2xQ Quadratic Models: Using GLM and Multiple Regression for Quadratic Models Including Interactions

Note: Analyses of the linear model for these data are in another handout that should be linked from the same website as this handout.

The data come from a sports psychology study of the motivational effects of labeling. During a week-long basketball camp for aspiring college players, there was an optional "tactical training" session that everyone took once, and was optional for the other nine offerings. Before the first session, the clinic coaches had identified who were the "starters" and who were the "bench players" among the attendees. For the first drill of each session, players were divided into the "starters" and the "bench players" – this was the labeling manipulation. The research question was, "How would this label influence tactical learning?"

GLM: Getting the Quadratic Model & Comparing Groups at 2, 5 & 9 Sessions

We need to compute the quadratic term as the square of the mean-centered number of sessions.

```
*getting the squared-centered quadratic term.
compute numsescen_sq = numsessions_cen**2.


*glm quadratic model -- use motv_s1b2 (= to motv_s1b0 from reg) -- will get simple regression line for bench (=2).
*use numsessions_cen -- will get group comparison at mean=0.
*with numsessions mean centered what was 5.28125 (mean) is now 0
what was 2  is now (2 - 5.28125) = -3.28125
what was 5  is now (5 - 5.28125) = -.28125
what was 9 is now (9 - 5.28125) = 3.71875.
UNIANOVA tacticalerrors   BY   motv_s1b2   WITH   numsessions_cen   numsescen_sq
  /METHOD=SSTYPE(3)
  /EMMEANS=TABLES(motv_s1b2) WITH(numsessions_cen = -3.28125 numsescen_sq = 10.7666) COMPARE (motv_s1b2)
  /EMMEANS=TABLES(motv_s1b2) WITH(numsessions_cen = -.28125   numsescen_sq = .07910)  COMPARE (motv_s1b2)
  /EMMEANS=TABLES(motv_s1b2) WITH(numsessions_cen = 3.71875  numsescen_sq = 13.8291) COMPARE (motv_s1b2)
  /PRINT=DESCRIPTIVE PARAMETER
  /DESIGN= motv_s1b2    numsessions_cen    numsescen_sq    motv_s1b2*numsessions_cen    motv_s1b2*numsescen_sq.
```

EMMEANS – estimated marginal means command

- TABLES – tells what grouping variable to use – will give you means of each group
- WITH -- tells the covariate variable and the specific value at which to compare the groups
  for the quadratic model this must also specify to control the quadratic term at the square of the value for which the centered quantitative variable is being controlled
- COMPARE -- requests significance test of the group means

DESIGN -- this is where you tell GLM to compute the interaction

The one limitation of the GLM procedure is that you can't get the regression slope and significance test of it for both groups in the same analysis. To get that weight and significance test for the Starters, you would need to compute a "motv_s2b1" variable and use it instead of the "motv_s1b2" variable. All of the model would be parallel, but you'd get that one new bit of info.

Tests of Between-Subjects Effects

Dependent Variable: tacticalerrors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	104.634[a]	5	20.927	14.771	.000
Intercept	277.771	1	277.771	196.066	.000
motv_s1b2	.485	1	.485	.342	.564
numsessions_cen	58.628	1	58.628	41.383	.000
numsescen_sq	16.600	1	16.600	11.717	.002
motv_s1b2 * numsessions_cen	3.633	1	3.633	2.564	.121
motv_s1b2 * numsescen_sq	12.898	1	12.898	9.104	.006
Error	36.835	26	1.417		
Total	873.000	32			
Corrected Total	141.469	31			

a. R Squared = .740 (Adjusted R Squared = .690)

This gives a very different picture of the relationships between number of sessions attended, whether players were identified as "starters" or "bench players" and their learning of tactical strategy!!

Parameter Estimates

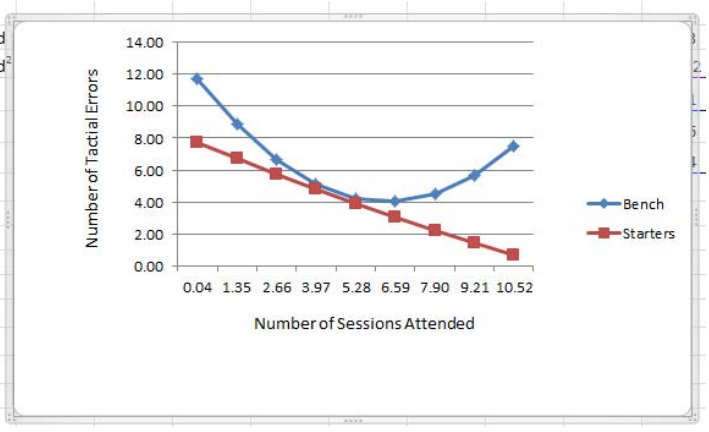
Dependent Variable: tacticalerrors

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Intercept	4.240	.407	10.430	.000	3.404	5.075
[motv_s1b2=1.00]	-.340	.581	-.585	.564	-1.535	.855
[motv_s1b2=2.00]	0[a]	.	.	.	.	.
numsessions_cen	-.405	.119	-3.400	.002	-.650	-.160
numsescen_sq	.195	.041	4.803	.000	.112	.279
[motv_s1b2=1.00] * numsessions_cen	-.268	.168	-1.601	.121	-.613	.076
[motv_s1b2=2.00] * numsessions_cen	0[a]	.	.	.	.	.
[motv_s1b2=1.00] * numsescen_sq	-.183	.061	-3.017	.006	-.308	-.058
[motv_s1b2=2.00] * numsescen_sq	0[a]	.	.	.	.	.

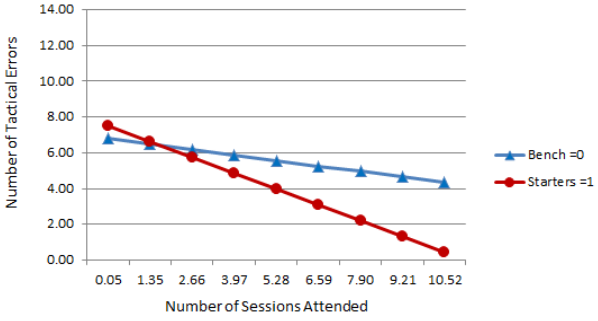
a. This parameter is set to zero because it is redundant.

Getting the simple regression models and plot for the quadratic model

height z=0	constant	4.24			x-centered
slope z=0	b(x)	-0.405			x-centered^2
curve z=0	b(x^2)	0.195		Practice	*z wt*
height dif z=1	b(z)	-0.34		Bench	*0*
slope dif z=1	b(xz)	-0.268		Starters	*1*
curve dif z=1	b(x^2z)	-0.183			
	x(mean)	5.281			
	x(std)	2.618			
	(slope * X) +		(curve * X^2) +		height
Bench	-0.405	* X +	0.195	* X^2 +	4.24
Starters	-0.673	* X +	0.012	* X^2 +	3.9



The linear model – for comparison…



Here are the results from the group comparisons at specific numbers of sessions from this model.

1. motv_s1b2

A much bigger group difference at 2 sessions than was obtained from the linear model.

Estimates

Dependent Variable: tacticalerrors

motv_s1b2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
starters	6.241[a]	.537	5.137	7.346
bench	7.670[a]	.554	6.531	8.809

a. Covariates appearing in the model are evaluated at the following values: numsessions_cen = -3.28, numsescen_sq = 10.77.

Pairwise Comparisons

Dependent Variable: tacticalerrors

(I) motv_s1b2	(J) motv_s1b2	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
starters	bench	-1.429	.772	.076	-3.015	.158
bench	starters	1.429	.772	.076	-.158	3.015

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: tacticalerrors

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	4.855	1	4.855	3.427	.076
Error	36.835	26	1.417		

The F tests the effect of motv_s1b2. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

2. motv_s1b2

No group difference at 5 sessions.

Estimates

Dependent Variable: tacticalerrors

motv_s1b2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
starters	4.090[a]	.412	3.244	4.936
bench	4.369[a]	.399	3.548	5.190

a. Covariates appearing in the model are evaluated at the following values: numsessions_cen = -.28, numsescen_sq = .08.

Pairwise Comparisons

Dependent Variable: tacticalerrors

(I) motv_s1b2	(J) motv_s1b2	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
starters	bench	-.279	.573	.631	-1.458	.900
bench	starters	.279	.573	.631	-.900	1.458

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: tacticalerrors

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	.335	1	.335	.237	.631
Error	36.835	26	1.417		

The F tests the effect of motv_s1b2. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Estimates

Dependent Variable: tacticalerrors

motv_s1b2	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
starters	1.566[a]	.603	.326	2.806
bench	5.434[a]	.537	4.330	6.538

a. Covariates appearing in the model are evaluated at the following values: numsessions_cen = 3.72, numsescen_sq = 13.83.

Pairwise Comparisons

Dependent Variable: tacticalerrors

(I) motv_s1b2	(J) motv_s1b2	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
starters	bench	-3.868*	.808	.000	-5.528	-2.207
bench	starters	3.868*	.808	.000	2.207	5.528

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: tacticalerrors

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	32.481	1	32.481	22.927	.000
Error	36.835	26	1.417		

The F tests the effect of motv_s1b2. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

A much bigger group difference at 9 sessions than was obtained from the linear model.

Same Quadratic Model - but testing Performance-#sessions Slope & Curve for Starters

*recoding original grouping variable with starters as the comparison group (=2).
*remember to make value label for this new version of variable – is used in GLM output.
*centering the original quant variable (again- just to keep it around).
if (motv_s1b2 = 1) motv_s2b1 = 2.
if (motv_s1b2 = 2) motv_s2b1 = 1.
compute numsessions_cen = numsessions - 5.28125.

```
UNIANOVA tacticalerrors   BY   motv_s2b1   WITH   numsessions_cen   numsescen_sq
  /METHOD=SSTYPE(3)
  /PRINT=DESCRIPTIVE PARAMETER
  /DESIGN= motv_s1b2    numsessions_cen    numsescen_sq    motv_s2b1*numsessions_cen    motv_s2b1*numsescen_sq.
```

Parameter Estimates

Dependent Variable: tacticalerrors

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Intercept	3.900	.416	9.385	.000	3.046	4.754
[motv_s2b1=1.00]	.340	.581	.585	.564	-.855	1.535
[motv_s2b1=2.00]	0[a]	.	.	.	.	.
numsessions_cen	-.673	.118	-5.709	.000	-.916	-.431
numsescen_sq	.012	.045	.273	.787	-.080	.105
[motv_s2b1=1.00] * numsessions_cen	.268	.168	1.601	.121	-.076	.613
[motv_s2b1=2.00] * numsessions_cen	0[a]	.	.	.	.	.
[motv_s2b1=1.00] * numsescen_sq	.183	.061	3.017	.006	.058	.308
[motv_s2b1=2.00] * numsescen_sq	0[a]	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

The only difference between this model from the last is the change in the reference group (now starters =2).

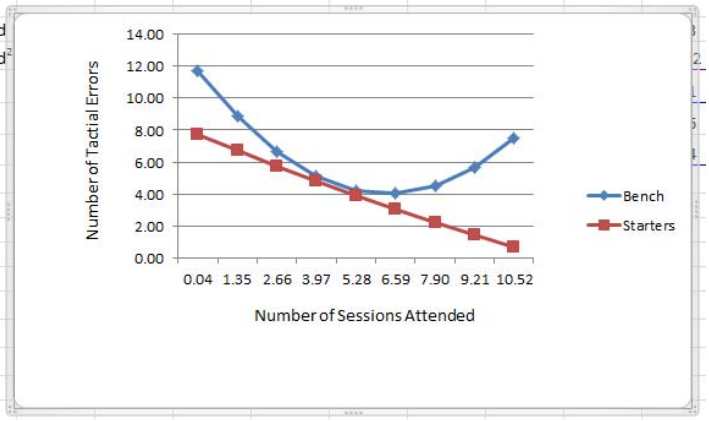
This produces the following changes in the regression weights

- The regression weight & significance test for numsessions_cen now refers to the slope of the tactical error-#sessions regression line for the starters **← our reason for running this analysis !!**
- The regression weight & significance test for numsescen_sq now refers to the curve of the tactical error-#sessions regression line for the starters **← our reason for running this analysis !!**
- The constant changes, because it now refers to the expected value for the starters with the average amount of practice **← our reason for running this analysis !!**
- The sign of the group comparison regression weight will switch, because it is comparing the same group differences, at the same (mean=0) value of #sessions, but with starters (=2) as the reference/comparison group
- The sign of the linear interaction regression weight will switch, because it is comparing the same two slopes, but with starters (=2) as the reference/comparison group
- The sign of the quadratic interaction regression weight will switch, because it is comparing the same two curves, but with starters (=2) as the reference/comparison group

What have we learned from these analyses of these data?

height z=0	constant	4.24		x-centered
slope z=0	b(x)	-0.405		x-centered^2
curve z=0	b(x^2)	0.195	Practice	*z wt*
height dif z=1	b(z)	-0.34	Bench	*0*
slope dif z=1	b(xz)	-0.268	Starters	*1*
curve dif z=1	b(x^2z)	-0.183		
	x(mean)	5.281		
	x(std)	2.618		

	(slope * X) +	(curve * X^2) +	height
Bench	-0.405 * X +	0.195 * X^2 +	4.24
Starters	-0.673 * X +	0.012 * X^2 +	3.9



Overall model

- The model has R^2 = .74, with F(5, 26) = 14.771 p < .001, MSe = 1.417

Interaction

- There is a non-significant linear interaction of number of sessions and motivational label they relate to number of tactical errors committed, F(1, 26) = 2.564, p = .121, MSe = 1.417
- There is a significant quadratic interaction of number of sessions and motivational label they relate to number of tactical errors committed, F(1, 26) = 9.104, p = .006, MSe = 1.417

- **Describing the interaction as the regression difference for the 2 groups**
  - The tactical error-number of sessions linear regression slope is negative for Bench players, b = -.405, t = -3.4, p = .002 & the there is a positive quadratic for this group, b = .195, t = 4.803, p < .001
  - The tactical error-number of sessions linear regression slope is negative for Starters, b = -.673, t = -5.709, p , .001 & there is no quadratic curve for this group, b = .012, t = .273, p = 787
  - From the non-significant linear interaction term, we know that the slope for the Starters is “not significantly more negative” than for the Bench players
  - From the significant quadratic interaction term, we know that the regression line for the Starters is “significantly more curved” than for the Bench players
- **Describing the interaction as the group difference at different #sessions**
  - At 2 sessions, the starters (6.241) had marginally fewer errors than the bench players (7.670), p = .076
  - At 5 sessions, the starters (4.090) had an equivalent average number of errors as the bench players (4.369), p = .631
  - At 9 sessions, the starters (1.566) had fewer average errors than the bench players (5.434), p < .001

Number of sessions

- There is an overall linear effect for number of sessions, F(1, 26) = 41.383, p < .001, MSe = 1.417
- There is an overall quadratic effect for number of sessions, F(1,26) = 11.717, o = .002, MSe = 1.417
- However, the differential pattern of the simple regression slopes of number of sessions for the two motivational groups means that the overall shape of this relationship is misleading

Motivational Labeling

- There is no overall effect of labeling, F(1, 26) = .342, p = .564, MSe = 1.417
- However, the differential pattern of the simple group differences for different numbers of sessions means that the overall pattern is misleading

Multiple Regression: Getting the Quadratic Model & Comparing Groups at 2, 5 & 9 Sessions

We need to compute several additional terms to obtain this model using multiple regression!

Comparing the groups at 2 sessions (with bench as the comparison group).

```
if (motv_s1b2 = 1) motv_s1b0 = 1.
if (motv_s1b2 = 2) motv_s1b0 = 0.

compute numsessions_2cen = numsessions – 2.
compute numses2cen_sq = numsessions_2cen **2.

compute numsess2cen_motvs1b0_int = motv_s1b0 * numsessions_2cen.
compute numsess2censq_motvs1b0_int = motv_s1b0 * numses2cen_sq.
```

Comparing the groups at 5 sessions (with bench as the comparison group).

```
if (motv_s1b2 = 1) motv_s1b0 = 1.
if (motv_s1b2 = 2) motv_s1b0 = 0.

compute numsessions_5cen = numsessions – 5.
compute numses5cen_sq = numsessions_5cen **2.

compute numsess5cen_motvs1b0_int = motv_s1b0 * numsessions_5cen.
compute numsess5censq_motvs1b0_int = motv_s1b0 * numses5cen_sq.
```

Comparing the groups at 9 sessions (with bench as the comparison group).

```
if (motv_s1b2 = 1) motv_s1b0 = 1.
if (motv_s1b2 = 2) motv_s1b0 = 0.

compute numsessions_9cen = numsessions – 9.
compute numses9cen_sq = numsessions_9cen **2.

compute numsess9cen_motvs1b0_int = motv_s1b0 * numsessions_9cen.
compute numsess9censq_motvs1b0_int = motv_s1b0 * numses9cen_sq.
```

Getting the regression slope & curve for the Starters (centering at 5 – any centering will get the same slope & curve**)**

```
if (motv_s1b2 = 1) motv_s2b1 = 2.
if (motv_s1b2 = 2) motv_s2b1 = 1.

compute numsessions_5cen = numsessions – 5.
compute numses5cen_sq = numsessions_5cen **2.

compute numsess5cen_motv s1b0_int = motv_s1b0 * numsessions_5cen.
compute numsess5censq_motv s1b0_int = motv_s1b0 * numses5cen_sq.
```