Multiple Regression Models: Some Details & Surprises

- Review of raw & standardized models
- Differences between r, b & β
- Bivariate & Multivariate patterns
- Suppressor Variables
- Colinearity
- MR Surprises:
  - Multivariate power
  - Null Washout
  - Extreme colinearity
- Missing Data

raw score regression $y' = b_1x_1 + b_2x_2 + b_3x_3 + a$

each b

• represents the unique and independent contribution of that predictor to the model

• for a quantitative predictor tells the expected direction and amount of change in the criterion for a 1-unit change in that predictor, while holding the value of all the other predictors constant

• for a binary predictor (with unit coding -- 0,1 or 1,2, etc.), tells direction and amount of group mean difference on the criterion variable, while holding the value of all the other predictors constant

a

• the expected value of the criterion if all predictors have a value of 0

standard score regression $Z_y' = \beta Z_{x1} + \beta Z_{x2} + \beta Z_{x3}$

each β

• for a quantitative predictor the expected Z-score change in the criterion for a 1-Z-unit change in that predictor, holding the values of all the other predictors constant

• for a binary predictor, tells size/direction of group mean difference on criterion variable in Z-units, holding all other variable values constant

As for the standardized bivariate regression model there is no “a” or “constant” because the mean of Zy’ always = Zy = 0

The most common reason to refer to standardized weights is when you (or the reader) is unfamiliar with the scale of the criterion. A second reason is to promote comparability of the relative contribution of the various predictors (but see the important caveat to this discussed below!!!).

Different kinds of correlations & regression weights

r -- simple correlation

tells the direction and strength of the linear relationship between two variables ($r = \beta$ for bivariate models)

b -- raw regression weight from a bivariate model

tells the expected change (direction and amount) in the criterion for a 1-unit increase in the predictor

β -- standardized regression weight from a bivariate model

tells the expected change (direction and amount) in the criterion in Z-score units for a 1-Z-score unit increase in that predictor

b_i -- raw regression weight from a multivariate model

tells the expected change (direction and amount) in the criterion for a 1-unit increase in that predictor, holding the value of all the other predictors constant

β_i -- standardized regression weight from a multivariate model

tells the expected change (direction and amount) in the criterion in Z-score units for a 1-Z-score unit change in that predictor, holding the value of all the other predictors constant

What influences the size of bivariate r, b & β ?????

r -- bivariate correlation range = -1.00 to +1.00
- -- strength of linear relationship with the criterion
- -- sampling "problems" (e.g., range restriction)

b -- raw-score regression weights range = $-\infty$ **to** ∞
- -- strength of linear relationship with the criterion
- -- scale differences between & criterion
- -- sampling "problems" (e.g., range restriction)

β -- standardized regression weights range = -1.00 to +1.00
- -- strength of linear relationship with the criterion
- -- sampling "problems" (e.g., range restriction)

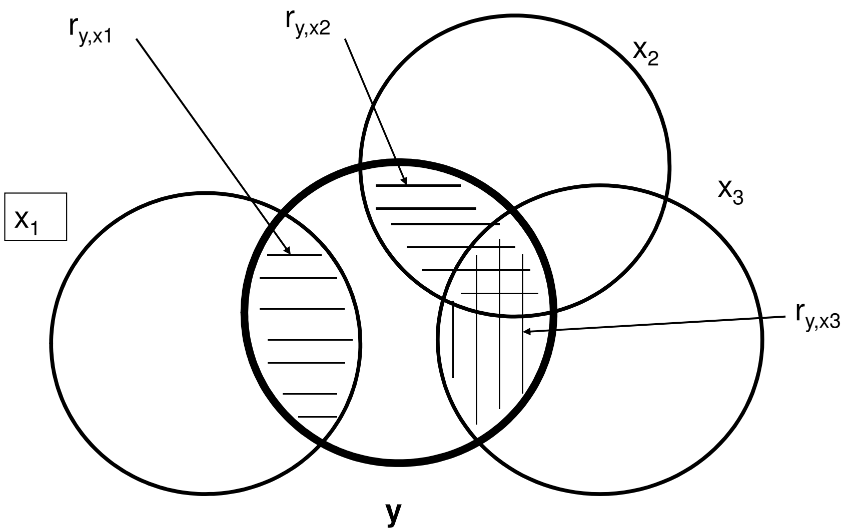
What influences the size of multivariate b_i & β_i

b (raw-score regression weights range = $-\infty$ **to** ∞
- -- strength of linear relationship with the criterion
- -- collinearity with the other predictors
- -- scale differences between predictor and criterion
- -- sampling "problems" (e.g., range restriction)

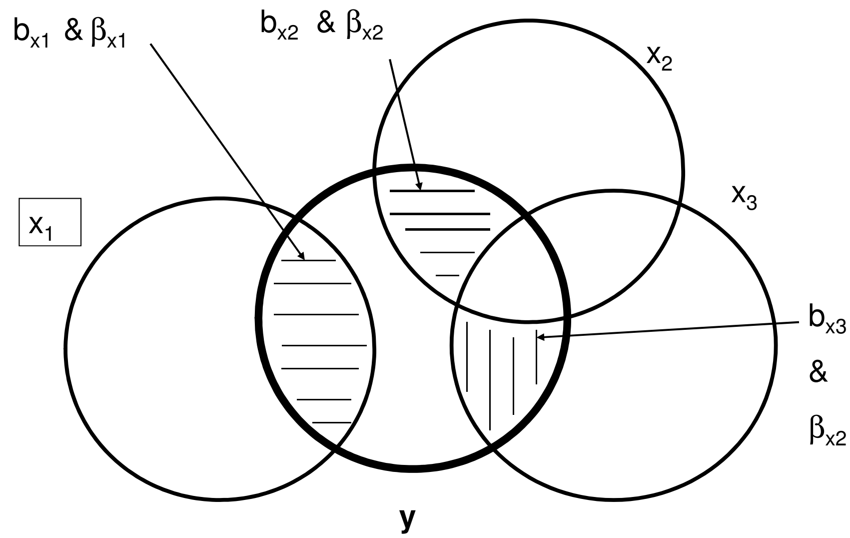
β -- standardized regression weights range = -1.00 to +1.00
- -- strength of relationship with the criterion
- -- collinearity with the other predictors
- -- sampling "problems" (e.g., range restriction)

Difficulties of determining "more important contributors"
-- b is not very helpful - scale differences produce b differences
-- β works better, but influenced by sampling variability and measurement influences (range restriction)
Only interpret "very large" β differences as evidence that one predictor is "more important" than another

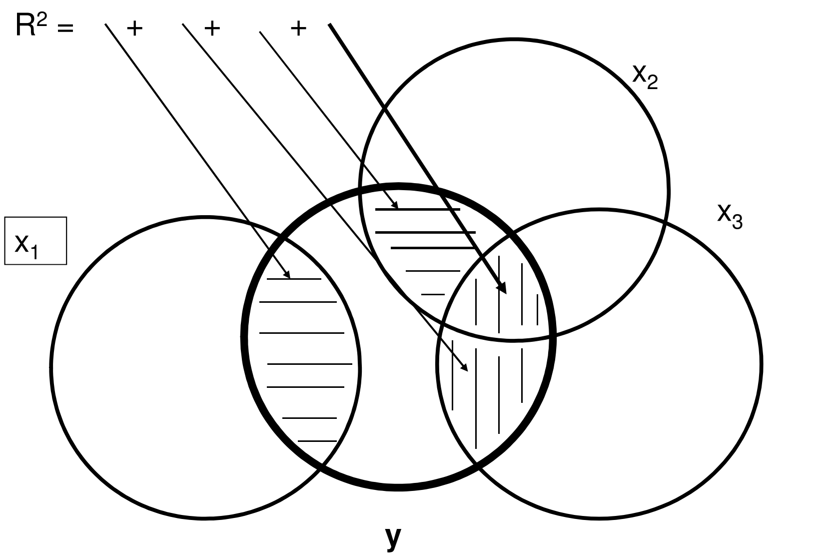
Venn diagrams representing r, b and R^2



Remember that the b of each predictor represents the part of that predictor shared with the criterion that is not shared with any other predictor -- the unique contribution of that predictor to the model



Remember R^2 is the total variance shared between the model (all of the predictors) and the criterion (not just the accumulation of the parts uniquely attributable to each predictor).



Bivariate vs. Multivariate Analyses & Interpretations

We usually perform both bivariate and multivariate analyses with the same set of predictors. Why?

Because they address different questions

- correlations ask whether variables each have a relationship with the criterion
- bivariate regressions add information about the details of that relationship (how much change in Y for how much change in that X)
- multivariate regressions tell whether variables have a unique contribution to a particular model (and if so, how much change in Y for how much change in that X after holding all the other Xs constant)

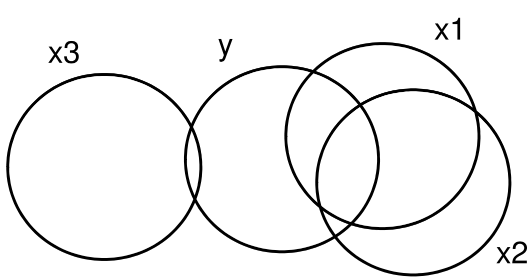
So, it is important to understand the different outcomes possible when performing both bivariate and multivariate analyses with the same set of predictors.

There are 5 patterns of bivariate/multivariate relationship

Multiple regression weight \ Simple correlation with the criterion	-	0	+
-	Bivariate relationship and multivariate contribution (to this model) have same sign	"Suppressor variable" – no bivariate relationship but contributes (to this model)	"Suppressor variable" – bivariate relationship & multivariate contribution (to this model) have different signs
0	Non-contributing – probably because colinearity with one or more other predictors	Non-contributing – probably because of weak relationship with the criterion	Non-contributing – probably because colinearity with one or more other predictors
+	"Suppressor variable" – bivariate relationship & multivariate contribution (to this model) have different signs	"Suppressor variable" – no bivariate relationship but contributes (to this model)	Bivariate relationship and multivariate contribution (to this model) have same sign

Here's a depiction of the two different reasons that a predictor might not be contributing to a multiple regression model...

1. the variable isn't correlated with the criterion – X3
2. the variable is correlated with the criterion, but is collinear with one or more other predictors (we can't tell which), and so, has no independent contribution to the multiple regression model



X1 has a substantial r with the criterion and has a substantial b

x2 has a substantial r with the criterion but has a small b because it is collinear with x1

x3 has neither a substantial r nor substantial b

Bivariate & Multivariate contributions – DV = Grad GPA

predictor→	age	UGPA	GRE	work hrs	#credits
r(p)	.11(.32)	.45(.01)	.38(.03)	-.15(.29)	.28(.04)
b(p)	.06(.67)	1.01(.02)	.002(.22)	.023(.01)	-.15(.03)

Bivariate relationship and multivariate contribution (to this model) have same sign	UGPA
Non-contributing – probably because colinearity with one or more other predictors	GRE
Non-contributing – probably because of weak relationship with the criterion	age
"Suppressor variable" – no bivariate relationship but contributes (to this model)	work hrs
"Suppressor variable" – bivariate relationship & multivariate contribution (to this model) have different signs	#credits

Bivariate & Multivariate contributions – DV = Pet Quality

predictor→	#fish	#reptiles	ft^2	#employees	#owners
r(p)	-.10(.31)	.48(.01)	-.28(.04)	.37(.03)	-.08(.54)
b(p)	-.96(.03)	1.61(.42)	1.02(.02)	1.823(.01)	-.65(.83)

#fish	"Suppressor variable" – no bivariate relationship but contributes (to this model)
#reptiles	Non-contributing – probably because colinearity with one or more other predictors
ft^2	"Suppressor variable" – bivariate relationship & multivariate contribution (to this model) have different signs
#employees	Bivariate relationship and multivariate contribution (to this model) have same sign
#owners	Non-contributing – probably because of weak relationship with the criterion

How to think about suppressor effects ?

To be a suppressor, the variable must contribute to the multivariate model AND

- not be correlated with the criterion OR
- be correlated with the criterion with the opposite sign of b_i

A suppressor effect means that "the part of the predictor that is not related to the other predictors," is better/differently related with the criterion than is "the whole predictor".

ft^2 from last example…

-r – fish quality is negatively correlated with store size

+b in mreg – fish quality is positively correlated with the part of store size that is not related to #fish, #reptiles, #employees & #owners

(the hard part is to figure out what to call "the part of store size that is not related to #fish, #reptiles, #employees & #owners")

What to do with a suppressor variable ??

One common response is to "simplify the model" by dumping any suppressor variables from the model...

Another is to label the suppressor variable and then ignore it...

A much better approach is to determine which other variables in the equation are involved

- Look at the collinearities among the predictors (predictors that are positively correlated with some predictors and negative correlated with others are the most likely to be involved in suppressor effects)
- Check each 2-predictor, 3-predictor, etc. model (ways including the target variable), to reproduce the suppressor effect (this is less complicated with variables you know well)
- Then you can (sometimes) figure out an interesting & informative interpretation of the suppression
  - suppression often indicates "mediational models" & sometimes interaction/moderation effects

While we're on this collinearity thing…

It is often helpful to differentiate between three "levels" of collinearity…

1. Collinearity -- correlations among predictors

   -- the stuff of life -- behaviors, attributes and opinions are related to each other

   -- consequences -- forces us to carefully differentiate between the question asked of simple correlation (whether or not a given predictor correlates with that criterion) vs. the question asked by multiple correlation (whether or not a given predictor contributes to a particular model of that criterion)

Collinearity can be assessed using the "tolerance" statistic, which, for each predictor, is 1 - R^2 predicting that predictor using all the other predictors (larger values are "better")

2. Extreme collinearity --

   -- one useful definition is when the collinearities are as large or larger than the validities (correlations between the predictors and the criterion)

   -- need to consider whether the collinearity is really between the predictor constructs, or the predictor measures (do predictors have overlapping elements?)

   -- may need to "select" or "aggregate" to form smaller set of predictors

3. Singularity -- when one or more predictors is perfectly correlated with one or more other predictors

   -- be sure not to include as predictors a set of variables and another that is their total (or mean)

   -- will need to "select" or "aggregate" to form smaller set of predictors

Another concern we have is “range restriction”

… when the variability of a predictor or criterion variable in the sample is less than the variability of the represented construct in the population -- the consequence is that the potential correlation between that variable and others will be less than 1.00

Two major sources of range restriction …

1. Sample doesn’t represent population of interest
   examples -- selection research, analog research

2. Poor fit between sample and measure used -- also called “floor” or “ceiling” effects
   examples -- MMPI with “normals”, BDI with inpatients

Range restriction will yield a sample correlation that under-estimates the population correlation !!

Range restriction issues in multiple regression

if the criterion is range restricted

-- the strength of the model will be underestimated
-- “good predictors” will be missed (Type II errors)

if all the predictors are range restricted

-- same as above

the real problem is .. (huge and almost impossible to avoid)
DIFFERENTIAL range restriction among the predictors

-- relative importance of predictors as single predictors and contributors to multiple regression models will be misrepresented in the sample (if is concern over this which will be why we don’t just inspect β weights to determine which predictors are “more important” in a multiple regression model)

As we talked about, collinearity among the multiple predictors can produce several patterns of bivariate-multivariate results. There are three specific combinations you should be aware of (none of which is really common, but each can be perplexing if they aren’t expected)…

1. Multivariate Power -- sometimes a set of predictors none of which are significantly correlated with the criterion can produce a significant multivariate model (with one or more contributing predictors)

How’s that happen?

- The error term for the multiple regression model and the test of each predictor’s b is related to 1-R^2 of the model
- Adding predictors will increase the R^2 and so lower the error term – sometimes leading to the model and one or more predictors being “significant”
- This happens most often when one or more predictors have “substantial” correlations, but the sample power is low

2. Null Washout -- sometimes a set of predictors with only one or two significant correlations to the criterion will produce a model that is not significant. Even worse, those significantly correlated predictors may or may not be significant contributors to the non-significant model

How's that happen?

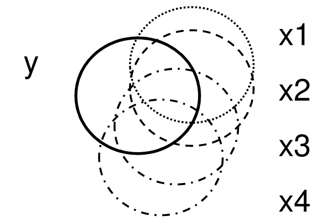
$$F = \frac{R^2 \; / \; k}{(1 - R^2) \; / \; (N - k - 1)}$$

- The F-test of the model R^2 really (mathematically) tests the average contribution of all the predictors in the model
- So, a model dominated by predictors that are not substantially correlated with the criterion might not have a large enough "average" contribution to be statistically significant
- This happens most often when the sample power is low and there are many predictors

3. Extreme collinearity -- sometimes a set of predictors all of which are significantly correlated with the criterion can produce a significant multivariate model with one or more contributing predictors

How's that happen?

- Remember that in a multiple regression model each predictor's b weight reflects the unique contribution of that predictor to that model
- If the predictors are more highly correlated with each other than with the criterion then the "overlap" each has with the criterion is shared with 1 or more other predictors, and so, no predictor has much unique contribution to that very successful (high R^2) model



Missing Data

Missing data happen for many different reasons and how you treat the missing values is likely to change the results you get

Casewise or Listwise Deletion

- Only cases that have "complete data" are used in any of the analyses
- Which cases those are can change as the variables used in the analysis change

Pairwise Analyses

- Use whatever cases have "complete data" for that analysis
- Which cases those are can change as the variables used in
- the analysis change

In particular → watch for results of different analyses reported with different sample sizes or no sample sizes