Meta-Analyses: Combining, Comparing & Modeling ESs

- inverse variance weight
- weighted mean ES – where it starts…
  - fixed v. random effect models
  - fixed effects ES mean, tests & CIs
- heterogeneity analyses
- single-variable fixed effects comparison model – "Q"
- modeling study attributes related to ES
  - fixed effects modeling
  - random effects modeling

The Inverse Variance Weight

- An ES based on 400 participants is assumed to be a "better" estimate of the population ES than one based on 50 participants.
- So, ESs from larger studies should "count for more" than ESs from smaller studies!
- Original idea was to weight each ES by its sample size.
- Hedges suggested an alternative…
  - we want to increase the precision of our ES estimates
  - he showed that weighting ESs by their inverse variance minimizes the variance of their sum (& mean), and so, minimizes the Standard Error of Estimate (SE)
  - the resulting smaller Standard Error leads to narrower CIs and more powerful significance tests!!!
  - The optimal weight is 1 / SE^2

Calculating Standard Error & Inverse Variance Weights for Different Effect Sizes

d***

$$se = \sqrt{\frac{n_1 + n_2}{n_1 n_2} + \frac{\overline{ES}_{sm}}{2(n_1 + n_2)}}$$

$$w = \frac{1}{se^2}$$

r***

$$se = \sqrt{\frac{1}{n-3}}$$

$$w = n - 3$$

Odds Ratio***

$$se = \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$

$$w = \frac{1}{se^2}$$

*** Note: Applied to ESs that have been transformed to normal distn

Weighted Mean Effect Size

The most basic "meta analysis" is to find the average ES of the studies representing the population of studies of "the effect".

The formula is pretty simple – the sum of the weighted ESs, divided by the sum of the weightings.

$$\overline{ES} = \frac{\sum (w \times ES)}{\sum w}$$

But much has happened to get to here!

- select & obtain studies to include in meta analyis
- code study for important attributes
- extract d, d_{gain}, r, OR
- ND transformation of d, r, or OR
- perhaps adjust for unreliability, range restriction or outliers
- **Note: we're about to assume there is a single population of studies represented & that all have the same effect size, except for sampling error !!!!!**

Weighted Mean Effect Size

One more thing…

Fixed Effects vs. Random Effects Meta Analysis

Alternative ways of computing and testing the mean effect sizes.

Which you use depends on….

How you conceptualize the source(s) of variation among the study effect sizes – why don't all the studies have the same effect size???

And leads to …
How you will compute the estimate and the error of estimate.

Which influences…

The statistical results you get!

Fixed Effect Models

•Assume each study in the meta-analysis used the same (fixed) operationalizations of the design conditions & same external validity elements (population, setting, task/stimulus)

(Some say they also assume that the IV in each study is manipulated (fixed), so the IV in every study is identical.)

•Based on this, the studies in the meta analysis are assumed to be drawn from a population of studies that all have the same effect size, except for sampling error

•So, the sampling error is inversely related to the size of the sample

- which is why the effect size of each study is weighted by the inverse variance weight (which is computed from sample size)

$$se = \sqrt{\frac{1}{n-3}} \qquad w_i = \frac{1}{se_i^2}$$

Random Effect Models

•Assume different studies in the meta-analysis used different operationalizations of the design conditions, and/or different external validity elements (population, setting, task/stimulus)

•Based on this, studies in the meta analysis are assumed to be drawn from a population of studies that have different effect sizes for two reasons:

- Sampling variability
- "Real" effect size differences between studies caused by the differences in operationalizations and external validity elements

•So, the sampling error is inversely related to the size of the sample and directly related to the variability across the population of studies

- Compute the inverse weight differently

$$se = \sqrt{\frac{1}{n-3}} \qquad \hat{v}_\theta = \frac{Q_T - k - 1}{\sum w - \left(\frac{\sum w^2}{\sum w}\right)} \qquad w_i = \frac{1}{se_i^2 + \hat{v}_\theta}$$

How do you choose between Fixed & Random Effect Models ???

•The assumptions of the Fixed Effect model are less likely to be met than those of the Random Effect model. Even "replications" don't use all the same external validity elements and operationalizations…

•The sampling error estimate of the Random Effect model is likely to be larger, and, so, the resulting statistical tests less powerful than for the Fixed Effect model

•It is possible to test to see if the amount of variability (heterogeneity) among a set of effect sizes is larger than would be expected if all the effect sizes came from the same population. Rejecting the null is seen by some as evidence that a Random Effect model should be used.

It is very common advice to compute mean effect sizes using both approaches, and to report both sets of results!!!

Computing Fixed Effects weighted mean ES

This example will use "r".

Step 1

There is a row or case for each effect size.

The study/analysis each effect size was taken from is noted.

The raw effect size "r" and sample size (n) is given for each of the effect sizes being analyzed.

	A	B	C	D
1	Study	Class	**r**	**n**
2	1	math	0.48	30
3	2a	science	-0.16	18
4	2b	science	-0.13	33
5	3	science	-0.2	19
6	4	math	0.42	25
7	5a	math	0.45	14
8	5b	math	0.33	22
9	5c	math	0.45	18
10	6	science	-0.11	17
11	7	math	0.49	17
12	8	science	-0.1	17

Step 2
Use Fisher's Z transform to normalize each "r"

1. Label the column
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)
4. Copy that cell into other cells in that column

All further computations will use ES(Zr)

E2 *fx* = FISHER(C2)

	A	B	C	D	E
1	Study	Class	r	n	ES (Zr)
2	1	math	0.48	30	0.5230
3	2a	science	-0.16	18	-0.1614
4	2b	science	-0.13	33	-0.1307
5	3	science	-0.2	19	-0.2027
6	4	math	0.42	25	0.4477
7	5a	math	0.45	14	0.4847
8	5b	math	0.33	22	0.3428
9	5c	math	0.45	18	0.4847
10	6	science	-0.11	17	-0.1104
11	7	math	0.49	17	0.5361
12	8	science	-0.1	17	-0.1003

Formula is

FISHER("r" cellref)

Step 3
Compute inverse variance weight

1. Label the column
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)
4. Copy that cell into other cells in that column
5. Also compute sum of ES

F2 *fx* =D2-3

A	B	C	D	E	F
Study	Class	r	n	ES (Zr)	w
1	math	0.48	30	0.5230	27
2a	science	-0.16	18	-0.1614	15
2b	science	-0.13	33	-0.1307	30
3	science	-0.2	19	-0.2027	16
4	math	0.42	25	0.4477	22
5a	math	0.45	14	0.4847	11
5b	math	0.33	22	0.3428	19
5c	math	0.45	18	0.4847	15
6	science	-0.11	17	-0.1104	14
7	math	0.49	17	0.5361	14
8	science	-0.1	17	-0.1003	14

Formula is "n" cellref - 3

Rem: the inverse variance weight (w) is computed differently for different types of ES

Step 4
Compute weighted ES

1. Label the column
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)
4. Copy that cell into other cells in that column

G2 *fx* =F2*E2

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047

Formula is

"ES (Zr)" cellref * "w" cellref

Step 5
Get sums of weights and weighted ES

1. Add the “Totals” label
2. Highlight cells containing “w” values
3. Click the “Σ”
4. Sum of those cells will appear below last cell
5. Repeat to get sum of weighted ES (shown)

G13 | *fx* =SUM(G2:G12)

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	totals					197	38.0529

Step 6
Compute weighted mean ES

1. Add the label
2. Highlight a cell
3. Type “=“ and the formula (will appear in the *fx* bar above the cells)

G15 | *fx* =G13/F13

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	totals					197	38.0529
14							
15					weighted Mean ES		0.1932

The formula is

“sum weightedES” cellref

“sum weights” cellref

Computing weighted mean r

Step 7
Transform mean ES → r

1. Add the label
2. Highlight a cell
3. Type “=“ and the formula (will appear in the *fx* bar above the cells)

G22 | *fx* = FISHERINV(G15)

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	totals					197	38.0529
14							
15					weighted Mean ES		0.1932
16							
17							
18							
19							
20							
21							
22					Meta-analytic r		0.1908

The formula is

FISHERINV(“meanES” cellref)

Ta Da !!!!

Z-test of mean ES (also test of r)

Step 1
Compute Standard Error of mean ES

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

G16 | *fx* = SQRT(1 / F13)

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	totals					197	38.0529
14							
15					weighted Mean ES		0.1932
16					SE Mean ES		0.0712

The formula is

SQRT(1 / "sum of weights" cellref)

Z-test of mean ES (also test of r)

Step 2
Compute Z

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

G18 | *fx* = G15/G16

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	totals					197	38.0529
14							
15					weighted Mean ES		0.1932
16					SE Mean ES		0.0712
17							
18					Z-test of H0: ES = 0		2.7112

The formula is

"weighted Mean ES cellref" / "SE mean ES cellref"

Z-test of mean ES (also test of r)

Step 3
Compute p-value

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

I18 | *fx* = NORMDIST(0,ABS(G18),1,TRUE) * 2

	A	B	C	D	E	F	G	H	I
1	Study	Class	r	n	ES (Zr)	w	w*ES	w*ES^2	
2	1	math	0.48	30	0.5230	27	14.1206	7.384839	
3	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685	
4	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787	
5	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608	
6	4	math	0.42	25	0.4477	22	9.8492	4.409419	
7	5a	math	0.45	14	0.4847	11	5.3317	2.584278	
8	5b	math	0.33	22	0.3428	19	6.5137	2.233093	
9	5c	math	0.45	18	0.4847	15	7.2705	3.524015	
10	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779	
11	7	math	0.49	17	0.5361	14	7.5048	4.02305	
12	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941	
13	totals					197	38.0529	26.03149	
14									
15					weighted Mean ES		0.1932		
16					SE Mean ES		0.0712		
17									
18					Z-test of H0: ES = 0		2.7112	p=	0.006705

The formula is

"= NORMDIST(0,ABS(G18),1,TRUE) * 2"

Ta Da !!!!

CIs

Step 1
Compute CI values for ES

1. Add the labels
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

G19 | *fx* = G15 - (1.96*G16)

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	**totals**					197	38.0529
14							
15					**weighted Mean ES**		**0.1932**
16					**SE Mean ES**		**0.0712**
17							
18					Z-test of H0: ES = 0		2.7112
19					95% CI	lower bound	0.0535
20						upper bound	0.3328
21							
22					Meta-analytic r		0.1908

The formulas are

Lower "wtdMean ES" cellref – (1.96 * "SE Mean ES" cellref)
Upper "wtdMean ES" cellref + (1.96 * "SE Mean ES" cellref)

CIs

Step 2
Convert ES bounds
→ r bounds

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

G23 | *fx* = FISHERINV(G19)

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	**totals**					197	38.0529
14							
15					**weighted Mean ES**		**0.1932**
16					**SE Mean ES**		**0.0712**
17							
18					Z-test of H0: ES = 0		2.7112
19					95% CI	lower bound	0.0535
20						upper bound	0.3328
21							
22					Meta-analytic r		0.1908
23					95% CI	lower bound	0.0535
24						upper bound	0.3210

The formula for each is

FISHERINV("CI boundary" cellref)

Ta Da !!!!

Here are the formulas we've used…

Mean ES

$$\overline{ES} = \frac{\sum (w \times ES)}{\sum w}$$

SE of the Mean ES

$$se_{\overline{ES}} = \sqrt{\frac{1}{\sum w}}$$

Z-test for the Mean ES

$$Z = \frac{\overline{ES}}{se_{\overline{ES}}}$$

95% Confidence Interval

$$Upper = \overline{ES} + 1.96(se_{\overline{ES}})$$

$$Lower = \overline{ES} - 1.96(se_{\overline{ES}})$$

What about computing a Random Effect weighted mean ES??

It is possible to compute a “w” value that takes into account both the random sampling variability among the studies and the systematic sampling variablity.

Then you would redo the analyses using this “w” value – and that would be a Random Effect weighted mean ES!

Doing either with a large set of effect sizes, using XLS, is somewhat tedious, and it is easy to make an error that is very hard to find.

Instead, find the demo of how to use the SPSS macros written by David Wilson.

When we compute the average effect sizes, with significance tests, Cis, etc. -- we **assume there is a single population of studies represtented & that all have the same effect size, except for sampling error !!!!!**

The alternative hypothesis is that there are systematic differences among effect sizes of the studies – **these differences are related to (caused by) measurement, procedural and statistical analysis differences among the studies!!!**

Measurement

- operationalizations of IV manipulations/measures & DV measures, reliability & validity,

Procedural

- sampling, assignment, tasks & stimuli , G/WG designs, exp/nonexp designs, operationalizations of controls

Statistical analysis

- bivariate v multivariate analyses, statistical control

Suggested Data to Code Along with the Effect Size

1. A label or ID so you can backtrack to the exact analysis from the exact study – you will be backtracking!!!
2. Sample size for each group *
3. Sample attributes (mean age, proportion female, etc.) #
4. DV construct & specific operationalization / measure #
5. Point in time (after/during TX) when DV was measured #
6. Reliability & validity of DV measure *
7. Standard deviation of DV measure *
8. Type of statistical test used *#
9. Between group or within-group comparison / design #
10. True, quasi-, or non-experimental design #
11. Details about IV manipulation or measurement #
12. External validity elements (pop, setting, task/stimulus) #
13. “Quality” of the study #
    - better yet → data about attributes used to eval quality!!!

We can test if there are effect size differences associated with any of these differences among studies !!!

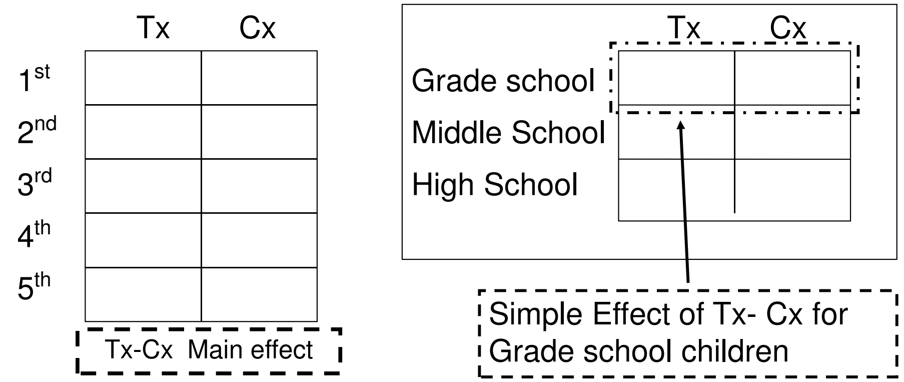
Remember that one goal of meta-analyses is to help us decide how to design and conduct future research. So, knowing what measurement, design, and statistical choices influence resulting effect sizes can be very helpful!

This also relates back to **External Validity** – does the selection of population, setting, task/stimulus & societal temporal "matter" or do basic finding generalize across these?

This also related to **Internal Validity** – does the selection of research design, assignment procedures, and control procedures "matter" or do basic finding generalize across these?

Does it matter which effect size you use – or are they generalizable???

This looks at population differences, but any "2nd variable" from a factorial design or multiple regression/ANCOVA might influence the resulting effect size !!!



We can test for homogeneity vs. heterogeneity among the effect sizes in our meta-analysis.

The "Q test" has a formulas much like a Sum of Squares, and is distributed as a X^2, so it provides a significance test of the Null Hypothesis that the heterogeneity among the effect sizes is no more than would be expected by chance,

We already have much of this computed, just one more step…

$$Q = \sum(w \times ES^2) - \frac{\left[\sum(w \times ES)\right]^2}{\sum w}$$

Please note: There is disagreement about the use of this statistical test, especially about whether it is a necessary pre-test before examining design features that may be related to effect sizes.

Be sure you know the opinion of "your kind" !!!

Computing Q

Step 1

You'll start with the w & s*ES values you computed as part of the mean effect size calculations.

G13 *fx* =SUM(G2:G12)

	A	B	C	D	E	F	G
1	Study	Class	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	2a	science	-0.16	18	-0.1614	15	-2.4208
4	2b	science	-0.13	33	-0.1307	30	-3.9222
5	3	science	-0.2	19	-0.2027	16	-3.2437
6	4	math	0.42	25	0.4477	22	9.8492
7	5a	math	0.45	14	0.4847	11	5.3317
8	5b	math	0.33	22	0.3428	19	6.5137
9	5c	math	0.45	18	0.4847	15	7.2705
10	6	science	-0.11	17	-0.1104	14	-1.5463
11	7	math	0.49	17	0.5361	14	7.5048
12	8	science	-0.1	17	-0.1003	14	-1.4047
13	totals					197	38.0529

Computing Q

Step 2

Compute weighted ES^2 for each study

1. Label the column
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)
4. Copy that cell into other cells in that column

H2 *fx* = F2*E2^2

	A	B	C	D	E	F	G	H
1	Study	Class	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
4	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
5	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
6	4	math	0.42	25	0.4477	22	9.8492	4.409419
7	5a	math	0.45	14	0.4847	11	5.3317	2.584278
8	5b	math	0.33	22	0.3428	19	6.5137	2.233093
9	5c	math	0.45	18	0.4847	15	7.2705	3.524015
10	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
11	7	math	0.49	17	0.5361	14	7.5048	4.02305
12	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
13	totals					197	38.0529	

Formula is

"w" cellref * "ES (Zr)" cellref ^{2}

Computing Q

Step 3

Compute sum of weighted ES^2

1. Highlight cells containing "w*ES^2" values
2. Click the "Σ"
3. Sum of those cells will appear below last cell

H13 *fx* =SUM(H2:H12)

	A	B	C	D	E	F	G	H
1	Study	Class	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
4	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
5	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
6	4	math	0.42	25	0.4477	22	9.8492	4.409419
7	5a	math	0.45	14	0.4847	11	5.3317	2.584278
8	5b	math	0.33	22	0.3428	19	6.5137	2.233093
9	5c	math	0.45	18	0.4847	15	7.2705	3.524015
10	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
11	7	math	0.49	17	0.5361	14	7.5048	4.02305
12	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
13	totals					197	38.0529	26.03149

Computing Q

Step 4

Compute Q

1. Add the label
2. Highlight a cell
3. Type “=“ and the formula (will appear in the *fx* bar above the cells)

F27 | *fx* = H13 - (G13^2)/F13

	A	B	C	D	E	F	G	H
1	Study	Class	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
4	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
5	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
6	4	math	0.42	25	0.4477	22	9.8492	4.409419
7	5a	math	0.45	14	0.4847	11	5.3317	2.584278
8	5b	math	0.33	22	0.3428	19	6.5137	2.233093
9	5c	math	0.45	18	0.4847	15	7.2705	3.524015
10	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
11	7	math	0.49	17	0.5361	14	7.5048	4.02305
12	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
13	totals					197	38.0529	26.03149
14								
15					weighted Mean ES		0.1932	
16					SE Mean ES		0.0712	
17								
18					Z-test of H0: ES = 0		2.7112	
19					95% CI lower bound		0.0535	
20					upper bound		0.3328	
21								
22					Meta-analytic r		0.1908	
23					95% CI lower bound		0.0535	
24					upper bound		0.3210	
25								
26								
27					Q =	18.68111		

The formula is

$$\text{“sum w*ES\^{}2” cellref} - \frac{\text{“sum weightedES” cellref}^2}{\text{“sum weights” cellref}}$$

Computing Q

Step 5

Add df & p

1. Add the labels
2. Add in df = #cases - 1
3. Calculate p-value using Chi-square p-value function

F29 | *fx* = CHIDIST(F27,F28)

	A	B	C	D	E	F	G	H
1	Study	Class	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
4	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
5	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
6	4	math	0.42	25	0.4477	22	9.8492	4.409419
7	5a	math	0.45	14	0.4847	11	5.3317	2.584278
8	5b	math	0.33	22	0.3428	19	6.5137	2.233093
9	5c	math	0.45	18	0.4847	15	7.2705	3.524015
10	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
11	7	math	0.49	17	0.5361	14	7.5048	4.02305
12	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
13	totals					197	38.0529	26.03149
14								
15					weighted Mean ES		0.1932	
16					SE Mean ES		0.0712	
17								
18					Z-test of H0: ES = 0		2.7112	
19					95% CI lower bound		0.0535	
20					upper bound		0.3328	
21								
22					Meta-analytic r		0.1908	
23					95% CI lower bound		0.0535	
24					upper bound		0.3210	
25								
26								
27					Q =	18.68111		
28					df =	10		
29					p =	0.044505		

Formula is CHIDIST(“Q” cellref , “df” cellref)

Interpreting the Q-test

p > .05

- effect size heterogeneity is no more than would be expected by chance
- Study attributes can not be systematically related to effect sizes, since there’s no systematic variation among effect sizes

p < .05

- Effect size heterogeneity is more than would be expected by chance
- Study attributes may be systematically related to effect sizes

Keep in mind that not everybody “likes” this test! Why???

- An alternative suggestion is to test theoretically meaningful potential sources of effect size variation without first testing for systematic heterogeneity.
- It is possible to retain the null and still find significant relationships between study attributes and effect sizes!!

Modeling Attributes Related to Effect Sizes

There are different approaches to testing for relationships between study attributes and effect sizes:

Fixed & Random Effects Q-test

These are designed to test whether groups of studies that are qualitatively different on some study attribute have different effect sizes

Fixed & Random Effects Meta Regression

These are designed to examine possible multivariate differences among the set of studies in the meta-analysis, using quantitative, binary, or coded study attribute variables.

Fixed Effects Q-test -- Comparing Subsets of Studies

Step 1

Sort the studies/cases into the subgroups

Different studies in this meta-analysis were conducted by teachers of different subjects – Math & Science. Were there different effect sizes from these two classes ??

N18 fx

	A	B	C	D	E	F	G
1	Study	Topic	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	4	math	0.42	25	0.4477	22	9.8492
4	5a	math	0.45	14	0.4847	11	5.3317
5	5b	math	0.33	22	0.3428	19	6.5137
6	5c	math	0.45	18	0.4847	15	7.2705
7	7	math	0.49	17	0.5361	14	7.5048
8							
9							
10							
11							
12							
13							
14							
15							
16	2a	science	-0.16	18	-0.1614	15	-2.4208
17	2b	science	-0.13	33	-0.1307	30	-3.9222
18	3	science	-0.2	19	-0.2027	16	-3.2437
19	6	science	-0.11	17	-0.1104	14	-1.5463
20	8	science	-0.1	17	-0.1003	14	-1.4047

All the values you computed earlier for each study are still good !

Computing Fixed Effects Q-test

Step 2

Compute weighted ES^2 for each study

1. Label the column
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)
4. Copy that cell into other cells in that column

H2 fx = F2*E2^2

	A	B	C	D	E	F	G	H
1	Study	Topic	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	4	math	0.42	25	0.4477	22	9.8492	4.409419
4	5a	math	0.45	14	0.4847	11	5.3317	2.584278
5	5b	math	0.33	22	0.3428	19	6.5137	2.233093
6	5c	math	0.45	18	0.4847	15	7.2705	3.524015
7	7	math	0.49	17	0.5361	14	7.5048	4.02305
8								
9								
10								
11								
12								
13								
14								
15								
16	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
17	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
18	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
19	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
20	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941

Formula is

"w" cellref * "ES (Zr)" cellref ^{2}

Computing Fixed Effects Q-test

Step 3

Get sums of weights, weighted ES & weighted ES^2

1. Add the “Totals” label
2. Highlight cells containing “w” values
3. Click the “Σ”
4. Sum of those cells will appear below last cell
5. Repeat to get sum of each value for each group

H8 | f_x =SUM(H2:H7)

	A	B	C	D	E	F	G	H
1	Study	Topic	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	4	math	0.42	25	0.4477	22	9.8492	4.409419
4	5a	math	0.45	14	0.4847	11	5.3317	2.584278
5	5b	math	0.33	22	0.3428	19	6.5137	2.233093
6	5c	math	0.45	18	0.4847	15	7.2705	3.524015
7	7	math	0.49	17	0.5361	14	7.5048	4.02305
8		totals				108	50.5906	24.15869
9								
10								
11								
12								
13								
14								
15								
16	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
17	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
18	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
19	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
20	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
21		totals				89	-12.5377	1.8728
22								

Computing Q

Step 4

Compute Q_{within} for each group

1. Add the label
2. Highlight a cell
3. Type “=“ and the formula (will appear in the *fx* bar above the cells)

H12 | f_x = H8 - (G8^2)/F8

	A	B	C	D	E	F	G	H
1	Study	Topic	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	4	math	0.42	25	0.4477	22	9.8492	4.409419
4	5a	math	0.45	14	0.4847	11	5.3317	2.584278
5	5b	math	0.33	22	0.3428	19	6.5137	2.233093
6	5c	math	0.45	18	0.4847	15	7.2705	3.524015
7	7	math	0.49	17	0.5361	14	7.5048	4.02305
8		totals				108	50.5906	24.15869
9								
10								
11								
12						Q (math) =		0.460475
13								
14								
15								
16	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
17	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
18	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
19	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
20	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
21		totals				89	-12.5377	1.8728
22								
23								
24								
25						Q (science) =		0.106585

The formula is

$$\text{“sum w*ES\^{}2” cellref} - \frac{\text{“sum weightedES” cellref}^2}{\text{“sum weights” cellref}}$$

Computing Q

Step 5

Compute $Q_{between}$

1. Add the label
2. Highlight a cell
3. Type “=“ and the formula (will appear in the *fx* bar above the cells)

H27 | f_x =18.6811 - (H12+H25)

	A	B	C	D	E	F	G	H
1	Study	Topic	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	4	math	0.42	25	0.4477	22	9.8492	4.409419
4	5a	math	0.45	14	0.4847	11	5.3317	2.584278
5	5b	math	0.33	22	0.3428	19	6.5137	2.233093
6	5c	math	0.45	18	0.4847	15	7.2705	3.524015
7	7	math	0.49	17	0.5361	14	7.5048	4.02305
8		totals				108	50.5906	24.15869
9								
10								
11								
12						Q (math) =		0.460475
13								
14								
15								
16	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
17	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
18	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
19	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
20	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
21		totals				89	-12.5377	1.8728
22								
23								
24								
25						Q (science) =		0.106585
26								
27						Q (Class) =		18.11404

The formula is $Q - (Q_{w1} + Q_{w2})$

Computing Q

Step 6

Add df & p

1. Add the labels
2. Add in df = #cases - 2
3. Calculate p-value using Chi-square p-value function

H29 | fx = CHIDIST(H27,H28)

	A	B	C	D	E	F	G	H
1	Study	Topic	r	n	ES (Zr)	w	w*ES	w*ES^2
2	1	math	0.48	30	0.5230	27	14.1206	7.384839
3	4	math	0.42	25	0.4477	22	9.8492	4.409419
4	5a	math	0.45	14	0.4847	11	5.3317	2.584278
5	5b	math	0.33	22	0.3428	19	6.5137	2.233093
6	5c	math	0.45	18	0.4847	15	7.2705	3.524015
7	7	math	0.49	17	0.5361	14	7.5048	4.02305
8		totals				108	50.5906	24.15869
9								
10								
11								
12						Q (math) =		0.460475
13								
14								
15								
16	2a	science	-0.16	18	-0.1614	15	-2.4208	0.390685
17	2b	science	-0.13	33	-0.1307	30	-3.9222	0.512787
18	3	science	-0.2	19	-0.2027	16	-3.2437	0.657608
19	6	science	-0.11	17	-0.1104	14	-1.5463	0.170779
20	8	science	-0.1	17	-0.1003	14	-1.4047	0.140941
21		totals				89	-12.5377	1.8728
22								
23								
24								
25						Q (science) =		0.106585
26								
27						Q (Class) =		18.11404
28							df =	9
29							p =	0.033873

Formula is CHIDIST(“Q” cellref , “df” cellref)

Interpreting the Fixed Effects Q-test

p > .05

- This study attribute is not systematically related to effect sizes

p < .05

- This study attribute is not systematically related to effect sizes

If you have group differences, you’ll want to compute separate effect size aggregates and significance tests for each group.

Computing weighted mean ES for @ group

Step 1

Compute weighted mean ES

1. Add the label
2. Highlight a cell
3. Type “=“ and the formula (will appear in the *fx* bar above the cells)

D10 | fx =G8/F8

	A	B	C	D	E	F	G
1	Study	Topic	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	4	math	0.42	25	0.4477	22	9.8492
4	5a	math	0.45	14	0.4847	11	5.3317
5	5b	math	0.33	22	0.3428	19	6.5137
6	5c	math	0.45	18	0.4847	15	7.2705
7	7	math	0.49	17	0.5361	14	7.5048
8		totals				108	50.5906
9							
10	weighted Mean ES			0.468431			
11							
12							
13							
14							
15							
16	2a	science	-0.16	18	-0.1614	15	-2.4208
17	2b	science	-0.13	33	-0.1307	30	-3.9222
18	3	science	-0.2	19	-0.2027	16	-3.2437
19	6	science	-0.11	17	-0.1104	14	-1.5463
20	8	science	-0.1	17	-0.1003	14	-1.4047
21		totals				89	-12.5377
22							
23	weighted Mean ES			-0.14087			

The formula is

“sum weightedES” cellref

“sum weights” cellref

Computing weighted mean r for @ group

Step 2

Transform mean ES → r

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

G10 | *fx* = FISHERINV(D10)

	A	B	C	D	E	F	G
1	Study	Topic	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	4	math	0.42	25	0.4477	22	9.8492
4	5a	math	0.45	14	0.4847	11	5.3317
5	5b	math	0.33	22	0.3428	19	6.5137
6	5c	math	0.45	18	0.4847	15	7.2705
7	7	math	0.49	17	0.5361	14	7.5048
8		totals				108	50.5906
9							
10	weighted Mean ES			0.468431	Meta-analytic r		0.436931
11							
12							
13							
14							
15							
16	2a	science	-0.16	18	-0.1614	15	-2.4208
17	2b	science	-0.13	33	-0.1307	30	-3.9222
18	3	science	-0.2	19	-0.2027	16	-3.2437
19	6	science	-0.11	17	-0.1104	14	-1.5463
20	8	science	-0.1	17	-0.1003	14	-1.4047
21		totals				89	-12.5377
22							
23	weighted Mean ES			-0.14087	Meta-analytic r		-0.13995

The formula is

FISHERINV("meanES" cellref)

Ta Da !!!!

Z-tests of mean ES (also test of r)

Step 1

Compute Standard Error of mean ES

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

D11 | *fx* =SQRT(1/F8)

	A	B	C	D	E	F	G
1	Study	Topic	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	4	math	0.42	25	0.4477	22	9.8492
4	5a	math	0.45	14	0.4847	11	5.3317
5	5b	math	0.33	22	0.3428	19	6.5137
6	5c	math	0.45	18	0.4847	15	7.2705
7	7	math	0.49	17	0.5361	14	7.5048
8		totals				108	50.5906
9							
10	weighted Mean ES			0.468431	Meta-analytic r		0.436931
11	SE Mean ES			0.096225			
12							
13							
14							
15							
16	2a	science	-0.16	18	-0.1614	15	-2.4208
17	2b	science	-0.13	33	-0.1307	30	-3.9222
18	3	science	-0.2	19	-0.2027	16	-3.2437
19	6	science	-0.11	17	-0.1104	14	-1.5463
20	8	science	-0.1	17	-0.1003	14	-1.4047
21		totals				89	-12.5377
22							
23	weighted Mean ES			-0.14087	Meta-analytic r		-0.13995
24	SE Mean ES			0.106			

The formula is

SQRT(1 / "sum of weights" cellref)

Z-test of mean ES (also test of r)

Step 2

Compute Z

1. Add the label
2. Highlight a cell
3. Type "=" and the formula (will appear in the *fx* bar above the cells)

D13 | *fx* =D10/D11

	A	B	C	D	E	F	G
1	Study	Topic	r	n	ES (Zr)	w	w*ES
2	1	math	0.48	30	0.5230	27	14.1206
3	4	math	0.42	25	0.4477	22	9.8492
4	5a	math	0.45	14	0.4847	11	5.3317
5	5b	math	0.33	22	0.3428	19	6.5137
6	5c	math	0.45	18	0.4847	15	7.2705
7	7	math	0.49	17	0.5361	14	7.5048
8		totals				108	50.5906
9							
10	weighted Mean ES			0.468431	Meta-analytic r		0.436931
11	SE Mean ES			0.096225			
12							
13	Z-test of H0: ES = 0			4.868082			
14							
15							
16	2a	science	-0.16	18	-0.1614	15	-2.4208
17	2b	science	-0.13	33	-0.1307	30	-3.9222
18	3	science	-0.2	19	-0.2027	16	-3.2437
19	6	science	-0.11	17	-0.1104	14	-1.5463
20	8	science	-0.1	17	-0.1003	14	-1.4047
21		totals				89	-12.5377
22							
23	weighted Mean ES			-0.14087	Meta-analytic r		-0.13995
24	SE Mean ES			0.106			
25							
26	Z-test of H0: ES = 0			-1.32899			

Ta Da !!!!

The formula is

"weighted Mean ES cellref" / "SE mean ES cellref"

Random Effect Q-test -- Comparing Subsets of Studies

Just as there is the random effects version of the mean ES, there is ransom effects version of the Q-test,

Like with the mean ES computation, the difference is the way the error term is calculated – based on the assumption that the variability across studies included in the meta-analysis comes from 2 sources;

- Sampling variability
- “Real” effect size differences between studies caused by the differences in operationalizations and external validity elements

Take a look at the demo of how to do this analysis using the SPSS macros written by David Wilson.

Meta Regression

Far more interesting than the Q-test for comparing subgroups of studies is meta regression.

These analyses allow us to look at how multiple study attributes are related to effect size, and tell us the unique contribution of the different attributes to how those effects sizes vary.

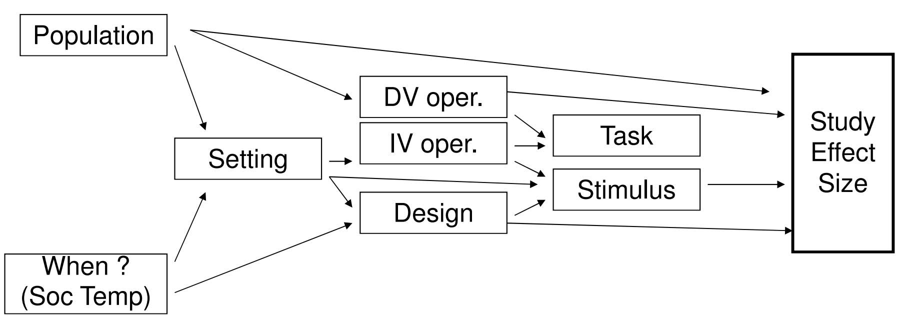
There are both “fixed effect” and “random effects” models.

Random effects meta regression models are more complicated, but have become increasingly popular because the assumptions of the model include the idea that differences in the effect sizes across studies are based on a combination of sampling variation and differences in how the studies are conducted (measurement, procedural & statistical analysis differences).

An example of random effects meta regression using Wilson’s SPSS macros is shown in the accompanying handout.

Meta Path Analyses

Meta analytic studies of what leads different studies to find different effect sizes can involve hundreds of studies, several study-difference variables, and sophisticated multivariate models!



The results of these studies help researchers:
- understand the rich research literature of an area of study
- decide the best ways to conduct future research studies!!!