2x2 BG Factorial Designs

- The structure, variables and effects of a factorial design
- 5 terms necessary to understand factorial designs
- 5 patterns of factorial results for a 2x2 factorial designs
- Descriptive & misleading main effects
- Causal Interpretability of Factorial Effects
- Variable Explication for Factorial Designs
- Line and Bar Graphs of Factorial Designs

Introduction to factorial designs

Factorial designs have 2 (or more) Independent Variables

An Example…
Forty clients at a local clinic volunteered to participate in a research project designed to examine the individual and combined effects of the client's Initial Diagnosis (either general anxiety or social anxiety and the Type of Therapy they receive (either group or individual). Twenty of the participants had been diagnosed with general anxiety and 20 had been diagnosed as having social anxiety. One-half of the clients with each diagnosis were assigned to receive group therapy and one-half received individual therapy. All clients underwent 6 months of the prescribed treatment, and then completed a battery of assessments which were combined into a DV score of "wellness from anxiety", for which larger scores indicate better outcome.
Here is a depiction of this design.

Showing this design is a 2x2 Factorial

	Type of Therapy	
Initial Diagnosis	Group	Individual
General Anxiety	clients diagnosed w/ general anxiety who received group therapy	clients diagnosed w/ general anxiety who received individual therapy
Social Anxiety	clients diagnosed w/ social anxiety who received group therapy	clients diagnosed w/ social anxiety who received individual therapy

Participants in each "cell" of this design have a unique combination of IV conditions.

What's involved in a 2x2 factorial design ?

There are 3 variables examined …

1-- the DV (dependent, outcome, response, measured, etc. variable)

2 -- one IV (independent, treatment, manipulated, grouping, etc. variable)

3 – second IV (independent, treatment, manipulated, grouping, etc. variable)

There are 3 effects examined …

1 -- the main effect of the one IV -- how it relates to the DV independently of the interaction and the other main effect

2 -- the main effect of the other IV -- how it relates to the DV independently of the interaction and the other main effect

3 -- the interaction of the two IVs -- how they jointly relate to DV

For the example…

1 -- the "main effect" of Initial Diagnosis

2 -- the "main effect" of Type of Therapy

3 -- the "interaction" of Initial Diagnosis & Type of Therapy

The difficult part of learning about factorial designs is the large set of new terms that must be acquired. Here's a summary;;

cell means -- the mean DV score of all the folks with a particular combination of IV treatments

marginal means -- the mean DV score of all the folks in a particular condition of the specified IV (aggregated across conditions of the other IV)

Main effects involve the comparison of marginal means.

Simple effects involve the comparison of cell means.

Interactions involve the comparison of simple effects.

- An interaction is defined as "different simple effects"
- when the simple effects of one variable are different in direction and/or size across the conditions of the other variable

Identifying Cell Means and Marginal Means

Initial Diagnosis	Type of Therapy: Group	Type of Therapy: Individual	
General Anxiety	50	50	50
Social Anxiety	90	10	50
	70	30	

Cell means ➔ mean DV of subjects in a design cell

Marginal means ➔ average mean DV of all subjects in one condition of an IV

Identifying Main Effects -- difference between the marginal means of that IV (ignoring the other IV)

Initial Diagnosis	Type of Therapy: Group	Individual	
General Anxiety	50	50	50
Social Anxiety	90	10	50
	70	30	

Main effect of Initial Diagnosis

Main effect of Type of Therapy

Identifying Simple Effects -- cell means differences between conditions of one IV for a specific level of the other IV

Initial Diagnosis	Type of Therapy: Group	Individual	
General Anxiety	50	50	1
Social Anxiety	90	10	2
	a	b	

Simple effects of Initial Diagnosis for each Type of Therapy

a Simple effect of Initial Diagnosis for group therapy

b Simple effect of Initial Diagnosis for individual therapy

Identifying Simple Effects -- cell means differences between conditions of one IV for a specific level of the other IV

Initial Diagnosis	Type of Therapy: Group	Individual	
General Anxiety	50	50	1
Social Anxiety	90	10	2
	a	b	

Simple effects of Type of Therapy for each Initial Diagnosis

1 Simple effect of Type of Therapy for general anxiety patients

2 Simple effect of Type of Therapy for social anxiety patients

Here are the three basic patterns of interactions

#1

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	90	=	90
Hard	40	<	70

one simple effect "null"

one simple effect

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Easy tasks are performed equally well using paper and using the computer (90 vs. 90), however, hard tasks are performed better using the computer than using paper (70 vs. 40).

#2

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	90	>	70
Hard	40	<	60

simple effects are

opposite directions

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Easy tasks are performed better using paper than using computer (90 vs. 70), whereas hard tasks are performed better using the computer than using paper (60 vs. 40).

#3

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	80	<	90
Hard	40	<	70

simple effects in the same direction,

but of different sizes

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Performance was better using the computer than using paper, however this effect was larger for hard tasks (70 vs. 40) than for easy tasks (90 vs. 80).

Here are the two basic patterns of NON-interactions

#1 Task Presentation

Task Difficulty	Paper		Computer
Easy	30	<	50
Hard	50	<	70

both simple effects are in the same direction and are the same size

There is no interaction of Task Presentation and Task Difficulty as they relate to performance. Performance is better for computer than for paper presentations (for both Easy and Hard tasks).

#2 Task Presentation

Task Difficulty	Paper		Computer
Easy	50	=	50
Hard	70	=	70

both simple effects are nulls

There is no interaction of Task Presentation and Task Difficulty as they relate to performance. Performance is the same for computer and paper presentations (for both Easy and Hard tasks).

So, there are 5 basic patterns of results from a 2x2 Factorial

Three patterns that have an interaction:

1. = vs. < one null simple effect and one simple effect
2. < vs. > simple effects in opposite directions
3. < vs. < simple effects in same direction, but different sizes

Two patterns that have no interaction:

4. < vs. < simple effects of the same size in the same direction
5. = vs. = both null simple effects

Interpreting main effects … When there is an interaction, the pattern of the interaction may influence the interpretability (generality) of the description of the marginal means.

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	80	<	90
Hard	40	<	70
	60	<	80

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation.

Notice: that the pattern of the main effect is consistent with both the simple effect of Task Presentation for easy tasks and the simple effect of Task Presentation for hard tasks.

Another example …

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	90	=	90
Hard	40	<	70
	65	<	80

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation. However, this pattern is descriptive for hard tasks, but not for easy tasks, for which there was no simple effect of Task Presentation.

Yet another example …

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	80	>	60
Hard	20	<	70
	50	<	65

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation. However, this pattern is descriptive for hard tasks, but not for easy tasks, for which performance was better using paper presentations than using computer presentation.

"Null" main effects can also be misleading….

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	90	>	70
Hard	40	<	60
	65	=	65

There is no main effect for Task Presentation, overall performance was equivalent using computer presentation and using paper presentation. However, this pattern is descriptive for neither hard tasks, for which computer presentations worked better than paper, nor for easy tasks, for which performance was better using paper presentations than using computer presentation.

Another way that Main effects can be "meaningless"…

Sex	Age: 5	Age: 25	
Female	90	70	80
Male	80	60	70
	85	65	

There is no interaction, so the main effects are "unconditional".

But are they "meaningful" ???

Consider the Sex ME – those marginal means are aggregated across 5 & 25 year olds. Who are represented – 15 year olds? Not unless there is a linear relationship between age and the DV, which we've certainly not tested for !!!

Consider the Age ME – those marginal means are aggregated across males and females. Who is the average of females and males?

Main effects often don't represent any existing population. So, ME patterns are most useful if they describe SE patterns !!!

Remember the **5** basic patterns of results from a 2x2 Factorial ?

Interaction -- simple effects of different size and/or direction	1. = vs. <	one null simple effect and one simple effect	Misleading main effects
	2. < vs. >	simple effects in opposite directions	
	3. < vs. <	simple effects in same direction, but different sizes	Descriptive main effects
No Interaction -- simple effects are null or same size	4. < vs. <	simple effects of the same size in the same direction	
	5. = vs. =	both null simple effects	

Choosing Among Tables, Line Graphs and Bar Graphs

Tables

- Provides more detail (exact means and standard deviations)
- Easier to see main effects (can include marginal means)
- Harder to see the interaction

Line Graphs

- Easier to see interaction pattern (than tables)
- Harder to see main effects (than tables)
- "Formally" limited to using when quantitative IV on X axis

Bar Graphs

- Interactions -- easier than tables, not as easy as line graphs
- Mains -- harder to see than tables

Note: Any of these can include std, or SEM "whiskers"

About the causal interpretation of effects of a factorial design…

Start by assessing the causal interpretability of each main effect

In order to causally interpret an interaction, you must be able to casually interpret BOTH main effects.

Study of Age and Gender	no casually interpretable effects (main effects nor interaction)
Study of Age and Type of Toy (RA + Manip)	only casually interpretable effect would be the main effect of Type of Toy (not the main effect of Age, nor the interaction).
Study Type of Toy (RA + Manip) and Playing Situation (RA + manip)	all effects are causally interpreted (both main effects and the interaction).

About the causal interpretation of the effects of a factorial design…

In order to causally interpret an interaction, we must be able to causally interpret the difference between causally interpretable simple effects.

Huh ?!?!?

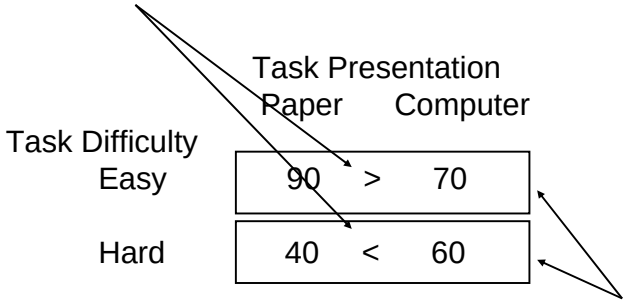
An interaction is the description of how the simple effects of one variable are different at different levels of another variable.

So, for that description to be causal, we must be able to causally interpret the simple effects…

… and to casually interpret the difference between the simple effects.

Example of a causally interpretable interaction …

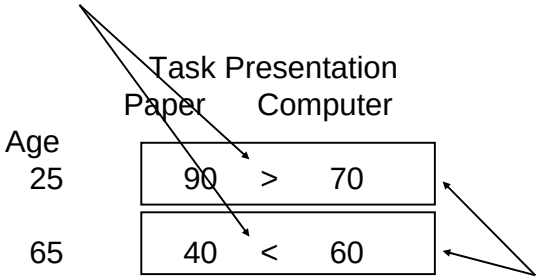
We can RA & Manip which Presentation someone gets, so we can causally interpret each simple effect (and the main effect) of Task Presentation. In other words – we know why these cell means are different, they ***were from*** different Task Presentations



We can RA & Manip which Task someone gets, so we can causally interpret the difference between the SEs of Task Presentation for Easy and for Hard Tasks (and the main effect). In other words – we know why these simple effects were different, they ***were from*** different Task Difficulties

Example of an interaction that is not causally interpretable …

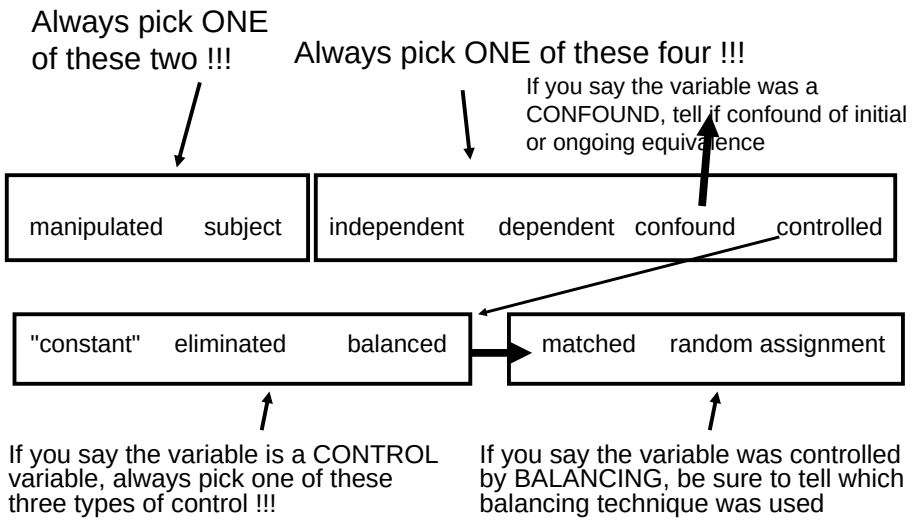
We can RA & Manip which Presentation someone gets, so we can causally interpret each simple effect (and the main effect) of Task Presentation. In other words – we know why these cell means are different, they ***were from*** different Task Presentations



We can not RA & Manip someone's Age, so we can not causally interpret the difference between the SEs of Task Presentation for the different Age groups In other words – we don't know why these simple effects were different, they ***could be because of age, or because of anything that is confounded with age.***

Explicating the role of variables in Factorial Designs …

Just like before… But now we must consider the role of each variable ***for each effect (2 mains & interaction)***



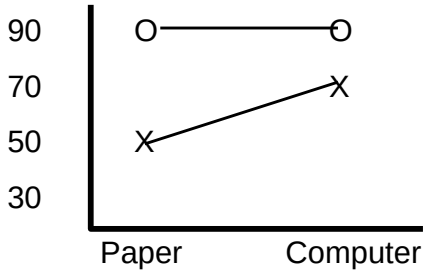
Inspecting a line graph …

Notice how the data in the table is displayed in the graph!

"Different differences" and "Differential Simple Effects" both translate into NONPARALLEL LINES in a figure.

Performance

Key for Task Difficulty
O = Easy X = Hard

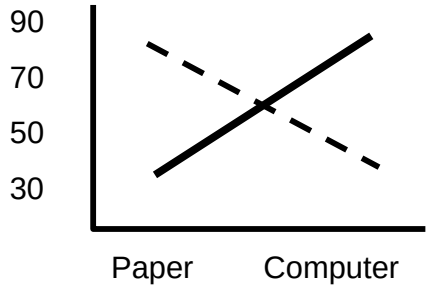


	P	C
Easy	90	90
Hard	50	70

How ***not*** to Inspect a line drawing to determine if there is an interaction…

This is a "cross-over" interaction -- it certainly IS an interaction but it IS NOT the only kind !!

Performance



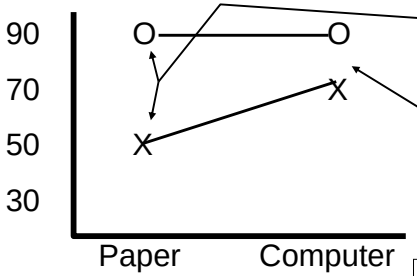
Key for Task Difficulty

Easy Hard

Inspecting a line graph to determine simple effects & interaction…

Performance

Simple Effects of Task Difficulty



SE Task Diff for Paper Pres.

90 vs. 50 SE = 40

SE Task Diff for Computer Pres.

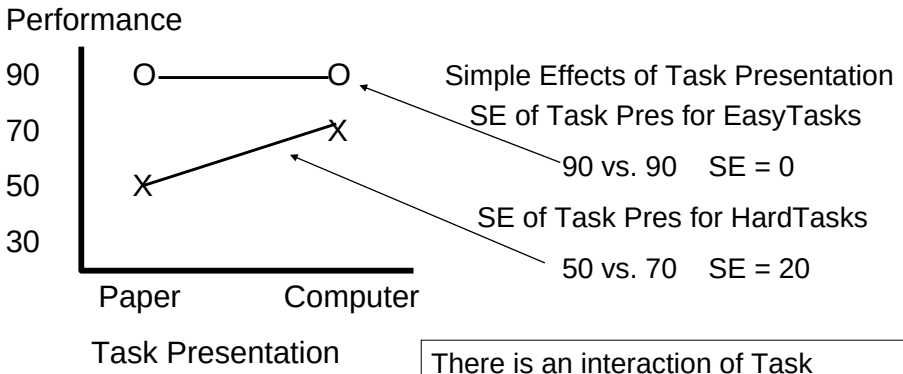
90 vs. 70 SE = 20

There is an interaction of Task Difficulty and Task Presentation as they relate to performance. Easy tasks are consistently performed better than hard tasks, however this effect is larger for paper presentations than for computer presentations.

Key for Task Difficulty

O = Easy X = Hard

Inspecting a line graph to determine simple effects & interaction…



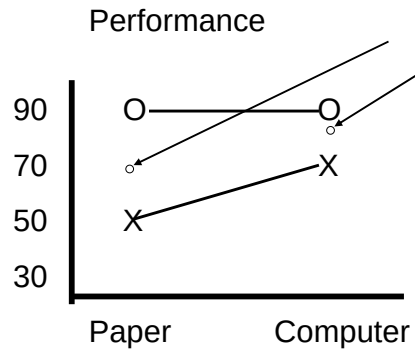
Key for Task Difficulty

O = Easy

X = Hard

There is an interaction of Task Difficulty and Task Presentation as they relate to performance. There is no effect of presentation for easy tasks, however for hard tasks computer presentations led to higher scores than did paper presentations.

Inspecting a line graph to determine if there are main effects…



This main effect is potentially misleading ...

Paper < Computer for hard tasks

but...

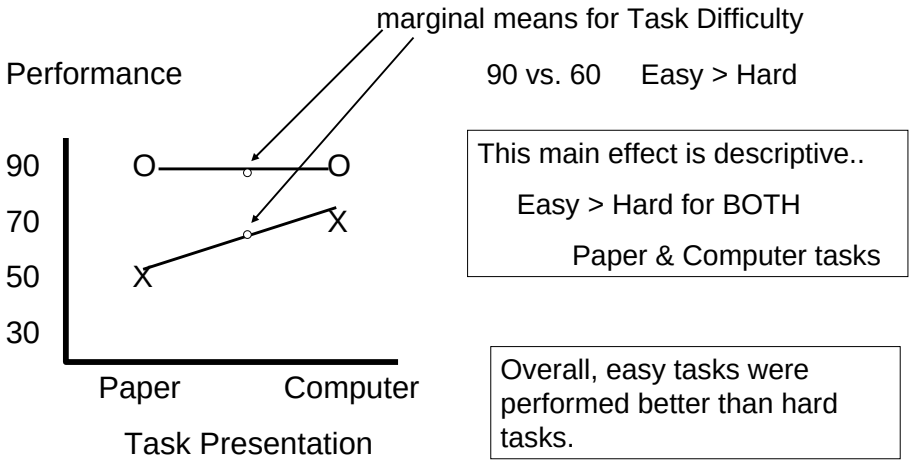
Paper = Computer for easy tasks

Key for Task Difficulty

O = Easy X = Hard

Overall, there was better performance on computer than paper tasks. However, this was not descriptive for easy tasks.

Inspecting a line graph to determine if there are main effects…



This main effect is descriptive..

Easy > Hard for BOTH

Paper & Computer tasks

Overall, easy tasks were performed better than hard tasks.

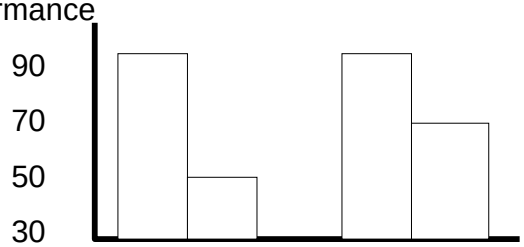
Key for Task Difficulty

O = Easy X = Hard

Inspecting a Bar Graph …

"Different differences" and "Differential Simple Effects" both translate into "different height differences" in a bar graph.

Performance

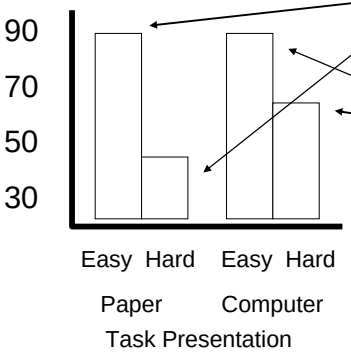


	P	C
Easy	90	90
Hard	50	70

Inspecting a Bar Graph to determine simple effects & interaction…
"Different differences" and "Differential Simple Effects" both translate into "different height differences" in a bar graph.

Performance

Simple Effects of Task Difficulty



SE Task Diff for Paper Pres.

90 vs. 50 SE = 40

SE Task Diff for Computer Pres.

90 vs. 70 SE = 20

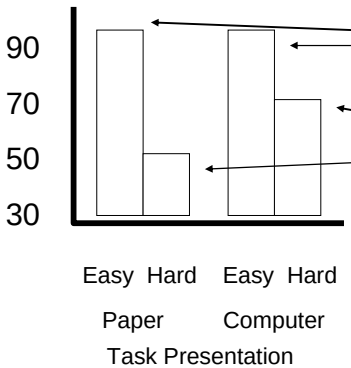
There is an interaction of Task Difficulty and Task Presentation as they relate to performance. Easy tasks are consistently performed better than hard tasks, however this effect is larger for paper presentations than for computer presentations.

Inspecting a Bar Graph to determine simple effects & interaction…

"Different differences" and "Differential Simple Effects" both translate into "different height differences" in a bar graph.

Performance

Simple Effects of Task Presentation



SE of Task Pres for EasyTasks

90 vs. 90 SE = 0

SE of Task Pres for Hard Tasks

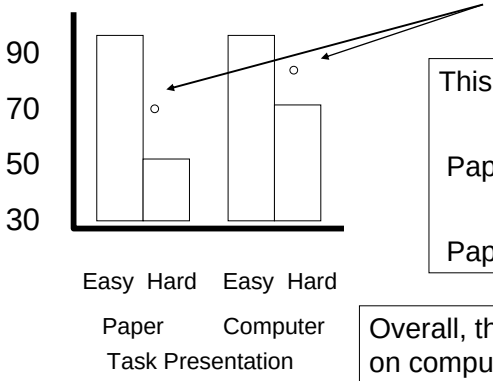
50 vs. 70 SE = 20

There is an interaction of Task Difficulty and Task Presentation as they relate to performance. There is no effect of presentation for easy tasks, however for hard tasks computer presentations led to higher scores than did paper presentations.

Inspecting a Bar graph to determine if there are main effects…

"Different differences" and "Differential Simple Effects" both translate into "different height differences" in a bar graph.

Performance



marginal means for Task Presentation
70 vs. 80 Paper < Computer

This main effect is potentially misleading ...

Paper < Computer for only for hard tasks

Paper = Computer for easy tasks

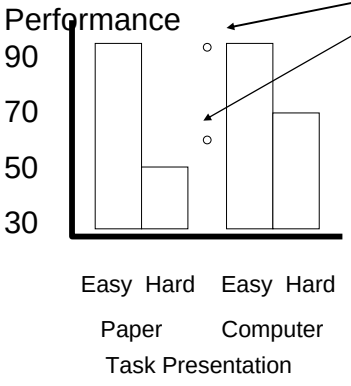
Overall, there was better performance on computer than paper tasks. However, this was not descriptive for easy tasks.

Inspecting a Bar graph to determine if there are main effects…

"Different differences" and "Differential Simple Effects" both translate into "different height differences" in a bar graph.

marginal means for Task Difficulty
90 vs. 60 Easy > Hard

Performance



This main effect is descriptive..

Easy > Hard for BOTH

Paper & Computer tasks

Overall, easy tasks were performed better than hard tasks.