Parametric & Nonparametric Univariate Statistics, Univariate Significance Tests & Tests of Association

- Statistics & models we will consider
- Univariate stats
- Univariate statistical tests
  - X^2 Tests for qualitative variables
- Parametric tests of Association
  - Pearson's correlation
- Nonparametric tests of Association
  - Spearman's rank order correlation (Rho)
  - Kendal's Tau

Statistics We Will Consider

DV →	Categorical	Parametric Interval/ND	Nonparametric Ordinal/~ND
univariate stats	mode, #cats	mean, std	median, IQR
univariate tests	gof X^2	1-grp t-test	1-grp Mdn test
association	X^2	Pearson's r	Spearman's r
2 bg	X^2	t- / F-test	M-W K-W Mdn
k bg	X^2	F-test	K-W Mdn
2wg	McNem Crn's	t- / F-test	Wil's Fried's
kwg	Crn's	F-test	Fried's

M-W -- Mann-Whitney U-Test Wil's -- Wilcoxin's Test Fried's -- Friedman's F-test
K-W -- Kruskal-Wallis Test
Mdn -- Median Test McNem -- McNemar's X^2 Crn's – Cochran's Test

Univariate Statistics for qualitative variables

Central Tendency – "best guess of next case's value"

- Mode -- the most common score(s)
- uni-, bi, multi-modal distributions are all possible

Variability – "index of accuracy of next guess"

- \# categories
- guessing whether or not the next person has a pet is more likely to be correct than guessing what kind of pet they have (including none)

Shape – symmetry & proportional distribution

- doesn't make sense for qualitative variables
- no prescribed value order

Parametric Univariate Statistics for ND/Int variables

Central Tendency – "best guess of next case's value"

- mean or arithmetic average → $M = \Sigma X / N$
- 1^{st} moment of the normal distribution formula
- since ND unimodal & symetrical → mode = mean = mdn

Variability – "index of accuracy of next guess"

- sum of squares → $SS = \Sigma(X – M)^2$
- variance → $s^2 = SS / (N-1)$
- standard deviation → $s = \sqrt{s^2}$
- std preferred because is on same scale as the mean
- 2^{nd} moment of the normal distribution formula
- average extent of deviation of each score from the mean

Parametric Univariate Statistics for ND/Int variables, cont.

Shape – "index of symmetry"

- skewness → $\frac{\Sigma (X - M)^3}{(N – 1) * s^3}$
- 3^{rd} moment of the normal distribution formula
- 0 = symmetrical, + = right-tailed, - = left-tailed
- can't be skewed & ND

Shape –"index of proportional distribution"

- kurtosis → M = ΣX / N $\qquad \frac{\Sigma (X - M)^4}{(N – 1) * s^4} - 3$
- 4th moment of the normal distribution formula
- 0 = prop dist as ND, + = leptokurtic, - = platakurtic

The four "moments" are all independent – all combos possible

- mean & std "are correct" as indices of central tendency & spread if skewness = 0 and kurtosic = 0

Nonparametric Univariate Statistics for ~ND/~Int variables

Central Tendency – "best guess of next case's value"

- **median** → middle-most value, 50^{th} percentile, 2^{nd} quartile

How to calculate the Mdn

1. Order data values — 11 13 16 18 18 21 22

2. Assign depth to each value, starting at each end

11	13	16	18	18	21	22
1	2	3	4	3	2	1

3. Calculate median depth $D_{mdn} = (N+1) / 2$ — $(7 + 1) / 2 = 4$

4. Median = value at D_{mdn} (or average of 2 values @ D_{mdn}, if odd number of values) — 18

Nonparametric Univariate Statistics for ~ND/~Int variables

Variability – "index of accuracy of next guess"

- Inter-quartile range (IQR)→ range of middle 50%, 3^{rd}-1^{st} quartile

How to calculate the IQR

	11 13 16 18 18 21 22
1. Order & assign depth to each value	1 2 3 4 3 2 1
2. Calculate median depth D_{Mdn} = (N+1) / 2	(7 + 1) / 2 = 4
3. Calculate quartile depth D_Q = (D_{Mdn} + 1) / 2	(4 + 1) / 2 = 2.5
4. 1^{st} Quartile value	Ave of 13 & 16 = 14.5
5. 3^{rd} Quartile value	Ave of 18 & 21 = 19.5
6. IQR – 3^{rd} - 1^{st} Q values	19.5 – 14.5 = 5

Univariate Parametric Statistical Tests for qualitative variables

Goodness-of-fit X^2 test

- Tests hypothesis about the distribution of category values of the population represented by the sample
- H0: is the hypothesized pop. distribution, based on either ...
  - theoretically hypothesized distribution
  - population distribution the sample is intended to represent
  - E.g., 65% ugrads & 35% grads or 30% Frosh, 45% Soph & 25% Juniors
- RH: & H0: often the same !
- binary and ordered category variables usually tested this way
- gof X^2 compares hypothesized distribution & sample dist.
- Retaining H0: -- sample dist. "equivalent to" population dist.
- Rejecting H0: -- sample dist. "is different from" population dist.

Data & formula for the gof X^2

Frequency of different class ranks in sample

Frosh	Soph	Junior
25	55	42

$$X^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Observed frequency – actual sample values (25, 55 & 42)

Expected frequency – based on a priori hypothesis

- however expressed (absolute or relative proportions, %s, etc)
- must be converted to expected frequencies

Example of a gof X^2

RH: "about ½ are sophomores and the rest are divided between frosh & juniors

Frosh	Soph	Junior
25	55	54

$$X^2 = \Sigma \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

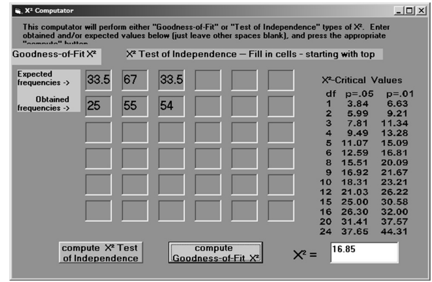
1. Obtain expected frequencies
- determine category proportions frosh .25 soph .5 junior .25
- determine category freq as proportion of total (N=134)
  - Frosh .25*122 = 33.5 Soph 67 Junior 33.5

2. Compute X^2
- $(25 - 33.5)^2/33.5 + (55-67)^2/67 + (54 - 33.5)^2/33.5 = 16.85$

3. Determine df & critical X^2
- df = k – 1 = 3 – 1 = 2
- $X^2_{2,.05} = 5.99$ $x^2_{2,.01} = 9.21$

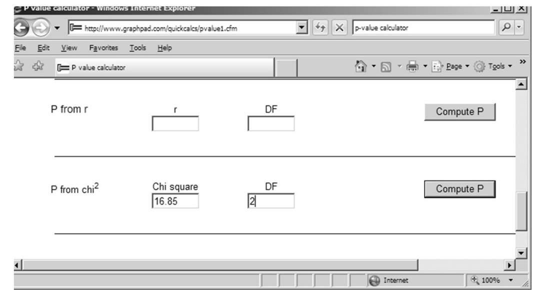
4. NHST & such
- $X^2 > X^2_{2,.01}$, so reject H0: at p = .01
- Looks like fewer Frosh – Soph & more Juniors than expected

Doing gof X^2 "by hand" – Computators & p-value calculators



The top 2 rows of the X^2 Computator will compute a gof X^2

If you want to know the p-value with greater precision, use one of the online p-value calculators



Univariate Parametric Statistical Tests for ND/Int

1-sample t-test

Tests hypothesis about the mean of the population represented by the sample (μ -- "mu")

- H0: value is the hypothesized pop. mean, based on either ...
  - theoretically hypothesized mean
  - population mean the sample is intended to represent
  - e.g., pop mean age = 19
- RH: & H0: often the same !
- 1-sample t-test compares hypothesized μ & x
- Retaining H0: -- sample mean "is equivalent to" population μ
- Rejecting H0: -- sample mean "is different from" population μ

Example of a 1-sample t-test

$$t = \frac{\overline{X} - \mu}{SEM} \qquad SEM = (s^2 / n)$$

The sample of 22 has a mean of 21.3 and std of 4.3

1. Determine the H0: μ value
   - We expect that the sample comes from a population with an average age of 19 $\mu = 19$
2. Compute SEM & t
   - $SEM = 4.3^2 / 22 = .84$
   - $t = (21.3 – 19) / .84 = 2.74$
3. Determine df & t-critical or p-value
   - df = N-1 = 22 – 1 = 21
   - Using t-table $t_{21,.05} = 2.08$ $t_{21,.01} = 2.83$
   - Using p-value calculator p = .0123
4. NHST & such
   - $t > t_{2,.05}$ but not t2,.05 so reject H0: at p = .05 or p = .0123
   - Looks like sample comes from population older than 19

Univariate Nonparametric Statistical Tests for ~ND/~In

1-sample median test

Tests hypothesis about the median of the population represented by the sample H0: value is the hypothesized pop. median, based on either ...

- theoretically hypothesized mean
- population mean the sample is intended to represent
- e.g., pop median age = 19

- RH: & H0: often the same !
- 1-sample median test compares hypothesized & sample mdns
- Retaining H0: -- sample mdn “is equivalent to” population mdn
- Rejecting H0: -- sample mdn “is different from” population mdn

Example of a 1-sample median test

age data → 11 12 13 13 14 16 17 17 18 18 18 20 20 21 22 22

1. Obtain obtained & expected frequencies
   - determine hypothesized median value → 19
   - sort cases in to above vs. below H0: median value
   - Expected freq for each cell = ½ of sample → 8
2. Compute X^2
   - $(11 – 8)^2/8 + (5 – 8)^2/8 = 2.25$

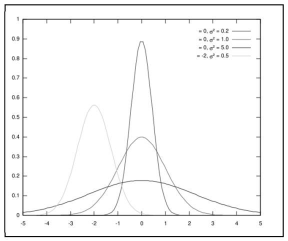
<19	>19
11	5

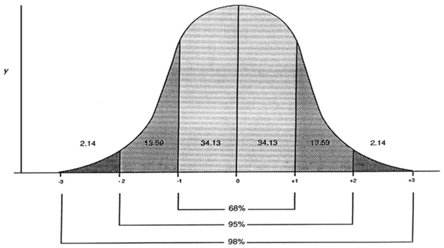
3. Determine df & X^2-critical or p-value
   - df = k-1 = 2 – 1 = 1
   - Using X^2-table $X^2_{1,.05} = 3.84$ $X^2_{1,.05} = 6.63$
   - Using p-value calculator p = .1336
4. NHST & such
   - X2 < X2 1, .05 & p > .05 so retain H0:
   - Looks like sample comes from population with median not different from 19

Tests of Univariate ND

One use of gof X^2 and related univariate tests is to determine if data are distributed as a specific distribution, most often ND.

No matter what mean and std, a ND is defined by symmetry & proportional distribution





Using this latter idea, we can use a gof X^2 to test if the frequencies in segments of the distribution have the right proportions

- here we might use a k=6 gof X2 with expected frequencies based on % of 2.14, 13.59, 34.13, 34.13, 13.59 & 2.14

Tests of Univariate ND

One use of t-tests is to determine if data are distributed as a specific distribution, most often ND.

ND have skewness = 0 and kurtosis = 0

Testing Skewness

t = skewness / SES

Standard Error of Skewness

$SES \approx \sqrt{(6 / N)}$

Testing Kurtosis

t = kurtosis / SEK

Standard Error of Kurtosis

$SEK \approx \sqrt{(24 / N)}$

Both of these are “more likely to find a significant divergence from ND, than that divergence is likely to distort the use of parametric statistics – especially with large N.”

Statistical Tests of Association w/ qualitative variables

Pearson’s X^2

$$X^2 = \sum \frac{(of - ef)^2}{ef}$$

Can be 2x2, 2xk or kxk – depending upon the number of categories of each qualitative variable

- H0: There is no pattern of relationship between the two qualitative variables.
- degrees of freedom df = (#colums - 1) * (#rows - 1)
- Range of values 0 to ∞
- Reject Ho: If $X^2_{obtained} > X^2_{critical}$

$$ef = \frac{\text{Row total} * \text{Column total}}{N}$$

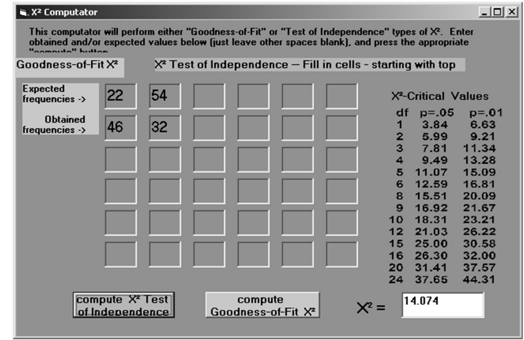
	Col 1	Col 2	
Row 1	22	54	76
Row 2	46	32	78
	68	86	154

The expected frequency for each cell is computed assuming that the H0: is true – that there is no relationship between the row and column variables.

If so, the frequency of each cell can be computed from the frequency of the associated rows & columns.

	Col 1	Col 2	
Row 1	(76*68)/154	(76*86)/154	76
Row 2	(78*68)/154	(78*86)/154	78
	68	86	154

$$X^2 = \Sigma \frac{(of - ef)^2}{ef}$$



df = (2-1) * (2-1) = 1

$X^2_{1, .05} = 3.84$

$X^2_{1, .01} = 6.63$

p = .0002 using online p-value calculator

So, we would reject H0: and conclude that there is a pattern of relationship between the variables.

Parametric tests of Association using ND/Int variables

Pearson's correlation

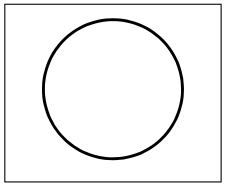
- H0: No linear relationship between the variables, in the population represented by the sample.
- degrees of freedom df = N - 2
- range of values - 1.00 to 1.00
- reject Ho: If $| r_{obtained} | > r_{critical}$

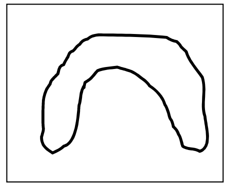
Pearson's correlation is an index of the direction and extent of the linear relationship between the variables.

It is important to separate the statements…

- there is no linear relationship between the variables
- there is no relationship between the variables
- correlation only addresses the former!

Correlation can not differentiate between the two bivariate distributions shown below – both have no linear relationship





One of many formulas for r is shown on the right.

$$r = \frac{\Sigma\ Z_X * Z_Y}{N}$$

- each person's "X" & "Y" scores are converted to Z-scores (M=0 & Std=1).
- r is calculated as the average Z-score cross product.

+r results when most of the cross products are positive (both Zs + or both Zs -)

-r results when most of the cross products are negative (one Z + & other Z-)

Nonparametric tests of Association using ~ND/~Int variables

Spearman's Correlation

- H0: No rank order relationship between the variables, in the population represented by the sample.
- degrees of freedom df = N - 2
- range of values - 1.00 to 1.00
- reject Ho: If $|r_{obtained}| > r_{critical}$

Computing Spearman's r

One way to compute Spearman's correlation is to convert X & Z values to ranks, and then correlate the ranks using Pearson's correlation formula, applying it to the ranked data. This demonstrates…

- rank data are "better behaved" (i.e., more interval & more ND) than value data
- Spearman's looks at whether or not there is a linear relationship between the ranks of the two variables

The most common formula for Spearman's Rho is shown on the right.

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

To apply the formula, first convert values to ranks.

	# practices	# correct	rank # practices	rank # correct	d	d^2
S1	6	21	4	5	-1	1
S2	2	18	1	4	-3	9
S3	4	7	2	1	1	1
S4	9	15	5	3	2	4
S5	5	10	3	2	1	1

$\Sigma d^2 = 16$

$$r = 1 - \frac{6 * 16}{5 * 24} = 1 - .80 = .20$$

For small samples (n < 20) r is compared to r-critical from tables.
For larger samples, r is transformed into t for NHSTesting.

Remember to express results in terms of the direction and extent of rank order relationship !

So, how does this strange-looking formula work? Especially the “6” ???

Remember that we’re working with “rank order agreement” across variable – a much simpler thing than “linear relationship” because there are a finite number of rank order pairings possible!

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

If there is complete rank order agreement between the variables …
→then, d = 0 for each case & $\Sigma d^2 = 0$
→so, r = 1-0
→r = 1 → indicating a perfect rank-order correlation

If the rank order of the two variables is exactly reversed…
→ Σd^2 can be shown to be $n(n^2-1)/3$
→ the equation numerator becomes $6 * n(n^2 – 1)/3 = 2 * n(n^2 – 1)$
→ so, r = 1 – 2
→ r = -1 → indicating a perfect reverse rank order correlation

If there is no rank order agreement of the two variables …
→Σd^2 can be shown to be $n(n^2-1)/6$
→the equation numerator becomes $6 * n(n^2 – 1)/6 = n(n^2 – 1)$
→so, r = 1 – 1
→ r = 0 → indicating no rank order correlation

Nonparametric tests of Association using ~ND/~Int variables

Kendall’s Tau

- H0: No rank order concordance between the variables, in the population represented by the sample.
- degrees of freedom df = N - 2
- range of values - 1.00 to 1.00
- reject Ho: If | robtained | > rcritical

All three correlations have the same mathematical range (-1, 1).

But each has an importantly different interpretation.

Pearson’s correlation
- direction and extent of the linear relationship between the variables

Spearman’s correlation
- direction and extent of the rank order relationship between the variables

Kendall’s tau
- direction and proportion of concordant & discordant pairs

The most common formula for Kendall’s Tau is shown on the right.**

$$tau = \frac{2(C-D)}{n(n-1)}$$

	# practices	# correct	rank # practices X	rank # correct Y
S1	6	21	4	5
S2	2	18	1	4
S3	4	7	2	1
S4	9	15	5	3
S5	5	10	3	2

To apply the formula, first convert values to ranks.

	# practices	# correct	rank # practices X	rank # correct Y
S2	2	18	1	4
S3	4	7	2	1
S5	5	10	3	2
S1	6	21	4	5
S4	9	15	5	3

Then, reorder the cases so they are in rank order for X.

**There are other forumlas for tau that are used when there are tied ranks.

	# practices ` X	# correct Y	rank # practices X	rank # correct Y	**C**	**D**
S2	2	18	1	4	1	3
S3	4	7	2	1	3	0
S5	5	10	3	2	2	0
S1	6	21	4	5	0	1
S4	9	15	5	3		
				sum	6	4

For each case…

C = the number of cases listed below it that have a larger Y rank

(e.g., for S2, C=1 → there is one case below it with a higher rank - S1)

D = the number of cases listed below it that have a smaller Y rank

(e.g., for S2, D=3 → there are 3 cases below it with a lower rank - S3 S5 S4)

$$\text{tau} = \frac{2(C-D)}{n(n-1)} = \frac{2(6-4)}{5(5-1)} = \frac{4}{20} = .20$$

For small samples (n < 20) tau is compared to tau-critical from tables.

For larger samples, tau is transformed into Z for NHSTesting.