## ANOVA

## Analysis of Variance

- Picking the correct bivariate statistical test
- When, Why to Use ANOVA
- Summarizing/Displaying Data -- 1 qual \& 1 quant var
- How ANOVA \& F Work
- Research and Null Hypotheses for ANOVA
- Making decisions about HO: and RH:
- Causal Interpretation
- Between Groups and Within-Groups ANOVA

So... what are the bivariate null hypothesis significance tests (NHSTs) we'll be using ???

What are the two kinds of variables that we've discussed?
What are the possible bivariate combinations?


We have separate bivariate statistics for each of these three data situations...

There is lots to learn about each of the statistical tests, but right now I want you to be sure you can tell when to use which one...
the "secret" is to figure out whether each variable is qualitative or quantitative, then you'll know which or the 3 stats to use !!

We want to know whether there is a relationship between someone's income and their amount of political campaign contributions.
IQ is ... quant Contributions is ... quant Stat? Pearson's $r$

We want to know whether employed and self-employed people make different amounts of political campaign contributions.
Employment...
qual Contributions is ...
quant
Stat?
F

We want to know whether Democrats or Republicans are more likely to make a political contribution.

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## Here's a few more...

"relationship" expressions of hypotheses
$\bullet$ expect there is a relationship between a person's height $r$ and their weight.

- I believe we'll find that there is a relationship between a person's athletic history (HS vs. not) and their weight.
- My hypothesis is that there is a relationship between a person's athletic history and whether or not currently work out.
"tend to..." expressions of hypotheses
- I expect that HS ath. tend to be heavier than non HS ath.
- My hypothesis is that taller folks also tend to be heavier
- I expect that folks who currently work out tend to have been HS athletes.
"if ... then more likely..." expressions of hypotheses
- If you currently work out, then you are more likely to have been a HS athlete
- If you are heavier, then you are more likely to be taller.


## When to use ANOVA ?

- Whenever you want to compare the means on a quantitative variable for two different groups or conditions.


## Said Statistically?

- When you are testing for a relationship between one quantitative variable and one qualitative variable -- by comparing the means of the quantitative variable for the categories of the qualitative variable
- Note: "groups" is often used when the qualitative variable is a subject variable, and "conditions" often used when it is a manipulated variable, but "categories", "situations", "subpopulations", "treatments" are all used.

To which of the following variable pairs would you apply ANOVA?

- GRE \& Public/Private school
- School Attended \& Type of Job Obtained
- Favorite Sport \& Age
- Age group \& Performance Speed
- Income \& Years of Education
- Income \& Highest Educational Degree

ANOVA

## Summarizing \& Reporting ANOVA Data

| Qual var -- When tested |  | Driving at Night | Driving <br> During Day |
| :---: | :---: | :---: | :---: |
| Quant -- Performance Score |  | 13 | 18 |
|  |  | 15 | 14 |
| Report the mean and Std of the quant variable for each condition of the qual variable. |  | 10 | 17 |
|  |  | 12 | 19 |
|  | Mean | 12.5 | 17.0 |
|  | Std | 2.1 | 2.2 |

We'd report the mean of which variable in each pair?

- GRE \&Ppublic/Private school


## GRE

- Favorite Sport \& Age

Age

- Failure Rate \& Brand of Computer

Failure rate

- Age group \& Performance Speed

Speed

- Income \& Highest Educational Degree

Income

## Displaying ANOVA Data

A Bar Graph can be used to display the data.

- The height of each bar is the mean of the quant var for that condition of the qual var.
- The "whiskers" show variability around each mean. Might be ...
- +/- ? std or SEM or CI?\%

- gotta be careful!!!

Table give more complete data, while graphs make it easier to "see" the means comparison quickly.

## Research Hypotheses for ANOVA

ANOVA RH: are ...

- Always about mean differences
- Always about the populations represented by the groups or conditions, not the groups or conditions themselves (remember, this is about inference)
- For 2-group designs, have only three possible RH: patterns

$$
\bar{X}_{\mathrm{G} 1}<\overline{\mathrm{X}}_{\mathrm{G} 2} \quad \overline{\mathrm{X}}_{\mathrm{G} 1}=\overline{\mathrm{X}}_{\mathrm{G} 2} \quad \overline{\mathrm{X}}_{\mathrm{G} 1}>\overline{\mathrm{X}}_{\mathrm{G} 2}
$$

Note: We’ll use the symbol "=" but we'll use phrases like "equivalent", "nearly equal", "not significantly different", "statistically equivalent", etc.

It is unlikely that the average of two populations is exactly the same. What we mean is that the mean difference "isn't large enough to be
"meaningful", "practical" or "important".

## Examples of Research Hypotheses for ANOVA

For a given pair of variables there are only three possible RH:s
Using Type of Therapy (group or individual) and Depression (measures on a 20-point scale) as an example...

- A population of psychiatric patients that receives group therapy will have lower average depression scores than a population of psychiatric patients that receives individual therapy.
- A population of psychiatric patients that receives group therapy will have statistically equivalent average depression scores as a population of psychiatric patients receiving individual therapy.
- A population of psychiatric patients that receives group therapy will have higher average depression scores than a population of psychiatric patients that receives individual therapy.


## Null Hypotheses for ANOVA

ANOVA HO: are ...

- Always about mean differences
- Always about the populations represented by the groups or conditions, not the groups or conditions themselves (remember, this is about inference)
- Always that $\bar{X}_{\mathrm{G} 1}=\bar{X}_{\mathrm{G} 2}$

Using the Type of Therapy (group or individual) and Depression (measures on a 20-point scale) example from before...

- A population of psychiatric patients that receives group therapy will have the same average depression scores as a population of psychiatric patients receiving individual therapy.

The H0: is a mathematical expression, so "equal" is appropriate -- unlike for the RH:

## How ANOVA "Works"

## ANOVA is from -- ANalysis Of VAriance

- Variance is a statistical term for variation or variability
- In ANOVA, variation among the scores on the quantitative variable is divided into
- variation between the groups / conditions
- variation within the groups / conditions
- These two types of variation are then combined into the ANOVA summary statistic -- $\mathbf{F}$
- F has a range of 0 to $\infty$
- We use regular "HO: testing logic"
- if the F is small, then we can't say the groups represent populations with different means
- if the F is "large enough" then the groups probably don't represent populations with the same mean on the duantitative variable


## Example of How ANOVA "Works"

Consider the following data set, with variation in the quantitative scores between \& within each condition of the qualitative variable


What "Retaining HO:" and "Rejecting HO:" means...

- When you retain H0: you're concluding...
- The mean difference between these groups/conditions in the sample is not large enough to allow me to conclude there is a mean difference between the populations represented by the groups/conditions.
- When you reject HO: you're concluding...
- The mean difference between these groups/conditions in the sample is large enough to allow me to conclude there is a mean difference between the populations represented by the groups/conditions.


## Mechanics of HO : testing with ANOVA

There are two different ways of making this decision, depending upon whether you are doing the analysis on the computer or performing the computations by hand. You must be familiar with each procedure.

## On the computer:

- Obtain the summary statistic and $p$-value

$$
F=5.21 \quad p=.024
$$

- Decide whether to retain or reject HO :
- if $p<.05$, reject HO : (decide the variables are related)
- if $p>.05$, retain HO : (decide there is no relationship)
- for the example, since $\mathrm{p}<.05 \ldots$ reject HO :

Remember: p tells you the probability of a Type I error (False Alarm) if you reject H0: -- we're only willing to take a $5 \%$ risk

Computing By Hand:

- Compute the "obtained value" of the summary statistic (based on the sample data -- sometimes called F-computed or -calculated)

$$
\text { F-obtained }=5.21
$$

- Look up the "critical value" of $F$ for the design on the "F-table"

$$
\text { F-critical }=4.41
$$

- Decide whether to retain or reject HO :
- If the "obtained value" is larger than the "critical value", reject HO:
- If the "obtained value" is smaller than the "critical value", retain HO :
- for the example $5.21>4.41$, so reject HO :

By-hand and computer analysis of the same data will always produce the same result, because if $p<.05$, then $F$-obtained $>$ F-critical

A little H0: and RH: practice ...
RH: 1st graders given more practice would have better performance on their math test.
Mean for 10-practice group $=72.4 \%$ $F=1.23, p=.15$
Mean for 30-practice group $=74.8 \%$
Retain or Reject HO: ? retain
Support for RH: ? No - no effect
-RH: Patients who receive psychotropic drugs will respond as well to "talk therapy" as those who don't.
Mean for drug group $=18.2 \quad \mathrm{~F}=6.24$, F-critical $=3.21$
Mean for no-drug group = 24.8
Retain or Reject H0: ? reject
Support for RH: ?
No - RH: = HO:, but got an effect
RH: Doing the WebEx will improve your scores on the computational homework.
Mean for WebEx group $=85.6 \% \quad F=8.98, p=.001$
Mean for no-WebEx group = 92.4\%
Retain or Reject HO: ? reject
Support for RH: ?

Statistical decisions \& errors with ANOVA...
In the Population

| Statistical | $\mathrm{X}_{\mathrm{G} 1}<\mathrm{X}_{\mathrm{G} 2}$ | $\mathrm{X}_{\mathrm{G} 1}=\mathrm{X}_{\mathrm{G} 2}$ | $\mathrm{X}_{\mathrm{G} 1}>\mathrm{X}_{\mathrm{G} 2}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & X_{G 1}<X_{G 2} \\ & (p<.05) \end{aligned}$ | Correct HO: Rejection \& Direction | Type I "False Alarm" | Type III <br> "Mis-specification" |
| $\begin{aligned} & X_{G 1}=X_{G 2} \\ & (p>.05) \end{aligned}$ | Type II "Miss" | Correct HO: <br> Retention | Type II "Miss" |
| $\begin{aligned} & X_{G 1}>X_{G 2} \\ & (p<.05) \end{aligned}$ | Type III <br> "Mis-specification" | Type I <br> "False Alarm" | Correct H0: Rejection \& Direction |

Remember that "in the population" is "in the majority of the literature" in practice!!

## About causal interpretation of ANOVA results ...

Like the other bivariate statistics we have studied, we can only give a causal interpretation of the results if the data were collected using a true experiment

- random assignment of subjects to conditions of the qualitative variable (IV) - gives initial eq.
- manipulation of the IV by the experimenter - gives temporal precedence
- control of procedural variables - gives ongoing eq.

Only then can differences between the condition means be taken as evidence that the IV causes the DV

Practice with causal interpretation of ANOVA results ...
Which of the following might and can't be given a causal interpretation?

- The 10 year olds spelled more words correctly than the 5 year olds.
- Those taught spelling using the computer did better than those taught by lecture.
- Democrats donate more to environmental causes than Republicans
- After instruction about the importance of our environment, school children increased the

Might amount they wanted to donate.

Can't tell ??? Remember, you need the operational definitions \& procedural info. to know if this is a True Exp, with RA, etc...

## ANOVA (F) - - - 2 Types

- Between Groups ANOVA
- we collect the quantitative variable from each participant, who is only ever in one condition of the qualitative variable
- also called "between subjects", "independent groups" or "independent subjects"
- Within-Groups ANOVA
- we collect the quantitative variable from each participant, who is in all conditions of the qualitative variable (one at a time, of course)
- also called "within-subjects", "dependent groups", "dependent subjects", or "repeated measures" designs

Between Subjects (Between Groups)


Within-subjects (Within-groups, Repeated Measures)

| Control | Experimental |  |
| :--- | :--- | :--- |
| Gissa Kate | Gissa Kate |  |
| John | Mo | John Mo |

## Comparison of BG \& WG ANOVA

- Data collection is different for the two
- BG - each participant is only in one condition
- WG - each participant will be in all conditions
- Same HO: (No mean difference)
- Same kind of RH (expected mean difference)
- Computation is slightly different for BG \& WG
- H0: testing is the same $\mathrm{p}<.05$ or F-crit $<\mathrm{F}$-obt
- Determining Causality is the Same
- random assignment (initial equivalence)
- manipulation of IV (temporal precedence and ongoing eq
- control of procedural variables (ongoing equivalence)


## Practice With Determining Whether Design is BG or WG

Here's two different versions of a study to test that "more practice leads to better performance"-- which is BG and which WG

- Each person is introduced to the task and either performs immediately (score is \# correct of 10), or is given 30 practices first and then performs (and is scored the same say).
- Each person is introduced to the task and immediately performs (score is \# correct out of 10). Is then given 30 \#correct out of 10).

Your turn - tell type of design \& pick which 2 variables to specify in SPSS ...

We wanted to know whether Psyc or Business majors studied more hours per week for this class.

Design ? BG What's the IV? major What's the DV?\# hours
Which two variables would go into SPSS (yes, pick 2 - only 2)

- major
- \# hours Psy study

Draw the boxes!


We wanted to know whether students studied more hours per week for the lecture or the laboratory of this class.

Design ? WG What's the IV? Lecture vs. Lab What's the DV? \# hours
Which two variables would go into SPSS (yes, pick 2 - only 2)

- lecture vs. lab
- \# hours study
- \# hours study for lecture
- \# hours study for lab



## What about t-tests?

- Whenever you want to compare the means from a 2- Between Groups design, you can use either a BG ANOVA or a BG t-test
- Whenever you want to compare the means from a 2-Within-Groups design, you can use either a WG ANOVA or a WG t-test

The two procedures will produce exactly the same:

- group means
- $p$-value \& NHST results
- $\mathrm{t}^{2}=\mathrm{F}$
- ANOVA dferror = t-test df

We will emphasize ANOVA in this class, because it is used somewhat more often, and because, unlike t-tests, can be used for larger designs (with more IV conditions - later!)


[^0]:    Registration is ... qual
    Contributions is ... qual
    Stat? Pearson's X

