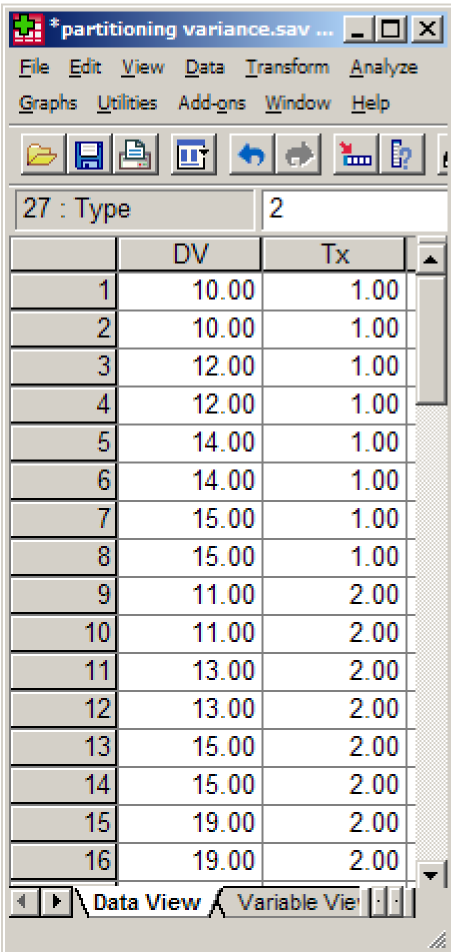
Factorial Designs: Partitioning Variation to Increase Power & "Control" Confounds

Starting with simple data set…



	DV	Tx
1	10.00	1.00
2	10.00	1.00
3	12.00	1.00
4	12.00	1.00
5	14.00	1.00
6	14.00	1.00
7	15.00	1.00
8	15.00	1.00
9	11.00	2.00
10	11.00	2.00
11	13.00	2.00
12	13.00	2.00
13	15.00	2.00
14	15.00	2.00
15	19.00	2.00
16	19.00	2.00

Descriptive Statistics

Dependent Variable: DV

Tx	Mean	Std. Deviation	N
1.00	12.7500	2.05287	8
2.00	14.5000	3.16228	8
Total	13.6250	2.72947	16

Tests of Between-Subjects Effects

Dependent Variable: DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Tx	12.250	1	12.250	1.724	.210
Error	99.500	14	7.107		
Total	111.750	15			

SStotal = SStx + SSerror ← Standard ANOVA w/ 2 variance sources

111.750 = 12.250 + 99.50

Partitioning existing variance (to add power) …

Whenever we have additional variables in the data set, we can incorporate them into the analysis. If an additional variable is also a categorical variable, we can use it as a second IV and analyze the data as a factorial design.

*Untitled1 [DataSet0] - SPSS Data Editor

19 : DV

	Tx	Kind	DV
1	1.00	1.00	10.00
2	1.00	1.00	10.00
3	1.00	1.00	12.00
4	1.00	1.00	12.00
5	1.00	2.00	14.00
6	1.00	2.00	14.00
7	1.00	2.00	15.00
8	1.00	2.00	15.00
9	2.00	2.00	11.00
10	2.00	2.00	11.00
11	2.00	2.00	13.00
12	2.00	2.00	13.00
13	2.00	1.00	15.00
14	2.00	1.00	15.00
15	2.00	1.00	19.00
16	2.00	1.00	19.00
17			

Data View | Variable View

Descriptive Statistics

Dependent Variable: DV

Kind	Tx	Mean	Std. Deviation	N
1.00	1.00	11.0000	1.15470	4
	2.00	17.0000	2.30940	4
	Total	14.0000	3.62531	8
2.00	1.00	14.5000	.57735	4
	2.00	12.0000	1.15470	4
	Total	13.2500	1.58114	8
Total	1.00	12.7500	2.05287	8
	2.00	14.5000	3.16228	8
	Total	13.6250	2.72947	16

This analysis is of the same 16 cases as the ANOVA, so the ME of Tx replicates the earlier result.

The SSiv is the same as in the ANOVA above → same 8 cases in each Tx group, so same means and same SSiv

Tests of Between-Subjects Effects

Dependent Variable: DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Tx	12.250	1	12.250	5.880	.032
Kind	2.250	1	2.250	1.080	.319
Tx * Kind	72.250	1	72.250	34.7	.000
Error	25.000	12	2.083		
Corrected Total	111.750	15			

However SSerror is much smaller in the factorial than in the ANOVA – see below.

SSerror from the ANOVA is partitioned into SSkind, SSint & SSerror in the factorial.

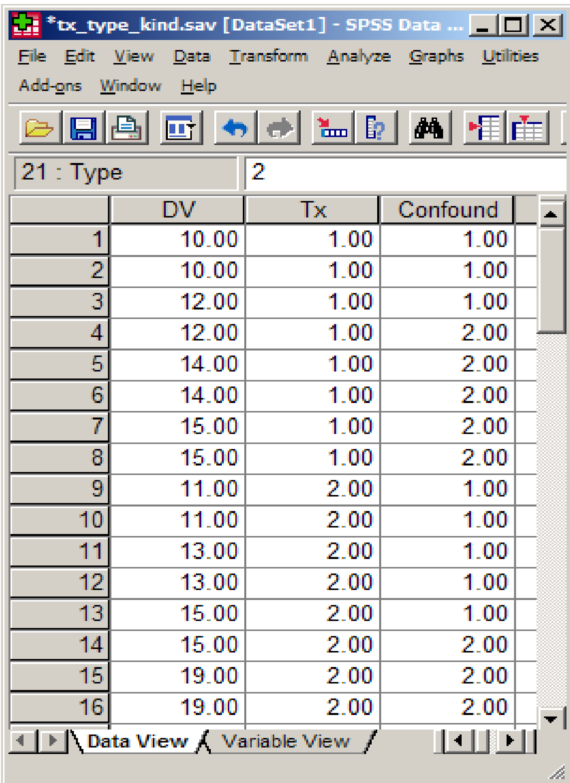
From this analysis we see that there is no main effect of Kind, but an interaction of Tx*Kind.

With the more powerful test (because of the smaller error term) we also find a significant Tx main effect that we "missed" in the original ANOVA (the ME is misleading).

1-factor	SStotal	=	SStx	+					SSerror
	111.750	=	12.250	+					99.50
2-factor	SStotal	=	SStx	+	SSkind	+	SSint	+	SSerror
	111.750	=	12.250	+	2.250	+	72.250	+	25.000

Partitioning existing variance to controlling a confound (& add power)

In the last case IV & Kind were orthogonal (4 of each Kind in each Tx group). But what if there was a confounding variable and we had data for it? Look below. Here Tx is confounded by Confound (Tx1 had 3 1s & 5 2s, whereas Tx2 has 5 1s & 3 2s).



	DV	Tx	Confound
1	10.00	1.00	1.00
2	10.00	1.00	1.00
3	12.00	1.00	1.00
4	12.00	1.00	2.00
5	14.00	1.00	2.00
6	14.00	1.00	2.00
7	15.00	1.00	2.00
8	15.00	1.00	2.00
9	11.00	2.00	1.00
10	11.00	2.00	1.00
11	13.00	2.00	1.00
12	13.00	2.00	1.00
13	15.00	2.00	1.00
14	15.00	2.00	2.00
15	19.00	2.00	2.00
16	19.00	2.00	2.00

Descriptive Statistics

Dependent Variable: DV

Confound	Tx	Mean	Std. Deviation	N
1.00	1.00	10.6667	1.15470	3
	2.00	12.6000	1.67332	5
	Total	11.8750	1.72689	8
2.00	1.00	14.0000	1.22474	5
	2.00	17.6667	2.30940	3
	Total	15.3750	2.44584	8
Total	1.00	12.7500	2.05287	8
	2.00	14.5000	3.16228	8
	Total	13.6250	2.72947	16

This analysis is of the same 16 cases as the ANOVA, so the ME of Tx replicates the earlier result**.**

The SSiv is different than in the ANOVA above → even though the same 8 cases in each Tx group and the same means.

Why? The factorial is re-partitioning the variance separating it into SS that represent the relationship between each effect and the DV, controlling for the other effects in the model (same as in multiple regression).

Tests of Between-Subjects Effects

Dependent Variable: DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Confound	66.150	1	66.150	25.998	.000
Tx	29.400	1	29.400	11.555	.005
Confound * Tx	2.817	1	2.817	1.107	.313
Error	30.533	12	2.544		
Corrected Total	111.750	15			

Which do we believe – ANOVA or factorial?

Since we have a confound, we know the ANOVA misrepresents the relationship between the Tx & DV.

The factorial ANOVA provides "statistical control" of the confound. While not as good as procedural control (constancy or balancing by matching or RA), it is "better than nothing."

Notice that we also get variance partitioning from this factorial. That is, with Confound and the Tx*Confound terms in the model the test of the Tx is not only "unconfounded" but it is also more powerful.

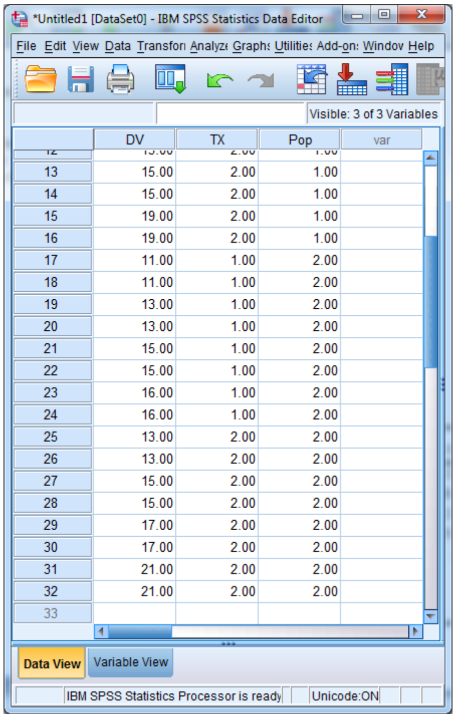
1-factor SStotal = SStx + SSerror

111.750 = 12.250 + 99.50

2-factor SStotal = SStx + SSconfound + SSint + SSerror

111.750 = 29.400 + 66.150 + 2.817 + 30.533

Adding variance → looking for "additional effects"



*Untitled1 [DataSet0] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

Visible: 3 of 3 Variables

	DV	TX	Pop	var
13	15.00	2.00	1.00	
14	15.00	2.00	1.00	
15	19.00	2.00	1.00	
16	19.00	2.00	1.00	
17	11.00	1.00	2.00	
18	11.00	1.00	2.00	
19	13.00	1.00	2.00	
20	13.00	1.00	2.00	
21	15.00	1.00	2.00	
22	15.00	1.00	2.00	
23	16.00	1.00	2.00	
24	16.00	1.00	2.00	
25	13.00	2.00	2.00	
26	13.00	2.00	2.00	
27	15.00	2.00	2.00	
28	15.00	2.00	2.00	
29	17.00	2.00	2.00	
30	17.00	2.00	2.00	
31	21.00	2.00	2.00	
32	21.00	2.00	2.00	
33				

Data View | Variable View

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These data include the original 16 from Pop=1 (8 each in TX=1 and TX=2), and also includes 16 from Pop=2.

Thus, we are adding cases (rather than just a variable) and "adding variance" to the model!

Descriptive Statistics

Dependent Variable: DV

Pop	TX	Mean	Std. Deviation	N
1.00	1.00	12.7500	2.05287	8
	2.00	14.5000	3.16228	8
	Total	13.6250	2.72947	16
2.00	1.00	13.7500	2.05287	8
	2.00	16.5000	3.16228	8
	Total	15.1250	2.94109	16
Total	1.00	13.2500	2.04939	16
	2.00	15.5000	3.22490	16
	Total	14.3750	2.89326	32

These are data from the same 16 cases as earlier.

Tests of Between-Subjects Effects

Dependent Variable: DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	60.500[a]	3	20.167	2.838	.056
Intercept	6612.500	1	6612.500	930.402	.000
Pop	18.000	1	18.000	2.533	.123
TX	40.500	1	40.500	5.698	.024
Pop * TX	2.000	1	2.000	.281	.600
Error	199.000	28	7.107		
Total	6872.000	32			
Corrected Total	259.500	31			

a. R Squared = .233 (Adjusted R Squared = .151)

The total variance is much larger.

The within-group standard deviations are similar to that from the original 2-group analysis, because those groups of 8 have not been "partitioned"!

The SS error is about twice as large – each group has about the same standard deviation as the original analysis, but now there are four groups instead of two.

1-factor	SStotal	=	SStx	+						SSerror
	111.750	=	12.250	+						99.50
2-factor	SStotal	=	SStx	+	SSpopulation	+	SSint	+		SSerror
	259.5	=	40.5	+	18.00	+	2.00	+		199.00