Plotting Non-linear & Complex Main Effects Models

- Plotting single-predictor non-linear models
- Plotting 2-predictor non-linear models – 2xQ, kxQ & QxQ
- Plotting complex non-linear main effects models

Models with a single centered quantitative predictor

$$X_{cen} = X – X_{mean}$$

$$y' = bX_{cen} + a$$

a → regression constant

- expected value of y when x = 0 (re-centered to mean=0)
- height of y-x regression line

b → regression weight

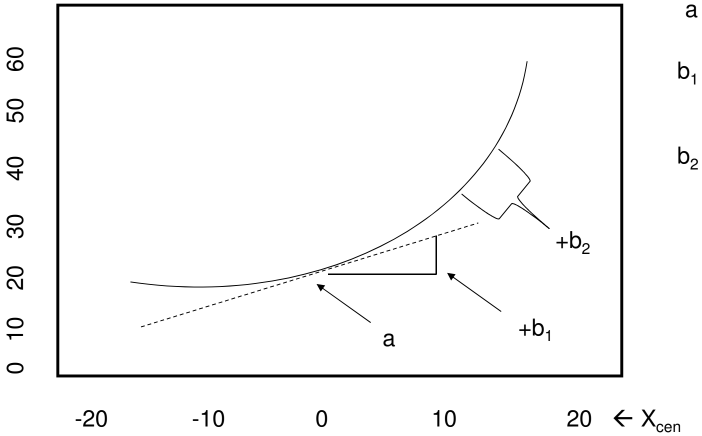
- expected direction and extent of change in y for a 1-unit increase in x
- slope of y-x regression line

X will be X_{cen} in all of the following models

Graphing & Interpreting
Non-linear Models with a single centered quantitative predictor

$X_{cen} = X – X_{mean}$ $\quad$ $X_{cen}^2 = (X – X_{mean})^2$

$y' = b_1X_{cen} + b_2X_{cen}^{2+}a$



a = ht of line

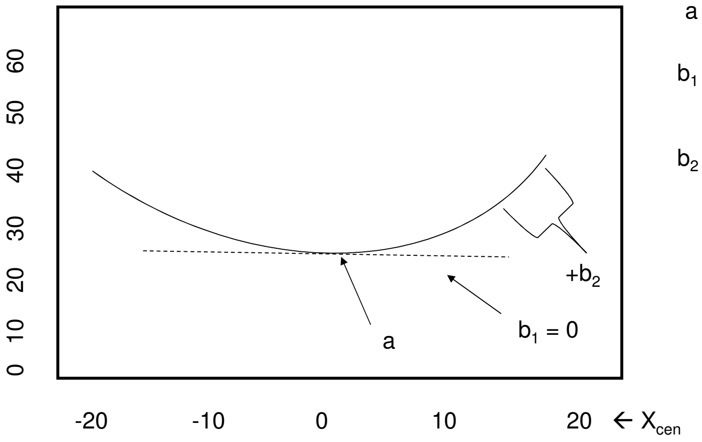
b_1 = slp of line at X=0

b_2 = curve of line

Graphing & Interpreting
Non-linear Models with a single centered quantitative predictor

$X_{cen} = X - X_{mean}$ $\qquad$ $X_{cen}^2 = (X - X_{mean})^2$

$y' = b_1X_{cen} + b_2X_{cen}^2 + a$



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Graphing & Interpreting
Non-linear Models with a single centered quantitative predictor

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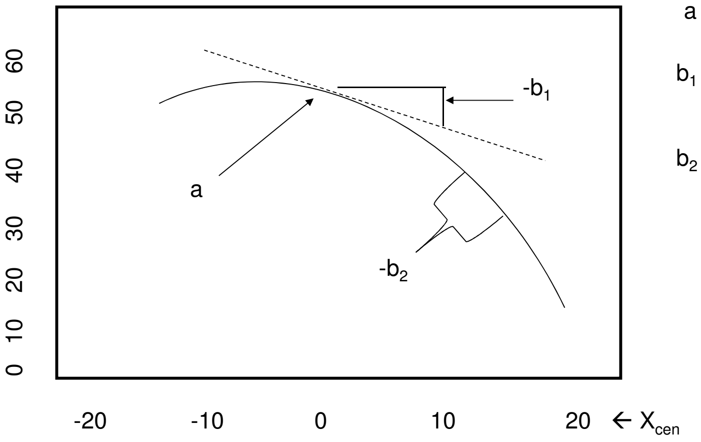


a = ht of line

b_1 = slp of line at X=0

b_2 = curve of line

Plotting 2-Predictor Non-linear Models

Now we get to the funer part – plotting multiple regression equations involving multiple variables.

Three kinds (for now, more later) …

- centered quant variable & dummy-coded binary variable
- centered quant variable & dummy-coded k-category variable
- 2 centered quant variables

What we’re trying to do is to plot these models so that we can see how both of the predictors are related to the criterion.

Like when we’re plotting data from a factorial design, we have to represent 3 variables -- the criterion & the 2 predictors X & Z -- in a 2-dimensional plot. We’ll use the same solution..

We’ll plot the relationship between one predictor and the criterion for different values of the other predictor

Non-linear Models with a centered quantitative predictor & a dummy coded binary predictor

$$y' = b_1X + b_2X^2 + b_3Z + a$$

This is called a main effects model → there are no interaction terms.

a → regression constant

- expected value of y if X=0 (mean) and Z=0 (comparison group)
- mean of the control group
- height of control group quant-criterion regression line

b_1 → regression weight for centered quant predictor – linear main effect of X

- expected direction and extent of change in y for a 1-unit increase in x , after controlling for the other variable(s) in the model
- expected direction and extent of change in y for a 1-unit increase in x , for the comparison group (coded 0)
- slope of quant-criterion regression line for the group coded 0 (comp)

Non-linear Models with a centered quantitative predictor & a dummy coded binary predictor

$$y' = b_1X + b_2X^2 + b_3Z + a$$

This is called a main effects model → there are no interaction terms.

b_2 → weight for centered & squared quant predictor – non-linear main effect of X

- expected direction and extent of change in y-x slope for a 1-unit increase in x , after controlling for the other variable(s) in the model
- expected direction and extent of change in y-x slope for a 1-unit increase in x , for the comparison group (coded 0)
- curve of quant-criterion regression line for the group coded 0 (comp)

b_3 → regression weight for dummy coded binary predictor – main effect of Z

- expected direction and extent of change in y for a 1-unit increase in x, after controlling for the other variable(s) in the model
- direction and extent of y mean difference between groups coded 0 & 1, after controlling for the other variable(s) in the model
- group mean/reg line height difference (when X = 0, the centered mean)

To plot the model we need to get separate regression formulas for each Z group. We start with the multiple regression model…

Model → $y' = b_1X + b_2X^2 + b_3Z + a$

For the Comparison Group coded Z = 0

Substitute the 0 in for Z
Simplify the formula

$$y' = b_1X + b_2X^2 + b_30 + a$$

$$y' = \underset{\text{slope}}{b_1}X + \underset{\text{curve}}{b_2}X^2 + \underset{\text{height}}{a}$$

For the Target Group coded Z = 1

Substitute the 1 in for Z
Simplify the formula

$$y' = b_1X + b_2X^2 + b_31 + a$$

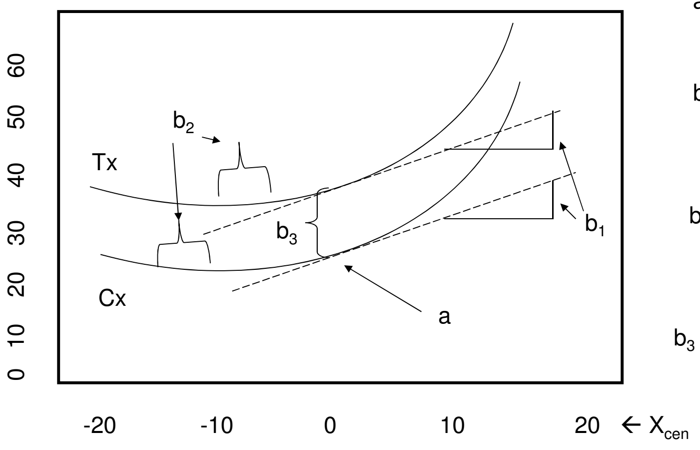
$$y' = \underset{\text{slope}}{b_1}X + \underset{\text{curve}}{b_2}X^2 + \underbrace{(b_3 + a)}_{\text{height}}$$

Plotting & Interpreting Non-linear Models
with a centered quantitative predictor & a dummy coded binary predictor

$$y' = b_1X + b_2X^2 + b_3Z + a$$

This is called a main effects model → no interaction → the regression lines are parallel.

$X_{cen} = X – X_{mean}$ $X_{cen}^2 = (X – X_{mean})^2$ Z = Tx(1) vs. Cx(0)



a = ht of Cx line
→ mean of Cx

b_1 = slp of Cx line
Cx slp = Tx slp
No interaction

b_2 = curve of Cx line
Cx curv = Tx curv
No interaction

b_3 = htdif Cx & Tx
→ Cx & Tx mean dif

Plotting & Interpreting Non-linear Models
with a centered quantitative predictor & a dummy coded binary predictor

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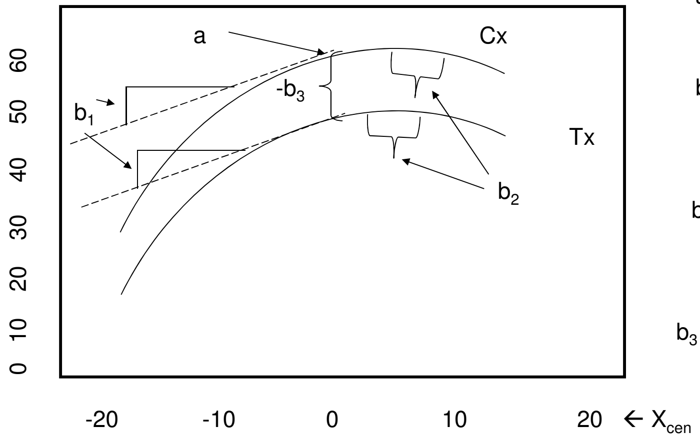

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→ mean of Cx

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Cx slp = Tx slp
No interaction

b_2 = curve of Cx line
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No interaction

b_3 = htdif Cx & Tx
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Plotting & Interpreting Non-linear Models
with a centered quantitative predictor & a dummy coded binary predictor

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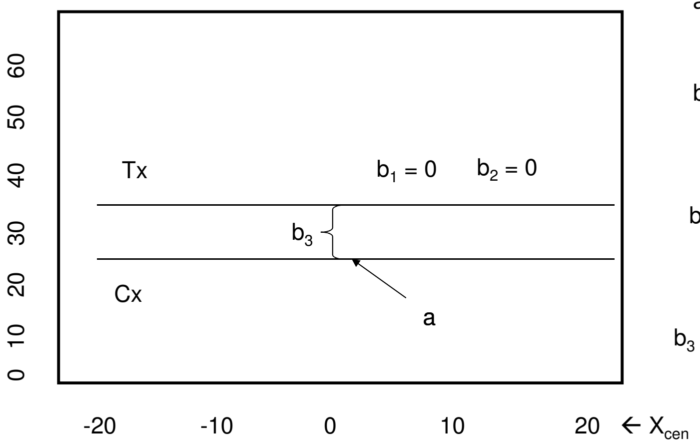


a = ht of Cx line
→ mean of Cx

b_1 = slp of Cx line
Cx slp = Tx slp
No interaction

b_2 = curve of Cx line
Cx curv = Tx curv
No interaction

b_3 = htdif Cx & Tx
→ Cx & Tx mean dif

Non-linear models with a centered quantitative predictor & a dummy coded k-category predictor

$$y' = b_1X + b_2X^2 + b_3 Z_1 + b_4 Z_2 + a$$

This is called a main effects model → there are no interaction terms.

Group	Z_1	Z_2
1	1	0
2	0	1
3*	0	0

a → regression constant
- expected value of y if Z_1 & $Z_2 = 0$ (the control group)
- mean of the control group
- height of control group quant-criterion regression line

b_1 → regression weight for centered quant predictor – main effect of X
- expected direction and extent of change in y for a 1-unit increase in x
- slope of quant-criterion regression line (for both groups)

b_2 → weight for centered & squared quant predictor – non-linear main effect of X
- expected direction and extent of change in y-x slope for a 1-unit increase in x , after controlling for the other variable(s) in the model
- expected direction and extent of change in y-x slope for a 1-unit increase in x , for the comparison group (coded 0)
- curve of quant-criterion regression line for the group coded 0 (comp)

Non-linear models with a centered quantitative predictor & a dummy coded k-category predictor

$$y' = b_1X + b_2X^2 + b_3 Z_1 + b_4 Z_2 + a$$

This is called a main effects model → there are no interaction terms.

Group	Z_1	Z_2
1	1	0
2	0	1
3*	0	0

b_3 → regression weight for dummy coded comparison of G1 vs G3 – main effect
- expected direction and extent of change in y for a 1-unit increase in x
- direction and extent of y mean difference between groups 1 & 3
- group height difference between comparison & target groups (3 & 1) (X = 0)

b_4 → regression weight for dummy coded comparison of G2 vs. G3 – main effect
- expected direction and extent of change in y for a 1-unit increase in x
- direction and extent of y mean difference between groups coded 0 & 1
- group height difference between comparison & target groups (3 & 2) (X = 0)

To plot the model we need to get separate regression formulas for each Z group. We start with the multiple regression model…

Model → $y' = b_1X + b_2X^2 + b_3 Z_1 + b_4 Z_2 + a$

Group	Z_1	Z_2
1	1	0
2	0	1
3*	0	0

For the Comparison Group - coded $Z_1 = 0$ & $Z_2 = 0$

Substitute the Z code values $y' = b1X + b_2X^2 + b_2{*}0 + b_3{*}0 + a$

Simplify the formula $y' = b_1X + b_2X^2 + a$

slope ↗ ↖ curve ← height

For the Target Group 1 - coded $Z_1 = 1$ & $Z_2 = 0$

Substitute the Z code values $y' = b1X + b_2X^2 + b_2{*}1 + b_3{*}0 + a$

Simplify the formula $y' = b_1X + b_2X^2 + \underbrace{(b_2 + a)}_{\text{height}}$

slope ↗ curve ↖

For the Target Group 2- coded $Z_1 = 0$ & $Z_2 = 1$

Substitute the Z code values $y' = b1X + b_2X^2 + b_2{*}0 + b_3{*}1 + a$

Simplify the formula $y' = b_1X + b_2X^2 + \underbrace{(b_3 + a)}_{\text{height}}$

slope ↗ curve ↖

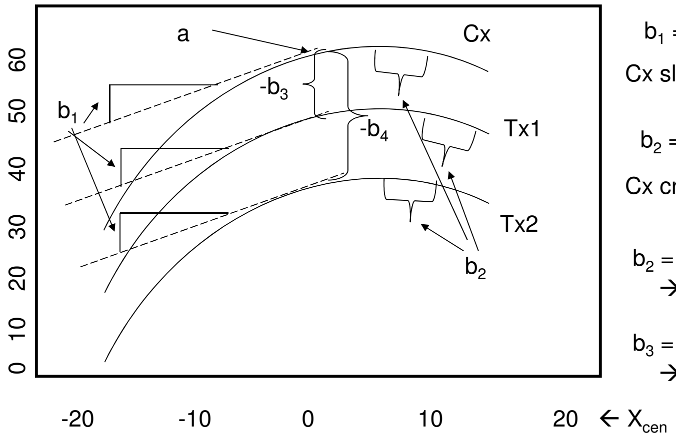
Plotting & Interpreting Non-linear Models with a centered quantitative predictor & a dummy coded k-category predictor

This is called a main effects model → no interaction → the regression lines are parallel.

$y' = b_1X + b_2X^2 + b_3 Z_1 + b_4 Z_2 + a$

Z_1 = Tx1 vs. Cx(0)

$X_{cen} = X - X_{mean}$ Z_2 = Tx2 vs. Cx (0)



a = ht of Cx line
→ mean of Cx

b_1 = slp of Cx line

Cx slp = Tx1 slp = Tx1 slp
No interaction

b_2 = curve of Cx line

Cx crv = Tx1 crv = Tx1 crv
No interaction

b_2 = htdif Cx & Tx_1
→ Cx & Tx_1 mean dif

b_3 = htdif Cx & Tx_2
→ Cx & Tx_2 mean dif

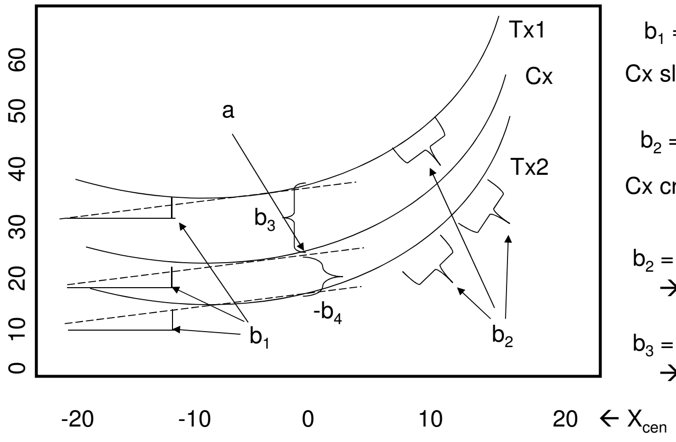
Plotting & Interpreting Non-linear Models with a centered quantitative predictor & a dummy coded k-category predictor

This is called a main effects model → no interaction → the regression lines are parallel.

$y' = b_1X + b_2X^2 + b_3 Z_1 + b_4 Z_2 + a$

Z_1 = Tx1 vs. Cx(0)

$X_{cen} = X - X_{mean}$ Z_2 = Tx2 vs. Cx (0)



a = ht of Cx line
→ mean of Cx

b_1 = slp of Cx line

Cx slp = Tx1 slp = Tx1 slp
No interaction

b_2 = curve of Cx line

Cx crv = Tx1 crv = Tx1 crv
No interaction

b_2 = htdif Cx & Tx_1
→ Cx & Tx_1 mean dif

b_3 = htdif Cx & Tx_2
→ Cx & Tx_2 mean dif

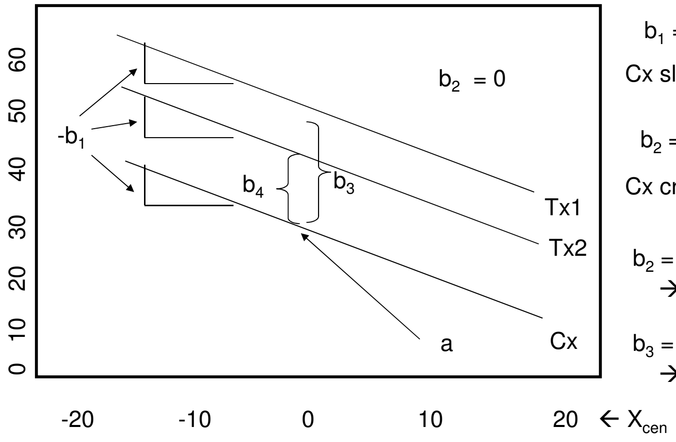
Plotting & Interpreting Non-linear Models with a centered quantitative predictor & a dummy coded k-category predictor

This is called a main effects model → no interaction → the regression lines are parallel.

$y' = b_1X + b_2X^2 + b_3 Z_1 + b_4 Z_2 + a$

Z_1 = Tx1 vs. Cx(0)

$X_{cen} = X - X_{mean}$ Z_2 = Tx2 vs. Cx (0)



a = ht of Cx line
→ mean of Cx

b_1 = slp of Cx line

Cx slp = Tx1 slp = Tx1 slp
No interaction

b_2 = curve of Cx line

Cx crv = Tx1 crv = Tx1 crv
No interaction

b_2 = htdif Cx & Tx_1
→ Cx & Tx_1 mean dif

b_3 = htdif Cx & Tx_2
→ Cx & Tx_2 mean dif

weights for non-linear main effects models w/ centered quantitative predictor **&** a dummy coded binary predictor **or** a dummy coded k-category predictor

Constant "a"

- the expected value of y when the value of all predictors = 0
- height of the Y-X regression line for the comparison group

b for a centered quantitative variable – main effect

- the direction and extent of the expected change in the value of y for a 1-unit increase in that predictor, holding the value of all other predictors constant at 0
- slope of the Y-X regression line for the comparison group

b for a centered & squared quantitative variable – main effect

- the direction and extent of the expected change in the y-x for a 1-unit increase in x, holding the value of all other predictors constant at 0
- curve of the Y-X regression line for the comparison group

b for a dummy coded binary variable -- main effect

- the direction and extent of expected mean difference of the Target group from the Comparison group, holding the value of all other predictors constant at 0
- difference in height of the Y-X regression lines for the comparison & target groups
- comparison & target groups Y-X regression lines have same slope – no interaction

b for a dummy coded k-group variable – main effect

- direction and extent of expected mean difference of Target group for that dummy code from Comparison group, holding the value of all other predictors constant at 0
- difference in height of the Y-X regression lines for the comparison & target groups
- comparison & target groups Y-X regression lines have same slope – no interaction

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Non-linear models with 2 centered quantitative predictors

$$y' = b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + a$$

This is called a main effects model → there are no interaction terms.

Same idea …

- we want to plot these models so that we can see how both of the predictors are related to the criterion

Different approach …

- when the second predictor was binary or had k-categories, we plotted the Y-X regression line for each Z group
- now, however, we don't have any groups – both the X_1 & X_2 variables are centered quantitative variables
- what we'll do is to plot the Y-X_1 regression line for different values of X_2
- the most common approach is to plot the Y-X_1 regression line for…
  - the mean of X_2
  - +1 std above the mean of X_2
  - -1 std below the mean of X_2

} We'll plot 3 lines

Non-linear Models with 2 centered quantitative predictors

$$y' = b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + a$$

This is called a main effects model → there are no interaction terms.

a → regression constant

- expected value of y if X_1=0 (mean) X_2=0 (mean)
- mean criterion score of those having X_1=0 (mean) X_2=0 (mean)
- height of quant-criterion regression line for those with X_2=0 (mean)

b_1 → regression weight for X_1 centered quant predictor

- expected direction and extent of change in y for a 1-unit increase in X_1, after controlling for the other variable(s) in the model
- expected direction and extent of change in y for a 1-unit increase in X_2 , for those with X_2=0 (mean)
- slope of quant-criterion regression line when X_2=0 (mean)

b_2 → weight for centered & squared quant predictor – non-linear main effect of X_1

- expected direction and extent of change in y-x slope for a 1-unit increase in X_1, after controlling for the other variable(s) in the model
- expected direction and extent of change in y-x slope for a 1-unit increase in X_1 , for those with X_2=0 (mean)
- curve of quant-criterion regression line when X_2 =0 (mean)

Non-linear Models with 2 centered quantitative predictors

$y' = b_1X_1 + b_2X_1^2 + b_3 X_2 + b_4X_2^2 + a$

This is called a main effects model → there are no interaction terms.

b_3 → regression weight for X_2 centered quant predictor

- expected direction and extent of change in y for a 1-unit increase in X_2, after controlling for the other variable(s) in the model
- expected direction and extent of change in y for a 1-unit increase in X_2, for those with $X_1=0$ (mean)
- expected direction and extent of change of the height of Y-X regression line for a 1-unit change in X_2 when $X_1=0$ (mean)

b_4 → weight for centered & squared quant predictor – non-linear main effect of X_2

- expected direction and extent of change in y-x slope for a 1-unit increase in X_2, after controlling for the other variable(s) in the model
- expected direction and extent of change in y-x slope for a 1-unit increase in X_2, for those with $X_1=0$ (mean)
- curve of quant-criterion regression line when $X_1 =0$ (mean)

To plot the model we need to get separate regression formulas for each chosen value of Z. Start with the multiple regression model..

$y' = b_1X_1 + b_2X_1^2 + b_3 X_2 + b_4X_2^2 + a$

For $X_2 = 0$ (the mean of centered X_2)

Substitute the 0 in for X_2 $\quad y' = b_1X_1 + b_2X_1^2 + b_3 0 + b_40 + a$

Simplify the formula $\quad y' = b_1X + b_2X_1^2 + a$

slope curve height

For X_2 = +1 std

Substitute the std for X_2 $\quad y' = b_1X + b_2X_1^2 + b_2{*}std + b_4std^2 + a$

Simplify the formula $\quad y' = b_1X + b_2X_1^2 + (\underbrace{b_2{*}std + b_4std^2 + a}_{height})$

slope curve

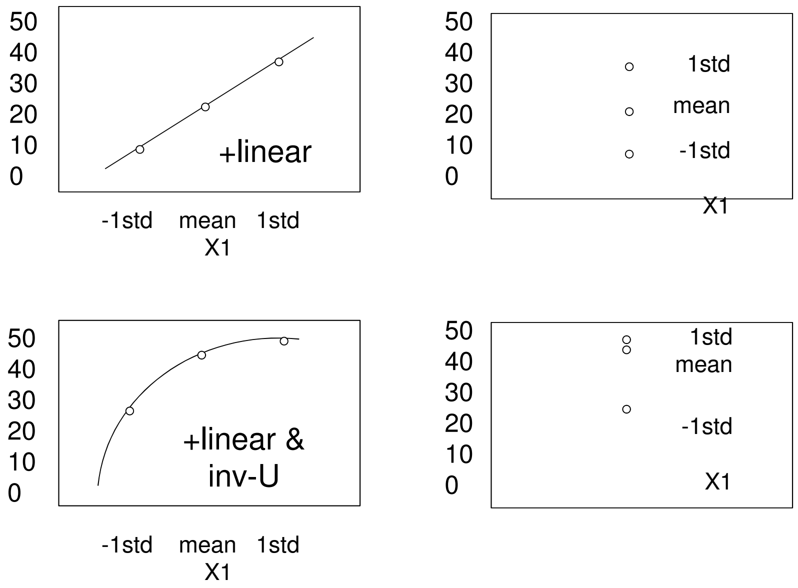
For X_2 = -1 std

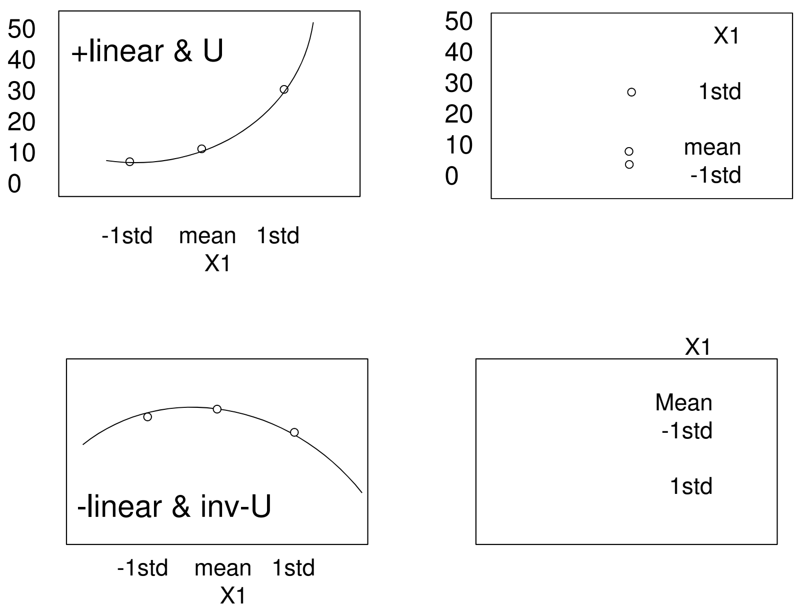
Substitute the std for X_2 $\quad y' = b_1X + b_2X_1^2 - b_2{*}std - b_4std^2 + a$

Simplify the formula $\quad y' = b_1X + b_2X_1^2 + (\underbrace{-b_2{*}std - b_4std^2 + a}_{height})$

slope curve

It can be a challenge to “see” the shape of the variable represented with the different lines for different values…



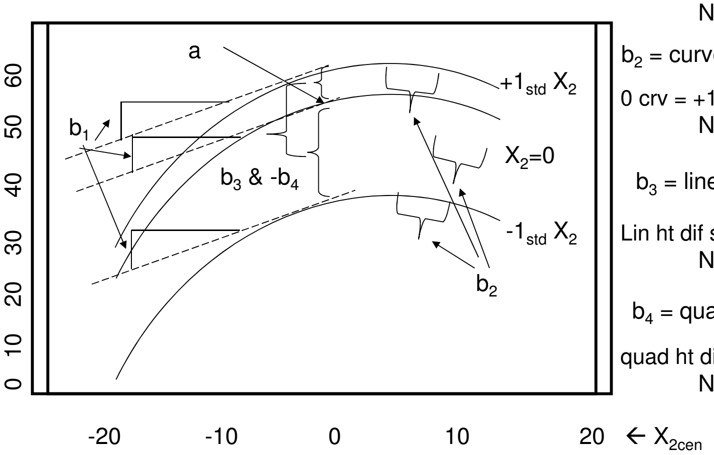


Plotting & Interpreting Non-linear Models with 2 centered quantitative predictors

This is called a main effects model → no interaction → the regression lines are parallel.

$y' = b_1X_{1cen} + b_2X_1{}^2{}_{cen} + b_3X_{2cen} + b_4X_2{}^2{}_{cen} + a$

$X_{1cen} = X_1 - X_{1mean}$ $X_{2cen} = X_2 - X_{2mean}$



a = ht of X_{2mean} line

b_1 = slp of X_{2mean} line

0 slp = +1std slp = -1std slp
No interaction

b_2 = curve of X_2=mean line

0 crv = +1crv slp = -1crv slp
No interaction

b_3 = linear htdifs among X_2-lines
Lin ht dif same all X_1 values
No interaction

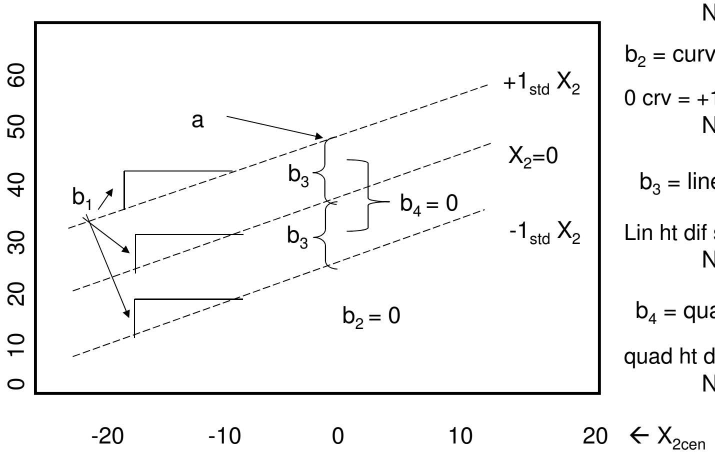
b_4 = quad htdifs among X_2-lines
quad ht dif same all X_1 values
No interaction

Plotting & Interpreting Non-linear Models with 2 centered quantitative predictors

This is called a main effects model → no interaction → the regression lines are parallel.

$y' = b_1X_{1cen} + b_2X_1{}^2{}_{cen} + b_3X_{2cen} + b_4X_2{}^2{}_{cen} + a$

$X_{1cen} = X_1 - X_{1mean}$ $X_{2cen} = X_2 - X_{2mean}$



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b_2 = curve of X_2=mean line

0 crv = +1crv slp = -1crv slp
No interaction

b_3 = linear htdifs among X_2-lines
Lin ht dif same all X_1 values
No interaction

b_4 = quad htdifs among X_2-lines
quad ht dif same all X_1 values
No interaction

Plotting Complex Linear Main Effects Models

Sometimes we have a model with more than 2 predictors, and sometimes we want to plot these more complex models.

There is only so much that can be put into a single plot and reasonably hope the reader will be able to understand it.

Remember, with main effects models (with no interactions), all lines are parallel!!!

So, our plots are a series of lines, each for a different combination of variable groups/values.

The most common version of these plots is to show how 2 variables relate to the criterion, for a given set of values for the other variables (usually their mean)

Here's an example!

So, we have this model…

$perf' = b_1 age + b_2 mar1 + b_3 mar2 + b_4 era + b_5 exp + a$

- age & exp(erience) are centered quantitative variables
- era is dummy coded 0 = 1980s 1 = 2010s
- mar(ital status) is dummy coded
  mar1 single = 1   mar2 divorced = 1   married = 0

To plot this we have to decide what we want to show, say…

How are experience & marital status related to performance for 25-year-olds from 2010s?

What we do is fill in the values of the "selection variables" and simplify the formula to end up with a plotting function for each regression line…

For this plot:

- performance (the criterion) is on the Y axis
- experience (a quantitative predictor) is on the X axis
- we'll have 3 regression lines, one each for single, married & divorced

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How are experience & marital status related to perf for 25-year-olds from 2010s?

$perf' = b_1 age + b_2 mar1 + b_3 mar2 + b_4 era + b_5 exp + a$

- era 0 = 1980s 1 = 2010s
- mar   mar1 single = 1   mar2 divorced = 1   married = 0

For married → $perf' = b_1*25 + b_2*0 + b_3*0 + b_4*1 + b_5 exp + a$

we plot $$perf' = \underbrace{b_5}_{\text{slope}} exp + \underbrace{(b_4*1 + b_1*25 + a)}_{\text{height}}$$

For singles → $perf' = b_1*25 + b_2*1 + b_3*0 + b_4*1 + b_5 exp + a$

we plot $$perf' = \underbrace{b_5}_{\text{slope}} exp + \underbrace{(b_2 + b_4*1 + b_1*25 + a)}_{\text{height}}$$

For divorced → $perf' = b_1*25 + b_2*0 + b_3*1 + b_4*1 + b_5 exp + a$

we plot $$perf' = \underbrace{b_5}_{\text{slope}} exp + \underbrace{(b_3 + b_4*1 + b_1*25 + a)}_{\text{height}}$$

How are experience & marital status related to perf for 25-year-olds from 2010s?

$perf' = b_1age + b_2 mar1 + b_3mar2 + b_4era + b_5exp + a$

- era 0 = 1980s 1 = 2010s
- mar mar1 single = 1 mar2 divorced = 1 married = 0

Let's look at the plotting formulas again

For married → $perf' = b_5exp + (b_4*1 + b_1*25 + a)$

For singles → $perf' = b_5exp + (b_2 + b_4*1 + b_1*25 + a)$

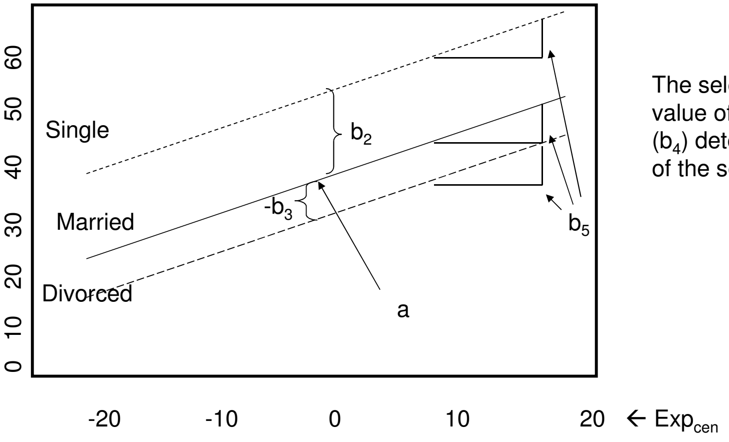
For divorced → $perf' = b_5exp + (b_3 + b_4*1 + b_1*25 + a)$

Notice that each group's regression line has the same slope (b_5), because there is no interaction.

The difference between the group's regression lines are their height – which differ based on the relationship between marital status and perf !!!

How are experience & marital status related to perf for 25-year-olds from 2010s?

$perf' = b_125 + b_2 mar1 + b_3mar2 + b_42010s + b_5exp + a$



The selection of the value of Age (b_1) & Era (b_4) determine the height of the set of lines!

Plotting Complex Non-Linear Main Effects Models

Sometimes we have a non-linear model with more than 2 predictors, and sometimes we want to plot these more complex models.

There is only so much that can be put into a single plot and reasonably hope the reader will be able to understand it.

Remember, with main effects models (with no interactions), all lines are parallel!!!

So, our plots are a series of lines, each for a different combination of variable groups/values.

The most common version of these plots is to show how 2 variables relate to the criterion, for a given set of values for the other variables (usually their mean)

Here's an example!

So, we have this model…

$perf' = b_1age + b_2\,mar1 + b_3mar2 + b_4era + b_5exp + a$

- age & exp(erience) are centered quantitative variables
- era is dummy coded 0 = 1980s 1 = 2010s
- mar(ital status) is dummy coded
  mar1 single = 1 mar2 divorced = 1 married = 0

To plot this we have to decide what we want to show, say…

How are experience & marital status related to performance for 25-year-olds from 1980s?

All we do is fill in the values of the "selection variables" and simplify the formula to end up with a plotting function for each regression line…

For this plot:

- performance (the criterion) is on the Y axis
- experience (a quantitative predictor) is on the X axis
- we'll have 3 regression lines, one each for single, married & divorced

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How are experience & marital status related to perf for 25-year-olds from 2010s?

$perf' = b_1age + b_2age^2 + b_3\,mar1 + b_4mar2 + b_5era + b_6exp + b_7exp^2 + a$

- era 0 = 1980s 1 = 2010s
- mar mar1 single = 1 mar2 divorced = 1 married = 0

For married → $perf' = b_125 + b_225^2 + b_3\,0 + b_40 + b_51 + b_6exp + b_7exp^2 + a$

we plot $perf' = b_6exp + b_7exp^2 + (\underbrace{b_5*1 + b_1*25 + b_225^2 + a}_{\text{height}})$

slope ↗ ↖ curve

For singles → $perf' = b_125 + b_225^2 + b_3\,1 + b_40 + b_51 + b_6exp + b_7exp^2 + a$

we plot $perf' = b_6exp + b_7exp^2 + (\underbrace{b_3 + b_5*1 + b_1*25 + b_225^2 + a}_{\text{height}})$

slope ↗ ↖ curve

For divorced → $perf' = b_125 + b_725^2 + b_3\,0 + b_41 + b_51 + b_6exp + b_7exp^2 + a$

we plot $perf' = b_6exp + b_7exp^2 + (\underbrace{b_4 + b_5*1 + b_1*25 + b_225^2 + a}_{\text{height}})$

slope ↗ ↖ curve

How are experience & marital status related to perf for 25-year-olds from 2010s?

$perf' = b_1age + b_2age^2 + b_3\,mar1 + b_4mar2 + b_5era + b_6exp + b_7exp^2 + a$

- era coded 0 = 1980s 1 = 2010s
- mar mar1 single = 1 mar2 divorced = 1 married = 0

Let's look at the plotting formulas again

For married → $perf' = b_6exp + b_7exp^2 + (b_5*1 + b_1*25 + b_225^2 + a)$

For singles → $perf' = b_6exp + b_7exp^2 + (b_3 + b_5*1 + b_1*25 + b_225^2 + a)$

For divorced → $perf' = b_6exp + b_7exp^2 + (b_4 + b_5*1 + b_1*25 + b_225^2 + a)$

Notice that each group's regression line has the same slope (b_6), because there is no interaction.

Notice that each group's regression line has the same curve (b_7), because there is no interaction.

The difference between the group's regression lines are their height – which differ based on the relationship between marital status and perf !!!