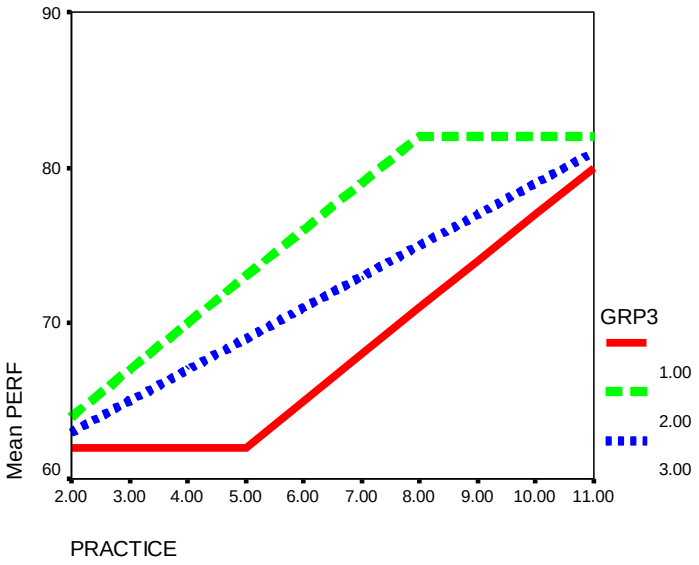
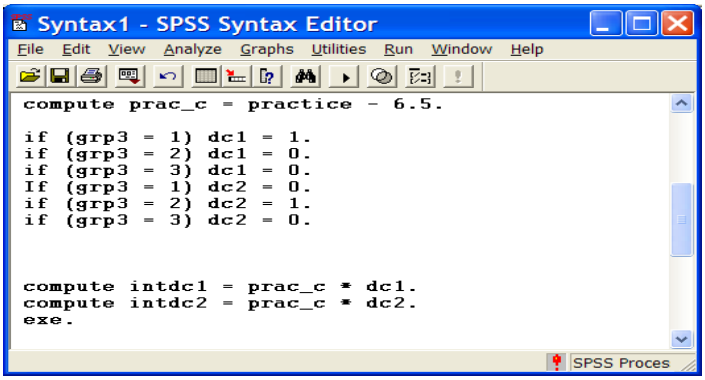
kxQ Models: Non-linear Model Example

Here are data from a 3-group design in which participants were assigned to three different feedback conditions (1 = intermittent feed, 2 = continuous feedback, 3 = corrective feedback) and completed an assigned number of practices with that type of feedback before performance testing.

Two of the groups show a quadratic component to their practice-performance function.

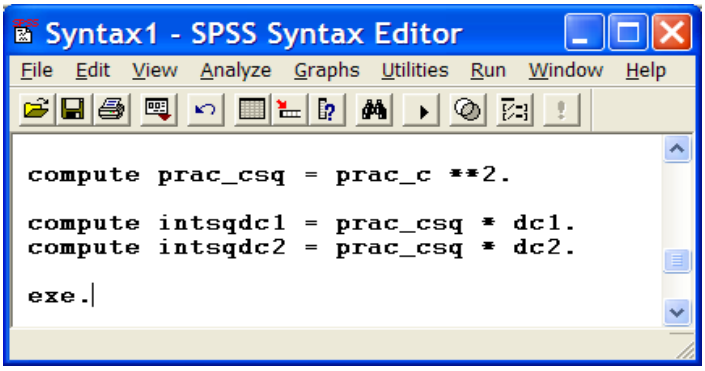
Below are analyses of the relationship between #practice, feedback type and their interaction with performance -- with and without the quadratic component.





Coding needed to run the linear model includes:

- Centering the quantitative practice (X) variable (mean = 6.5, std = 2.89)
- Dummy coding the grouping variable (here the highest-coded group – corrective feedback - was set as the comparison group)
- Interaction term computed as the product of each dummy code and the centered quantitative variable



Additional coding needed for the quadratic model includes:

- Quadratic term computed as the square of the centered practice (X) variable (nonlinear main effect)
- Quadratic interaction terms computed as the product of each dummy code and the quadratic term (nonlinear interaction)

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*hierarchical model – linear terms first then add quadratic terms.
REGRESSION
  /STATISTICS COEFF R ANOVA CHANGE
  /DEPENDENT perf
  /METHOD=ENTER dc1  dc2  prac_c   intdc1   intdc2
  /METHOD-ENTER  pract_csq  intsqdc1   intsqdc2.
```

Results from this model…

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.950[a]	.902	.895	2.42173	.902	118.288	5	64	.000
2	.974[b]	.949	.943	1.78765	.047	18.818	3	61	.000

a. Predictors: (Constant), INTDC2, INTDC1, DC2, DC1, PRAC_C

b. Predictors: (Constant), INTDC2, INTDC1, DC2, DC1, PRAC_C, PRAC_CSQ, INTSQDC1, INTSQDC2

ANOVA[c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3468.655	5	693.731	118.288	.000[a]
	Residual	375.345	64	5.865		
	Total	3844.000	69			
2	Regression	3649.064	8	456.133	142.734	.000[b]
	Residual	194.936	61	3.196		
	Total	3844.000	69			

a. Predictors: (Constant), INTDC2, INTDC1, DC2, DC1, PRAC_C

b. Predictors: (Constant), INTDC2, INTDC1, DC2, DC1, PRAC_C, PRAC_CSQ, INTSQDC1, INTSQDC2

c. Dependent Variable: PERF

Coefficients [a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	72.000	.766		94.017	.000
	DC1	-3.700	.884	-.247	-4.184	.000
	DC2	3.700	.884	.247	4.184	.000
	PRAC_C	2.000	.267	.775	7.501	.000
	INTDC1	.164	.308	.042	.532	.597
	INTDC2	.164	.308	.042	.532	.597
2	(Constant)	72.000	.855		84.183	.000
	DC1	-5.669	.988	-.379	-5.740	.000
	DC2	5.669	.988	.379	5.740	.000
	PRAC_C	2.000	.197	.775	10.162	.000
	INTDC1	.164	.227	.042	.720	.474
	INTDC2	.164	.227	.042	.720	.474
	PRAC_CSQ	9.122E-16	.078	.000	.000	1.000
	INTSQDC1	.239	.090	.202	2.656	.010
	INTSQDC2	-.239	.090	-.202	-2.656	.010

a. Dependent Variable: PERF

Model is

$$Perf' = \underbrace{b_0}_{\text{Constant}} + \underbrace{b1*DC_1 + b_2*DC_2}_{\text{group ht differences}} + \underbrace{b_3*prac\_c + b_4*prac\_csq}_{\text{slope \& curve}} + \underbrace{b_5*intdc1 + b_6*intdc1}_{\text{linear interaction}} + \underbrace{b_7*intsqdc1 + b_8*intsqdc2}_{\text{quadratic interaction}}$$

Reorganizing the regression model to show how the groups differ…

$$Perf' = \underbrace{b_0 + b_3*prac\_c + b_4*prac\_csq}_{\text{Ht, slp \& crv of Corrective FB}} + \underbrace{b1*DC_1 + b_5*intdc1 + b_7*intsqdc1}_{\text{Ht, slp \& crv dif of Intermittent from Corrective FB}} + \underbrace{b_2*DC_2 + b_6*intdc1 + b_8*intsqdc2}_{\text{Ht, slp \& crv dif of Conttinuous from Corrective FB}}$$

b_0 – constant – expected performance for those in comparison group with the mean (0) amount of practice

b_1 - the simple effect of intermittent vs. corrective feedback for the mean (0) amount of practice
- expected direction and extent of change in performance for those in the target group (intermittent) for that dummy code, compared to those in the comparison group (corrective), holding all other predictors constant at the value 0

b_2 - the simple effect of continuous vs. corrective feedback for the mean (0) amount of practice
- expected direction and extent of change in performance for those in the target group (continuous) for that dummy code, compared to those in the comparison group (corrective), holding all other predictors constant at the value 0

b_3 - the simple linear effect of practice for those in the comparison group (corrective feedback)
- expected direction and extent of change in performance for a 1-unit increase in practice holding all other predictors constant at 0

b_4 - simple quadratic effect of practice for those in the comparison group (corrective feedback)
- expected direction and extent of change in performance for a 1-unit change in performance, holding all other predictors constant at 0

b_5 - linear interaction - how the linear effect of practice for the target (intermittent feedback) differs from the linear effect of practice for the comparison group (corrective feedback)
- how the difference between target and comparison group performances changes for different amounts of practice
- expected direction and extent of change in effect of one predictor for a 1-unit increase in the value of the other predictor, holding all other predictors constant at 0, for the involved conditions of the categorical variable

b_6 - linear interaction - how the linear effect of practice for the target (continuous feedback) differs from the linear effect of practice for the comparison group (corrective feedback)
- how the difference between target and comparison group performances changes for different amounts of practice
- expected direction and extent of change in effect of one predictor for a 1-unit increase in the value of the other predictor, holding all other predictors constant at 0, for the involved conditions of the categorical variable

b_7 - quadratic interaction - how the quadratic effect of practice for the target (intermittent feedback) differs from quadratic effect of practice for the comparison group (corrective feedback)
- how how the difference between target and comparison group performances changes for different amounts of practice, for different amounts of practice
- difference in expected direction and extent of change in effect of one predictor for a 1-unit increase in the value of the other predictor, holding all the other predictors constant, for a 1-unit change in practice, for the involved conditions of the categorical variable

b_8 - quadratic interaction - how the quadratic effect of practice for the target (continuous feedback) differs for quadratic effect of practice for the comparison group (corrective feedback)
- how how the difference between target and comparison group performances changes for different amounts of practice, for different amounts of practice
- difference in expected direction and extent of change in effect of one predictor for a 1-unit increase in the value of the other predictor, holding all the other predictors constant, for a 1-unit change in practice, for the involved conditions of the categorical variable

Linear Model

This model, though it accounts for a significant 90% of the variance, doesn't much resemble the plot of the original data!

Interactions and nonlinear trends can both meaningfully change the model and the interpretation of the behavioral relationships, without always adding large incremental variance. Why?

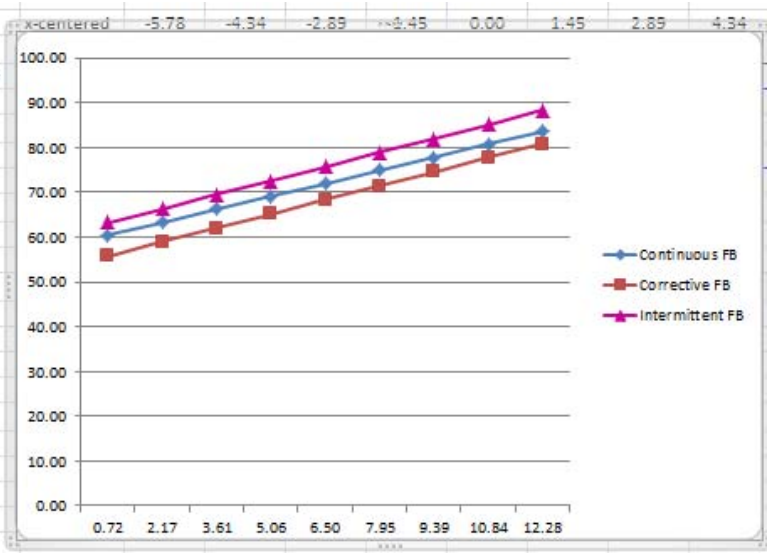
Much of the shape of the data pattern is well-fit by the linear model, but parts are more poorly fit. Notice that the linear and non-linear models make very similar predictions for

- participants in the Continuous FB condition (which has no nonlinear regression component)
- participants in the Intermitttent and Corrective conditions who practice fewer than 4 or more than 10 times are well-predictted by the linear model,

However, for between 5 and 9 practices, performance in the Intermitent condition will be underestimated by the linear model and performance in the Corrective conditon will be overestimated by the linear model.

height z1=0 z2=0	constant	72	Practice	*z1 wt*	*Z2 wt*
slope z1=0 z2=0	b(x)	2	Continuous FB	*0*	*0*
curve z1=0 z2=0	b(x^2)	0	Corrective FB	*1*	*0*
height dif z1=1 z2=0	b(z1)	-3.7	Intermittent FB	0	1
slope dif z1=1 z2=0	b(xz1)	0.164			
curve dif z1=1 z2=0	b(x^2z1)	0			
height dif z1=0 z2=1	b(z2)	3.7			
slope dif z1=0 z2=1	b(xz2)	0.164			
curve dif z1=0 z2=1	b(x^2z2)	0			
	x(mean)	6.5			
	x(std)	2.89			

	(slope * X)+		(curve * X^2)+	height
Continuous FB	2	* X +	0 * XX +	72
Corrective FB	2.164	* X +	0 * XX +	68.3
Intermittent FB	2.164	* X +	0 * XX +	75.7



Non-linear Model

height z1=0 z2=0	constant	72	Practice	*z1 wt*	*Z2 wt*
slope z1=0 z2=0	b(x)	2	Continuous FB	*0*	*0*
curve z1=0 z2=0	b(x^2)	0	Corrective FB	*1*	*0*
height dif z1=1 z2=0	b(z1)	-5.669	Intermittent FB	0	1
slope dif z1=1 z2=0	b(xz1)	0.164			
curve dif z1=1 z2=0	b(x^2z1)	0.239			
height dif z1=0 z2=1	b(z2)	5.669			
slope dif z1=0 z2=1	b(xz2)	0.164			
curve dif z1=0 z2=1	b(x^2z2)	-0.239			
	x(mean)	6.5			
	x(std)	2.89			

	(slope * X)+		(curve * X^2)+	height
Continuous FB	2	* X +	0 * XX +	72
Corrective FB	2.164	* X +	0.239 * XX +	66.331
Intermittent FB	2.164	* X +	-0.239 * XX +	77.669