Between Groups & Within-Groups ANOVA

- BG & WG ANOVA
  - Partitioning Variation
  - "making" F
  - "making" effect sizes

ANOVA → ANalysis Of VAriance

Variance means "variation"
- Sum of Squares (SS) is the most common variation index
- SS stands for, “Sum of squared deviations between each of a set of values and the mean of those values”
  \[ SS = \sum (\text{value} - \text{mean})^2 \]

So, Analysis Of Variance translates to "partitioning of SS"
In order to understand something about "how ANOVA works" we need to understand how BG and WG ANOVAs partition the SS differently and how F is constructed by each.

Variance partitioning for a BG design

- Variation among all the participants – represents variation due to "treatment effects" and "individual differences"
- Variation between the conditions – represents variation due to "treatment effects"
- Variation among participants within each condition – represents "individual differences"
- Called "error" because we can't account for why the folks in a condition -- who were all treated the same -- have different scores.

\[
SS_{\text{Total}} = SS_{\text{Effect}} + SS_{\text{Error}}
\]
How a BG $F$ is constructed

Mean Square is the SS converted to a “mean” → dividing it by “the number of things”

$SS_{Total} = SS_{Effect} + SS_{Error}$

$df_{effect} = k - 1$

represents # conditions in design

$F = \frac{MS_{effect}}{MS_{error}} = \frac{SS_{effect} / df_{effect}}{SS_{error} / df_{error}}$

$df_{error} = \sum n - k$

represents # participants in study

How a BG $r$ is constructed

$r^2 = \frac{effect}{(effect + error)}$ ← conceptual formula

$= \frac{SS_{effect}}{(SS_{effect} + SS_{error})}$ ← definitional formula

$= \frac{F}{(F + df_{error})}$ ← computational formula

$SS_{Total} = SS_{Effect} + SS_{Error}$

$1757.574 = 605.574 + 1152.000$

$r^2 = \frac{SS_{effect}}{(SS_{effect} + SS_{error})}$

$= 605.574 / (605.574 + 1152.000) = .34$

$r^2 = \frac{F}{(F + df_{error})}$

$= 9.462 / (9.462 + 18) = .34$

An Example …
Variance partitioning for a WG design

\[ \text{SS}_{\text{Total}} = \text{SS}_{\text{Effect}} + \text{SS}_{\text{Subj}} + \text{SS}_{\text{Error}} \]

- Variation among participants — estimable because “S” is a composite score (sum)
- Variation among participant's difference scores — represents "individual differences"
- Called “error” because we can't account for why folks who were in the same two conditions — who were all treated the same two ways — have different difference scores.

How a WG F is constructed

Mean Square is the SS converted to a “mean” → dividing it by “the number of things”

\[ \text{SS}_{\text{Total}} = \text{SS}_{\text{Effect}} + \text{SS}_{\text{Subj}} + \text{SS}_{\text{Error}} \]

\[ \text{df}_{\text{effect}} = k - 1 \] represents # conditions in design

\[ \text{df}_{\text{error}} = (k-1)*(n-1) \] represents # data points in study

\[ F = \frac{\text{MS}_{\text{effect}}}{\text{MS}_{\text{error}}} = \frac{\frac{\text{SS}_{\text{effect}}}{\text{df}_{\text{effect}}}}{\frac{\text{SS}_{\text{error}}}{\text{df}_{\text{error}}}} \]

How a WG r is constructed

\[ r^2 = \frac{\text{effect}}{\text{effect} + \text{error}} \] ← conceptual formula

\[ = \frac{\text{SS}_{\text{effect}}}{\text{SS}_{\text{effect}} + \text{SS}_{\text{error}}} \] ← definitional formula

\[ = \frac{\text{F}}{\text{F} + \text{df}_{\text{error}}} \] ← computational formula

\[ F = \frac{\text{MS}_{\text{effect}}}{\text{MS}_{\text{error}}} = \frac{\frac{\text{SS}_{\text{effect}}}{\text{df}_{\text{effect}}}}{\frac{\text{SS}_{\text{error}}}{\text{df}_{\text{error}}}} \]
An Example …

This uses the data from the BG design!!!!!!

<table>
<thead>
<tr>
<th>Group</th>
<th>Score</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>25</td>
</tr>
</tbody>
</table>

Tests of Within-Subjects Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>281.676</td>
<td>9</td>
<td>31.307</td>
<td>.68</td>
<td>.50</td>
</tr>
</tbody>
</table>

Tests of Between-Subjects Effects

<table>
<thead>
<tr>
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<th>Mean Square</th>
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<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>870.325</td>
<td>1</td>
<td>870.325</td>
<td>.68</td>
<td>.50</td>
</tr>
</tbody>
</table>

This is just an example -- don’t ever do this with real data !!!!!!

\[
\text{SS}_{\text{total}} = \text{SS}_{\text{effect}} + \text{SS}_{\text{subj}} + \text{SS}_{\text{error}}
\]

\[
1757.574 = 605.574 + 281.676 + 870.325
\]

\[
r^2 = \frac{\text{SS}_{\text{effect}}}{\text{SS}_{\text{effect}} + \text{SS}_{\text{error}}}
\]

\[
r^2 = \frac{605.574}{605.574 + 281.676} = .68
\]

What happened????? Same data. Same means & Std.

Same total variance. Different F ???

BG ANOVA \[ \text{SS}_{\text{Total}} = \text{SS}_{\text{Effect}} + \text{SS}_{\text{Error}} \]

WG ANOVA \[ \text{SS}_{\text{Total}} = \text{SS}_{\text{Effect}} + \text{SS}_{\text{Subj}} + \text{SS}_{\text{Error}} \]

The variation that is called “error” for the BG ANOVA is divided between “subject” and “error” variation in the WG ANOVA.

Thus, the WG F is based on a smaller error term than the BG F \rightarrow and so, the WG F is generally larger than the BG F.

\[
r^2 = \frac{\text{SS}_{\text{effect}}}{\text{SS}_{\text{effect}} + \text{SS}_{\text{error}}}
\]

\[
r^2 = \frac{605.574}{605.574 + 281.676} = .68
\]

What happened????? Same data. Same means & Std.

Same total variance. Different r ???

\[
r^2 = \frac{\text{effect}}{(\text{effect} + \text{error})}
\]

\[
r^2 = \frac{\text{SS}_{\text{effect}}}{(\text{SS}_{\text{effect}} + \text{SS}_{\text{error}})}
\]

\[
r^2 = \frac{F}{(F + df_{\text{error}})}
\]

The variation that is called “error” for the BG ANOVA is divided between “subject” and “error” variation in the WG ANOVA.

Thus, the WG r is based on a smaller error term than the BG r \rightarrow and so, the WG r is generally larger than the BG r.