2xQ Models: Using Regression & GLM for Linear Models Including Interactions

The data come from a sports psychology study of the motivational effects of labeling. During a week-long basketball camp for aspiring college players, there was an optional "tactical training" session that everyone took once, and was optional for the other nine offerings. Before the first session, the clinic coaches had identified who were the "starters" and who were the "bench players" among the attendees. For the first drill of each session, players were divided into the "starters" and the "bench players" – this was the labeling manipulation. The research question was, "How would this label influence tactical learning?"

Regression: Basic model

*recoding original grouping variable to bench (=0) as comparison/reference.
*centering original quant variable.
*computing the interaction.
if (motv_s1b2 = 1) motv_s1b0 = 1.
if (motv_s1b2 = 2) motv_s1b0 = 0.
compute numsessions_cen = numsessions - 5.28125.
compute numsesscen_motvs1b0_int = numsessions_cen * motv_s1b0.

*regression -- use motv_s1b0 -- will get simple regression line for bench(=0).
*use numsessions_cen – will get group comparison at mean=0.
REGRESSION
 /STATISTICS COEFF OUTS R ANOVA
 /DEPENDENT tacticalerrors
 /METHOD=ENTER numsessions_cen   motv_s1b0   numsesscen_motvs1b0_int.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.713[a]	.508	.455	1.57691

a. Predictors: (Constant), numsesscen_motvs1b0_int, motv_s1b0, numsessions_cen

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	71.843	3	23.948	9.631	.000[b]
	Residual	69.626	28	2.487		
	Total	141.469	31			

a. Dependent Variable: tacticalerrors
b. Predictors: (Constant), numsesscen_motvs1b0_int, motv_s1b0, numsessions_cen

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.570	.394		14.128	.000
	numsessions_cen	-.236	.151	-.289	-1.565	.129
	motv_s1b0	-1.591	.558	-.378	-2.853	.008
	numsesscen_motvs1b0_int	-.434	.216	-.370	-2.005	.055

a. Dependent Variable: tacticalerrors

Numsessions_cen – there is a nonsignificant negative regression slope for the bench group.

Constant – at the average number of sessions attended (5.28) Bench players have 5.57 errors.

Motv_s1b0 – at the average number of sessions attended (5.28) the Starters have 1.591 fewer tactical errors than the Bench players, and this difference is statistically significant. Starters have 5.570 – 1.151 = 3.979 tactical errors on average.

Numsesscen_motvs1b0 – while not quite significant… the slope of the Starter regression line (-.67) is .434 less positive than that of the Bench regression line (-.236).
or
for each additional session attended, the number of tactical errors made by the Starters is .434 less than made by the Bench.

height z=0	constant	5.57
slope z=0	b(x)	-0.236
height dif z=1	b(z)	-1.591
slope dif z=1	b(xz)	-0.434
	x(mean)	5.281
	x(std)	2.618

	(slope * X) +		height
Bench =0	-0.236	* X +	5.57
Starters =1	-0.67	* X +	3.979
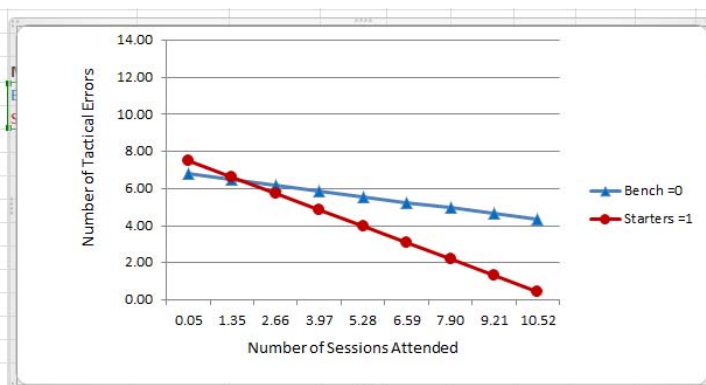
Remember – all the analyses shown in the following pages produce the same model!!! We may recode this or re-center that to change the specific information available from a regression weight and a significance test, but they are all the same model!

Regression: Group Differences at other Numbers of Sessions

We know there is a group difference for the average number of sessions attended. We can test for a group difference at any number of sessions we like, just by re-centering number of sessions at that value and computing a new interaction term.

Comparing Starter & Bench Groups for 2 sessions (attending 1 optional session).

```
*getting group difference for 2 sessions.
if (motv_s1b2 = 1) motv_s1b0 = 1.
if (motv_s1b2 = 2) motv_s1b0 = 0.
compute numsessions_2cen = numsessions - 2.
compute numsesscen2_motvs1b0_int = numsessions_2cen * motv_s1b0.
exe.
REGRESSION
  /STATISTICS COEFF OUTS R ANOVA
  /DEPENDENT tacticalerrors
  /METHOD=ENTER numsessions_2cen   motv_s1b0   numsesscen2_motvs1b0_int.
```

Showing only the non-redundant parts…

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	6.344	.636		9.972	.000
	numsessions_2cen	-.236	.151	-.289	-1.565	.129
	motv_s1b0	-.167	.903	-.040	-.185	.855
	numsesscen2_motvs1b0_int	-.434	.216	-.500	-2.005	.055

a. Dependent Variable: tacticalerrors

At 2 sessions, the Bench players had an average of 6.344 errors.

The Starters had an average number of tactical errors .167 fewer than the Benchplayers, which is not statistically significant. Starters have 6.344 – .167 = 6.175 tactical errors on average.

Notice the regression weights for number of sessions and the interaction are the same values as before.

Comparing Starter & Bench Groups for 9 sessions

```
*getting group difference for 9 sessions.
if (motv_s1b2 = 1) motv_s1b0 = 1.
if (motv_s1b2 = 2) motv_s1b0 = 0.
compute numsessions_9cen = numsessions - 9.
compute numsesscen9_motvs1b0_int = numsessions_9cen * motv_s1b0.
exe.
REGRESSION
  /STATISTICS COEFF OUTS R ANOVA
  /DEPENDENT tacticalerrors
  /METHOD=ENTER numsessions_9cen   motv_s1b0   numsesscen9_motvs1b0_int.
```

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.693	.681		6.886	.000
	numsessions_9cen	-.236	.151	-.289	-1.565	.129
	motv_s1b0	-3.205	.979	-.762	-3.272	.003
	numsesscen9_motvs1b0_int	-.434	.216	-.536	-2.005	.055

a. Dependent Variable: tacticalerrors

At 9 sessions, the Bench players had an average of 4.693 errors.

The Starters had an average number of tactical errors 3.205 fewer than the Bench players, which is statistically significant. Starters have 4.693 – 3.205 = 1.488 tactical errors on average.

We could get the group mean tactical errors and the mean difference for any number of sessions, using this approach.

Regression: Getting the Numbers of Sessions Regression Slope & Significance Test for the Starters

We know the slope of the regression line for the Bench is -.236 and that it isn't different from 0 (p=.129). We know that the slope of the regression line for the Starters is .434 less than the slope for the Bench (-.236 – (-.434) = .67). And we know that these regression slopes are marginally significantly different (p = .055). But, is the regression slope for the Starters significantly different from 0??

To get this, we just have to recode the dummy code representing the Motivational groups, with Starters = 0.

```
*same model with the starters as the reference/comparison group.
*recoding original grouping variable to starters as comparison/reference.
*centering original quant variable.
*computing the interaction.
if (motv_s1b2 = 1) motv_s0b1 = 0.
if (motv_s1b2 = 2) motv_s0b1 = 1.
compute numsessions_cen = numsessions - 5.28125.
compute numsesscen_motvs0b1_int = numsessions_cen * motv_s0b1.
exe.

*regression -- use motv_s0b1 -- will get simple regression line for starters.
*use numsessions_cen  – will get group comparison at mean=0.
REGRESSION
 /STATISTICS COEFF OUTS R ANOVA
 /DEPENDENT tacticalerrors
 /METHOD=ENTER numsessions_cen   motv_s0b1   numsesscen_motvs0b1_int
```

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.979	.394		10.093	.000
	numsessions_cen	-.670	.155	-.821	-4.311	.000
	motv_s0b1	1.591	.558	.378	2.853	.008
	numsesscen_motvs0b1_int	.434	.216	.382	2.005	.055

a. Dependent Variable: tacticalerrors

For the Starters, the regression slope is -.670 (just like we expected from the plotting xls), which is statistically significant.

Notice that the group comparisons regression weight is the same value, but opposite sign, since we swapped 0 & 1 code values.

GLM: Getting the Model & Comparing Groups at 2 & 9 Sessions

*glm -- use motv_s1b2 (= to motv_s1b0 from reg) -- will get simple regression line for bench (=2).
*use numsessions_cen -- will get group comparison at mean=0.
*with numsessions mean centered what was 5.28125 (mean) is now 0
what was 2 is now (2 - 5.28125) = -3.28125
what was 9 is now (9 - 5.28125) = 3.71875.

UNIANOVA tacticalerrors   BY   motv_s1b2   WITH   numsessions_cen
 /METHOD=SSTYPE(3)
 /EMMEANS=TABLES(motv_s1b2) WITH(numsessions_cen = -3.28125) COMPARE (motv_s1b2)
 /EMMEANS=TABLES(motv_s1b2) WITH(numsessions_cen = 3.71875) COMPARE (motv_s1b2)
 /PRINT=DESCRIPTIVE PARAMETER
 /DESIGN= motv_s1b2   numsessions_cen   motv_s1b2*numsessions_cen.

EMMEANS – estimated marginal means command
        TABLES   – tells what grouping variable to use – will give you means of each group
        WITH   -- tells the covariate variable and the specific value at which to compare the groups
        COMPARE   -- requests significance test of the group means

DESIGN   -- this is where you tell GLM to compute the interaction

Descriptive Statistics

Dependent Variable: tacticalerrors

motv_s1b2	Mean	Std. Deviation	N
starters	4.0000	2.03306	16
bench	5.5625	1.99896	16
Total	4.7813	2.13624	32

These are the raw or uncorrected group means. The group means corrected for the covariate and the interaction that are tested in the ANCOVA model are shown down below.

Remember that SPSS dummy codes the grouping variable with the highest-valued group as the comparison group (Bench=2).

Tests of Between-Subjects Effects

Dependent Variable: tacticalerrors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	71.843ᵃ	3	23.948	9.631	.000
Intercept	729.350	1	729.350	293.307	.000
motv_s1b2	20.242	1	20.242	8.140	.008
numsessions_cen	43.532	1	43.532	17.506	.000
motv_s1b2 * numsessions_cen	9.996	1	9.996	4.020	.055
Error	69.626	28	2.487		
Total	873.000	32			
Corrected Total	141.469	31			

a. R Squared = .508 (Adjusted R Squared = .455)

SPSS computes the interaction codes as the product of the dummy code of the grouping variable and of the centered continuous variable -- *you must remember to use the centered version of the quant variable when you submit the analysis.*

These "b" values are the same as from the regression analysis. GLM does not give "beta" values. However they can be calculated.
$$\beta = (b * Std_{pred}) / Std_{crit}$$

Parameter Estimates

Dependent Variable: tacticalerrors

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Intercept	5.570	.394	14.128	.000	4.762	6.377
[motv_s1b2=1.00]	-1.591	.558	-2.853	.008	-2.733	-.449
[motv_s1b2=2.00]	0ᵃ	.	.	.	.	.
numsessions_cen	-.236	.151	-1.565	.129	-.545	.073
[motv_s1b2=1.00] * numsessions_cen	-.434	.216	-2.005	.055	-.877	.009
[motv_s1b2=2.00] * numsessions_cen	0ᵃ	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

Using GLM in this way provides an F-test for each specific "effect" in the model.

The $F_{interaction} = t^2$ from the interaction regression weight.

The $F_{motv} = t^2$ from the motv group regression weight

The $F_{sessions} \neq t^2$ from the sessions regression weight !!!

Why? GLM uses dummy coding (0,1) to compute the regression b-weight & t-test for motv, but it uses effect coding (-.5, .5) to compute the SS & F-test for motv.

When motv is dummy coded, the regression weight in the parameter estimates for sessions tells the simple effect slope of the regression line for those in the Bench group (coded 0 – which was "2" in the original variable).

But, when motv is effect coded, -.5 & .5 as in the ANOVA table, the quant variable is testing the slope of the regression line for those with grp = 0 (which is no one because of effects coding). However grp=0 does represent the two groups "on average," and so, this F tests the "main effect" slope of the DV-cov_c regression line (i.e., "on average" for those coded -.5 & .5). Looking back at the plot of the model, we see that the "average" the two regression lines would likely have a significant negative slope.

To summarize:
- ANOVA F-tests are of the main effects & interaction
- Regression t-tests are of simple effects & interaction

▶ Estimated Marginal Means

1. motv_s1b2

Estimates

Dependent Variable: tacticalerrors

motv_s1b2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
starters	6.177[a]	.641	4.865	7.489
bench	6.344[a]	.636	5.041	7.647

a. Covariates appearing in the model are evaluated at the following values: numsessions_cen = -3.28.

Pairwise Comparisons

Dependent Variable: tacticalerrors

(I) motv_s1b2	(J) motv_s1b2	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
starters	bench	-.167	.903	.855	-2.016	1.683
bench	starters	.167	.903	.855	-1.683	2.016

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: tacticalerrors

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	.085	1	.085	.034	.855
Error	69.626	28	2.487		

The F tests the effect of motv_s1b2. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

2. motv_s1b2

Estimates

Dependent Variable: tacticalerrors

motv_s1b2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
starters	1.488[a]	.704	.047	2.929
bench	4.693[a]	.681	3.297	6.089

a. Covariates appearing in the model are evaluated at the following values: numsessions_cen = 3.72.

Pairwise Comparisons

Dependent Variable: tacticalerrors

(I) motv_s1b2	(J) motv_s1b2	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
starters	bench	-3.205*	.979	.003	-5.211	-1.199
bench	starters	3.205*	.979	.003	1.199	5.211

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: tacticalerrors

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	26.625	1	26.625	10.707	.003
Error	69.626	28	2.487		

The F tests the effect of motv_s1b2. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Here are the adjusted (estimated) group means when numsessions_cen is held constant at -3.28, which corresponds to 2 sessions.

Here is the significance test of the mean differences – notice that the mean difference is the same -.167 that was obtained as the regression weight for the dummy code comparing the groups, when number of sessions was centered at 2.

This gives the F-test of the corrected/adjusted/estimated mean differences at this specific value of number of sessions.

Here are the adjusted (estimated) group means when numsessions_cen is held constant at 3.72 which corresponds to 9 sessions.

Here is the significance test of the mean differences – notice that the mean difference is the same -3.205 that was obtained as the regression weight for the dummy code comparing the groups, when number of sessions was centered at 9.

This gives the F-test of the corrected/adjusted/estimated mean differences at this specific value of number of sessions.

Same GLM Model - but testing Performance-#sessions regression for Starters

*recoding original grouping variable with starters as the comparison group (=2).
*remember to make value label for this new version of variable – is used in GLM output.
*centering the original quant variable (again- just to keep it around).
if (motv_s1b2 = 1) motv_s2b1 = 2.
if (motv_s1b2 = 2) motv_s2b1 = 1.
compute numsessions_cen = numsessions - 5.28125.

*glm -- use motv_s2b1 (= to motv_s0b1 from reg) -- will get simple regression line for starters (=2).
*use numsessions_cen -- will get group comparison at mean=0.
UNIANOVA tacticalerrors   BY   motv_s2b1   WITH   numsessions_cen
 /METHOD=SSTYPE(3)
 /PRINT=DESCRIPTIVE PARAMETER
 /DESIGN= motv_s2b1   numsessions_cen   motv_s2b1*numsessions_cen.

Parameter Estimates

Dependent Variable: tacticalerrors

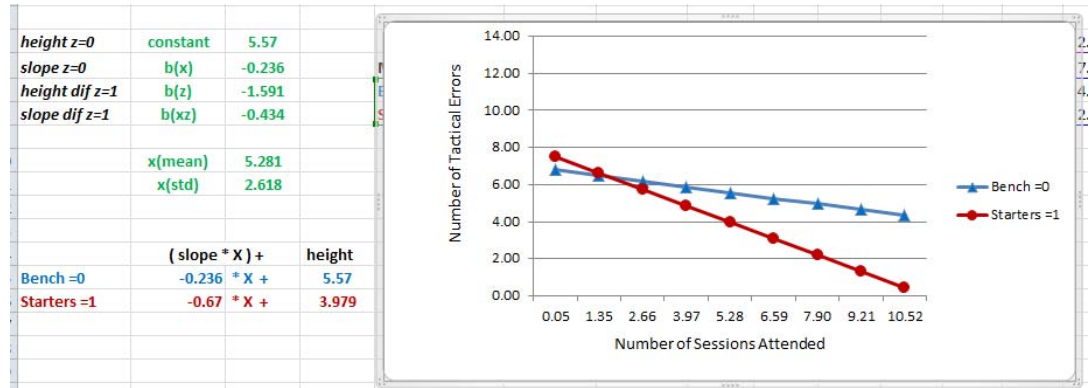| Parameter	B	Std. Error	t	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Intercept	3.979	.394	10.093	.000	3.171	4.787
[motv_s2b1=1.00]	1.591	.558	2.853	.008	.449	2.733
[motv_s2b1=2.00]	0ᵃ	.	.	.	.	.
numsessions_cen	-.670	.155	-4.311	.000	-.988	-.352
[motv_s2b1=1.00] * numsessions_cen	.434	.216	2.005	.055	-.009	.877
[motv_s2b1=2.00] * numsessions_cen	0ᵃ	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

The only difference between this model from the last is the change in the reference group (now starters =2).

This produces the following changes in the regression weights
* The regression weight & significance test for numsessions_cen now refers to the slope of the tactical error-#sessions regression line for the starters ← **our reason for running this analysis !!**
* The constant changes, because it now refers to the expected value for the starters with the average amount of practice ← **our reason for running this analysis !!**
* The sign of the group comparison regression weight will switch, because it is comparing the same group differences, at the same (mean=0) value of #sessions, but with starters (=2) as the reference/comparison group
* The sign of the interaction regression weight will switch, because it is comparing the same two slopes, but with starters (=2) as the reference/comparison group

What have we learned from these analyses of these data?

height z=0	constant	5.57
slope z=0	b(x)	-0.236
height dif z=1	b(z)	-1.591
slope dif z=1	b(xz)	-0.434
	x(mean)	5.281
	x(std)	2.618
	(slope * X) +	height
Bench =0	-0.236 *X +	5.57
Starters =1	-0.67 *X +	3.979

Overall model

- The model has R^2 = .455, with $F(3, 28)$ = 9.631, p < .001, MSe = 2.487

Interaction
- There is a (marginally) significant interaction of number of sessions and motivational label they relate to number of tactical errors committed, $F(1, 28)$ = 4.020, p = .055, MSe = 2.487
- **Describing the interaction as the slope difference for the 2 groups**
  - The tactical error-number of sessions regression slope is non-significant for Bench players, b = -.236, t = -1.565, p = .129
  - The tactical error-number of sessions regression slope is negative for Starters, b = -.670, t = -4.311, p , .001
- **Describing the interaction as the group difference at different #sessions**
  - At 2 sessions, the starters (6.177) had an equivalent average number of errors as the bench players (6.344), p = .855
  - At 9 sessions, the starters (1.488) had fewer average errors than the bench players (4.693), p = .003

Number of sessions
- There is an overall effect for number of sessions, $F(1, 28)$ = 17.506, p < .001, MSe = 2.487.
- However, the differential pattern of the simple regression slopes of number of sessions for the two motivational groups means that the overall pattern is misleading

Motivational Labeling
- There is an overall effect of labeling, $F(1, 28)$ = 8.140, p = .008, MSe = 2.487
- However, the differential pattern of the simple group differences for different numbers of sessions means that the overall pattern is misleading