## Basic Analysis of Factorial Designs

- The F-tests of a Factorial ANOVA
- Using LSD to describe the pattern of an interaction


## Statistical Analysis of a $2 \times 2$ Design



## Statistical Analysis of $2 \times 2$ Factorial Designs

Like a description of the results based upon inspection of the means, formal statistical analyses of factorial designs has five basic steps:

1. Tell IVs and DV 2. Present data in table or figure
2. Determine if the interaction is significant

- if it is, describe it in terms of one of the sets of simple effects.

4. Determine whether or not the first main effect is significant

- if it is, describe it
- determine if that main effect is descriptive or misleading

5. Determine whether or not the second main effect is significant

- if it is, describe it
- determine if that main effect is descriptive or misleading


## Constructing F-tests for a $2 \times 2$ Factorial

$$
\begin{aligned}
& \mathrm{F}_{\text {Presentation }}=\frac{\left(\mathrm{SS}_{\text {Presentation }} / \mathrm{df}_{\text {Presentation }}\right)}{\left(\mathrm{SS}_{\text {Error }} / \mathrm{df}_{\text {Error }}\right)} \\
& \mathrm{F}_{\text {Difficulty }}=\frac{\left(\mathrm{SS}_{\text {Difficulty }} / \mathrm{df}_{\text {Difficulty }}\right)}{\left(\mathrm{SS}_{\text {Error }} / \mathrm{df}_{\text {Error }}\right)} \\
& \mathrm{F}_{\text {Interaction }}=\frac{\left(\mathrm{SS}_{\text {Interaction }} / \mathrm{df}_{\text {Interaction }}\right)}{\left(\mathrm{SS}_{\text {Error }} / \mathrm{df}_{\text {Error }}\right)}
\end{aligned}
$$

## Statistical Analyses Necessary to Describe the Interaction of a $2 \times 2$ Design

The F-test of the interaction only tells us whether or not there is a "statistically significant" interaction...

- it does not tell use the pattern of that interaction
- to determine the pattern of the interaction we have to compare the simple effects
- to describe each simple effect, we must be able to compare the cell means
we need to know how much of a cell mean difference is "statistically significant"

Using Pairwise Comparisons (PrC) to Compare cell means to describe the simple effects of a $2 \times 2$ Factorial design

- LSD PrC can be used to determine how large of a cell mean difference is required to treat it as a "statistically significant mean difference"
- Will need to know three values to use the computator
- df error -- look on the printout or use $\mathrm{N}-4$
- $\mathrm{MS}_{\text {error }}$ - look on the printout
- $\mathrm{n}=\mathrm{N} / 4$-- use the decimal value - do not round to the nearest whole number!

Remember - for a $2 \times 2$ Design, only use the Isdmmd PrC to compare cell means. Marginal means are compared using the man effect F -tests.

## Applying Isd $_{\text {mmd }}$ to $2 \times 2$ BG ANOVA

Task Presentation Paper Computer
Task Difficulty
Easy
Hard

| 60 | 90 |
| :--- | :--- |
| 60 | 70 |

for the interaction $F(1,56)=6.5$, $\mathrm{Mse}=300$, $\mathrm{p}=.023$

Is there an Interaction? Based on what?
Yes! F-test of Int
What info do we need to compute the LSDmmd?
$\mathrm{k}=4$ groups
$\mathrm{n}=(\mathrm{df}+\mathrm{k}) / \mathrm{k}=(56+4) / 4=15$
$\mathrm{MSe}=6.5$
df error $=56$ (round down to 50)

|  | LSD \& HSD Minimum Mean Diffe |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{k}=4 \text { groups } \\ & \mathrm{n}=(\mathrm{dff}+\mathrm{k}) / \mathrm{k}=(56+4) / 4=15 \\ & \mathrm{Mse}=6.5 \\ & \mathrm{dif} \text { error } 56 \text { (round down to } 50 \text { ) } \end{aligned}$ | Enter $\mathbf{k}$ (number of conditions in the effect) $=>$ Enter $\mathbf{n}$ (average number of data points upon which each mean is based - N/k) => |  |  |
|  |  |  | 4 |
|  |  |  | 15 |
|  | Enter M | (Mean Square Error) => | 300 |
|  | Select dferror (erri "next sma | degrees of freedom - use st" if no exact match) => | 50. |
| With an LSDmmd $=12.7$ |  |  |  |
|  |  |  |  |
|  | LSD minimum mean difference $=$ |  | 12.70 |
|  | SE mean dif |  | sig? |
| Simple effect of Task Presentation |  |  |  |
| SE of Task Presentation | for Easy Tasks | 30 | > |
| SE of Task Presentation for Hard Tasks |  | 10 | = |
| Simple effects of Task Difficulty |  |  |  |
| SE of Task Difficulty for Paper Pres |  | 0 |  |
| SE of Task Difficulty for Comp. Pres |  | 20 | V |

## Statistical Analyses Necessary to Describe Main Effects of a $2 \times 2$ Design

In a $2 \times 2$ Design, the Main effects F-tests are sufficient to tell us about the relationship of each IV to the DV...

- since each main effect involves the comparison of two marginal means -- the corresponding significance test tells us what we need to know ...
- whether or not those two marginal means are
"significantly different"
- Don't forget to examine the means to see if a significant difference is in the hypothesized direction !!!


## Support for Main effect RH:s

A RH: about a Main effect is only fully supported if that Main effect is descriptive.
RH: Electric Toys are preferred to Puzzles - tell if each of the following give full, partial or no support ...


Partial


Partial


None


None? / Partial ?


None? / Partial?


Full

What statistic is used for which factorial effects????

| Task Experience <br> Yes <br> Age <br> 5 |  |  |  |  | 30 | 30 | 30 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

What statistic is used for which factorial effects????
Task Experience

| Age | Task Experience |  |
| :---: | :---: | :---: |
|  | Yes | No |
| 5 | 50 | 30 |
| 10 | 60 | 80 |
|  | 25 | 30 |

1. $\mathrm{F}_{\text {Age }} \mathrm{p}=.021$
2. $\mathrm{F}_{\text {TExp }} \mathrm{P}=.082$
3. $\mathrm{F}_{\text {lnt }} \quad \mathrm{p}=.001$
4. $\operatorname{PrC} L S D_{m m d}=15$

40 Are 40 \& 70 different? $\quad F_{\text {Age }}$ Are 50 \& 30 different? LSDmmd Are 30 \& 80 different? LSDmmd Are $50 \& 60$ differently
different than $30 \& 80$ ? Are 50 \& 60 different? LSDmmd
Are 25 \& 30 different ? $\quad \mathrm{F}_{\text {TExp }}$
Are $50 \& 30$ differently
different than 60 \& 80 ? $\quad \mathrm{F}_{\text {Int }}$ Are 60 \& 80 different? LSDmmd

## Applying Isd $_{\text {mmd }}$ to $2 \times 2$ BG ANOVA

Task Presentation
Paper Computer

Task Difficulty
Easy
Hard

| 60 | 90 |
| :--- | :--- |
| 60 | 70 |

for the interaction

$$
F(1,56)=6.5, p=.023
$$

$$
\mathrm{Isd}_{\mathrm{mmd}}=14
$$

Is there an interaction effect? Based on what?
Yes! F-test of Int
for the following, tell the mean difference and apply the $\operatorname{Isd} \mathrm{d}_{\text {mid }}$ Simple effect of Task Presentation

SE of Task Presentation for Easy Tasks
SE of Task Presentation for Hard Tasks
$10=$

Simple effects of Task Difficulty
SE of Task Difficulty for Paper Pres.
SE of Task Difficulty for Comp. Pres. 20 v

## Applying Isd $_{\text {mmd }}$ to $2 \times 2$ BG ANOVA

Task Presentation
Paper Computer

Task Difficulty
Easy
Hard

| 60 | 90 |
| :--- | :--- |
| 60 | 70 | for Difficulty ME

$F(1,56)=4.5, p=.041$

$$
\mathrm{Isd}_{\mathrm{mmd}}=14
$$

Is there a Task Difficulty main effect? Based on what?
Yes! F-test of ME

Is main effect descriptive (unconditional) or potentially misleading (conditional)?
Simple effects of Task Difficulty

$$
\begin{array}{lcc}
\text { SE of Task Difficulty for Paper Pres. } & 0 & \text { II } \\
\text { SE of Task Difficulty for Comp. Pres. }
\end{array}
$$

Descriptive only for Computer presentation; misleading for Paper presentations.

## Applying Isd $_{\text {mmd }}$ to $2 \times 2$ BG ANOVA

Task Presentation

|  | Task Presentation <br> Paper |  |
| :---: | :---: | :---: |
| Computer |  |  |
| Task Difficulty <br> Easy | 60 | 90 |
| Hard Presentation ME |  |  |
| H $(1,56)=7.2, \mathrm{p}=.011$ |  |  |

Is there a Task Presentation main effect? Based on whatYes! F-test of ME
Is main effect descriptive (unconditional) or potentially misleading (conditional)? Simple effects of Task Difficulty

> SE of Task Presentation for Easy Tasks
> SE of Task Presentation for Hard Tasks

Descriptive only for Easy tasks; misleading for Difficult tasks.

Here's one to watch out for...
Task Presentation
Paper Computer

| Comp ComfortLow |  |  | for the interaction$F(1,86)=4.2, p=.044$ |
| :---: | :---: | :---: | :---: |
|  | 70 | 60 |  |
|  | II | II |  |
| High | 60 | 70 | apply $\mathrm{Isd}_{\mathrm{mmd}}=13$ |

Huh ??? But...

The interaction F-tests $\rightarrow$ tests whether SEs are "different from each other"!!
It doesn't test if either of them is different from " 0 "!!!
" 10 " \& "-10" are "different from each other", but neither is different from " 0 "!

You can't use the LSDmmd to say that -10 \& 10 are sig dif! Rem!!! This is based on the F-test!!

## Effect Sizes for 2x2 BG Factorial designs

For Main Effects \& Interaction (each w/ df=1)

$$
r=\sqrt{ }\left[F /\left(F+d f_{e r r o r}\right)\right]
$$

Rem: This effect size can only be compared with other interaction effects from exactly the same factorial design

$$
\begin{gathered}
\text { For Simple Effects } \\
d=(\text { M1 - M2 }) / \sqrt{ } \text { Mserror } \\
r=\sqrt{ }\left[\begin{array}{c}
d^{2}-\cdots------1 \\
d^{2}+4
\end{array}\right]
\end{gathered} \begin{gathered}
\text { (An "approximation formula") }
\end{gathered}
$$

Rem: The effects size for a pairwise comparison can be compared with that pair of conditions from any study.

