an IV

Identifying Main Effects -- difference between the marginal means of that IV (ignoring the other IV)

Initial Diagnosis	Type of Therapy		
	Group	Individual	
General Anxiety	50	50	50
Social Anxiety	90	10	50
	70	30	

Main effect of Initial Diagnosis

Main effect of Type of Therapy

Identifying Simple Effects -- cell means differences between conditions of one IV for a specific level of the other IV

Initial Diagnosis	Type of Therapy		
	Group	Individual	
General Anxiety	50	50	1
Social Anxiety	90	10	2
	a	b	

- Simple effects of Initial Diagnosis for each Type of Therapy
- a Simple effect of Initial Diagnosis for group therapy
 - b Simple effect of Initial Diagnosis for individual therapy

Identifying Simple Effects -- cell means differences between conditions of one IV for a specific level of the other IV

Initial Diagnosis	Type of Therapy		
	Group	Individual	
General Anxiety	50	50	1
Social Anxiety	90	10	2
	a	b	

- Simple effects of Type of Therapy for each Initial Diagnosis
- 1 Simple effect of Type of Therapy for general anxiety patients
 - 2 Simple effect of Type of Therapy for social anxiety patients

Here are the three basic patterns of interactions

#1

Task Difficulty	Task Presentation		
	Paper	Computer	
Easy	90	= 90	one simple effect "null"
Hard	40	< 70	one simple effect

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Easy tasks are performed equally well using paper and using the computer (90 vs. 90), however, hard tasks are performed better using the computer than using paper (70 vs. 40).

#2

Task Difficulty	Task Presentation		
	Paper	Computer	
Easy	90	> 70	simple effects are
Hard	40	< 60	opposite directions

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Easy tasks are performed better using paper than using computer (90 vs. 70), whereas hard tasks are performed better using the computer than using paper (60 vs. 40).

#3

Task Difficulty	Task Presentation		
	Paper	Computer	
Easy	80	< 90	simple effects in the same direction,
Hard	40	< 70	but of different sizes

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Performance was better using the computer than using paper, however this effect was larger for hard tasks (70 vs. 40) than for easy tasks (90 vs. 80).

Here are the two basic patterns of NON-interactions

#1

Task Presentation

Paper Computer

Task Difficulty

Easy

30	<	50
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both simple effects are in the same direction and are the same size

Hard

50	<	70
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There is no interaction of Task Presentation and Task Difficulty as they relate to performance. Performance is better for computer than for paper presentations (for both Easy and Hard tasks).

#2

Task Presentation

Paper Computer

Task Difficulty

Easy

50	=	50
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both simple effects

Hard

70	=	70
----	---	----

are nulls

There is no interaction of Task Presentation and Task Difficulty as they relate to performance. Performance is the same for computer and paper presentations (for both Easy and Hard tasks).

So, there are 5 basic patterns of results from a 2x2 Factorial

Three patterns that have an interaction:

1. = vs. < one null simple effect and one simple effect
2. < vs. > simple effects in opposite directions
3. < vs. < simple effects in same direction, but different sizes

Two patterns that have no interaction:

4. < vs. < simple effects of the same size in the same direction
5. = vs. = both null simple effects

Interpreting main effects ... When there is an interaction, the pattern of the interaction may influence the interpretability (generality) of the description of the marginal means.

Task Difficulty	Task Presentation	
	Paper	Computer
Easy	80	90
Hard	40	70
	60	80

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation.

Notice: that the pattern of the main effect is consistent with both the simple effect of Task Presentation for easy tasks and the simple effect of Task Presentation for hard tasks.

Another example ...

Task Difficulty	Task Presentation	
	Paper	Computer
Easy	90	90
Hard	40	70
	65	80

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation. However, this pattern is descriptive for hard tasks, but not for easy tasks, for which there was no simple effect of Task Presentation.

Yet another example ...

Task Difficulty	Task Presentation	
	Paper	Computer
Easy	80	60
Hard	20	70
	50	65

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation. However, this pattern is descriptive for hard tasks, but not for easy tasks, for which performance was better using paper presentations than using computer presentation.

“Null” main effects can also be misleading....

Task Difficulty	Task Presentation	
	Paper	Computer
Easy	90 >	70
Hard	40 <	60
	65 =	65

There is no main effect for Task Presentation, overall performance was equivalent using computer presentation and using paper presentation. However, this pattern is descriptive for neither hard tasks, for which computer presentations worked better than paper, nor for easy tasks, for which performance was better using paper presentations than using computer presentation.

Remember the **5** basic patterns of results from a 2x2 Factorial ?

Interaction -- simple effects of different size and/or direction	$\left\{ \begin{array}{l} 1. = \text{ vs. } < \\ 2. < \text{ vs. } > \\ 3. < \text{ vs. } < \end{array} \right.$	$\left\{ \begin{array}{l} \text{one null simple effect and one simple effect} \\ \text{simple effects in opposite directions} \\ \text{simple effects in same direction, but different sizes} \end{array} \right.$	} Misleading main effects			
				No Interaction -- simple effects are null or same size	$\left\{ \begin{array}{l} 4. < \text{ vs. } < \\ 5. = \text{ vs. } = \end{array} \right.$	} Descriptive main effects

Statistical Analysis of 2x2 Factorial Designs

1. Tell IVs and DV
2. Present data in table or figure
3. Determine if the interaction is significant
 - if it is, describe it in terms of one of the sets of simple effects.
4. Determine whether or not the first main effect is significant
 - if it is, describe it
 - determine if that main effect is descriptive or misleading
5. Determine whether or not the second main effect is significant
 - if it is, describe it
 - determine if that main effect is descriptive or misleading

Interpreting Factorial Effects

Important things to remember:

- main effects and the interaction are 3 separate effects each must be separately interpreted -- three parts to the “story”
 - most common error -- “interaction is different main effects”
 - best thing -- be sure to carefully separate the three parts of the story and tell each completely
- Be careful of “causal words” when interpreting main effects and interactions (only use when really appropriate).
 - caused, effected influenced, produced, changed
- Consider more than the “significance”
 - consider effect sizes, confidence intervals, etc. when describing the results

Statistical Analysis of a 2x2 Design

Task Difficulty (b)	Task Presentation (a)		SE of Presentation for Easy Tasks
	Paper	Computer	
Easy	90	70	80
Hard	40	60	50
	65	65	SE for Presentation for Hard Tasks

Presentation Main Effect

$SS_{\text{Presentation}}$

65 vs. 65

Difficulty Main Effect

$SS_{\text{Difficulty}}$

80 vs. 50

Interaction Effect

$SS_{\text{Interaction}}$

SE_{Easy} vs. SE_{Hard}

Constructing F-tests for a 2x2 Factorial

$$F_{\text{Presentation}} = \frac{(SS_{\text{Presentation}} / df_{\text{Presentation}})}{(SS_{\text{Error}} / df_{\text{Error}})}$$

$$F_{\text{Difficulty}} = \frac{(SS_{\text{Difficulty}} / df_{\text{Difficulty}})}{(SS_{\text{Error}} / df_{\text{Error}})}$$

$$F_{\text{Interaction}} = \frac{(SS_{\text{Interaction}} / df_{\text{Interaction}})}{(SS_{\text{Error}} / df_{\text{Error}})}$$

Statistical Analyses Necessary to Describe Main Effects of a 2x2 Design

In a 2x2 Design, the Main effects F-tests are sufficient to tell us about the relationship of each IV to the DV...

- since each main effect involves the comparison of two marginal means -- the corresponding significance test tells us what we need to know ...
- whether or not those two marginal means are “significantly different”
- Don't forget to examine the means to see if a significant difference is in the hypothesized direction !!!

Statistical Analyses Necessary to Describe the Interaction of a 2x2 Design

However, the F-test of the interaction only tells us whether or not there is a “statistically significant” interaction...

- it does not tell us the pattern of that interaction
- to determine the pattern of the interaction we have to compare the simple effects
- to describe each simple effect, we must be able to compare the cell means

we need to know how much of a cell mean difference is “statistically significant”

Using LSD to Compare cell means to describe the simple effects of a 2x2 Factorial design

- LSD can be used to determine how large of a cell mean difference is required to treat it as a “statistically significant mean difference”
- Will need to know three values to use the computer
 - df_{error} -- look on the printout or use $N - 4$
 - MS_{error} -- look on the printout
 - $n = N / 4$ -- use the decimal value -- do not round to the nearest whole number!

Remember – only use the lsdmmd to compare cell means. Marginal means are compared using the main effect F-tests.

What statistic is used for which factorial effects????

Age	Gender		
	Male	Female	
5	30	30	30
10	20	30	25
	25	30	

This design as 7 "effects"

1. Main effect of age
2. Main effect of gender
3. Interaction of age & gender
4. SE of age for males
5. SE of age for females
6. SE of gender for 5 yr olds
7. SE of gender for 10 yr olds

There will be 4 statistics

1. F_{Age}
2. F_{Gender}
3. F_{Int}
4. LSD_{mmd}

What statistic is used for which factorial effects????

Age	Gender		
	Male	Female	
5	50	30	40
10	60	80	70
	25	30	

- Are 40 & 70 different ? F_{Age}
- Are 50 & 30 different ? LSD_{mmd}
- Are 30 & 80 different ? LSD_{mmd}
- Are 50 & 60 differently different than 30 & 80 ? F_{Int}
- Are 50 & 60 different ? LSD_{mmd}
- Are 25 & 30 different ? F_{Gender}
- Are 50 & 30 differently different than 60 & 80 ? F_{Int}
- Are 60 & 80 different ? LSD_{mmd}

1. $F_{Age} \quad p = .021$
2. $F_{Gender} \quad p = .082$
3. $F_{Int} \quad p = .001$
4. $LSD_{mmd} = 15$

Applying $l_{sd_{mmd}}$ to 2x2 BG ANOVA

Task Difficulty	Task Presentation	
	Paper	Computer
Easy	60	90
Hard	60	70

for the interaction
 $F(1,56) = 6.5, p = .023$

$l_{sd_{mmd}} = 14$

Is there an Interaction effect? Based on what?

Yes! F-test of Int
for the following, tell the mean difference and apply the $l_{sd_{mmd}}$

- | | | |
|--|----|---|
| Simple effect of Task Presentation | 30 | > |
| SE of Task Presentation for Easy Tasks | | |
| SE of Task Presentation for Hard Tasks | 10 | = |
| Simple effects of Task Difficulty | | |
| SE of Task Difficulty for Paper Pres. | 0 | |
| SE of Task Difficulty for Comp. Pres. | 20 | v |

Applying $l_{sd_{mmd}}$ to 2x2 BG ANOVA

Task Difficulty	Task Presentation		ME	for Difficulty ME $F(1,56) = 4.5, p = .041$
	Paper	Computer		
Easy	60	90	75	$l_{sd_{mmd}} = 14$
Hard	60	70	65	

Is there a Task Difficulty main effect? Based on what?

Yes! F-test of ME

Is main effect descriptive (unconditional) or potentially misleading (conditional)?

Simple effects of Task Difficulty

SE of Task Difficulty for Paper Pres.	0	
SE of Task Difficulty for Comp. Pres.	20	v

Descriptive only for Computer presentation; misleading for Paper presentations.

Applying $l_{sd_{mmd}}$ to 2x2 BG ANOVA

Task Difficulty	Task Presentation		ME	for Presentation ME $F(1,56) = 7.2, p = .011$
	Paper	Computer		
Easy	60	90	80	$l_{sd_{mmd}} = 14$
Hard	60	70		

Is there a Presentation main effect? Based on what?

Yes! F-test of ME

Is main effect descriptive (unconditional) or potentially misleading (conditional)?

Simple effects of Task Difficulty

SE of Task Presentation for Easy Tasks	30	<
SE of Task Presentation for Hard Tasks	10	=

Descriptive only for Easy tasks; misleading for Difficult tasks.

Remember the definition of an interaction...

- An interaction is when the simple effects are different in direction and/or size

Remember about the differential power of the different tests...

- Interaction significance tests are usually less powerful than main effects significance tests.
- Simple effect significance tests are usually more powerful than interaction significance tests, but less powerful than main effect significance tests.

A couple of data patterns you should know about...

#1 – significant interaction with no significant SEs

Task Difficulty	Task Presentation		for the interaction
	Paper	Computer	
Easy	60	= 70	$F(1,56) = 6.5, p = .023$
Hard	80	= 70	$LDS_{mmd} = 14$

Huh???

The significant interaction tells us → the simple effects are different from each other (not that → either is different from 0)

- The 10-point SEs in opposite directions
- neither is different from 0
- but, they are different from each other!

A couple of data patterns you should know about...

#2 – non-significant interaction with a significant SE

Task Difficulty	Task Presentation		for the interaction
	Paper	Computer	
Easy	60	< 75	$F(1,56) = 2.5, p = .13$
Hard	60	= 70	$LDS_{mmd} = 14$

Huh???

The non-significant interaction tells us → the simple effects are not different from each other (not that they are both the same)

- The 10-point & 15-point SEs in the same direction
- one is different from 0 and one is not
- but, they are not different from each other!

Effect Sizes for 2x2 BG Factorial designs

For Main Effects & Interaction (each w/ df=1)

$$r = \sqrt{[F / (F + df_{error})]}$$

For Main Effects & Simple Effects

$$d = (M_1 - M_2) / \sqrt{M_{error}}$$

$$r = \sqrt{\left[\frac{d^2}{d^2 + 4} \right]} \quad (\text{This is an "approximation formula"})$$