Non-linear Components for Multiple Regression

- Why we might need non-linear components
- Type of non-linear components
- Squared-term components & their disadvantages
- Centered-squared-term components & their advantages
- Quadratic terms, interactions & regression weights

Non-linear Components

Although linear regression models do a very good job of capturing or approximating most of the relationships between quantitative variables that psychologists study, there are several notable exceptions.

• Study of skills acquisition or other accrual tasks often show ...

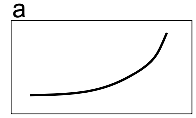
• initial learning is slow in earlier trials but then speeds up (a) …

• learning rates seem to reach a point of diminishing returns (b) …

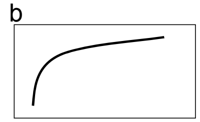
• or both (c) …

• depending upon the novelty of the learning task and the number of trials examined

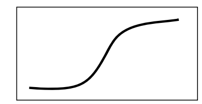
a



b



c



"Modeling" these non-linear relationships has two major forms…

• actually fitting the mathematical function of the relationship

• including nonlinear terms into linear regression

  • literally asking if there is a linear relationship between the criterion and a non-linear transformation of the predictor

  • very similar to the process of "trend analysis" in ANOVA

As you might imagine, the latter is more common (and easier)

The most common non-linear term (and the only one we'll cover here) is the quadratic term -- has two forms …

U-shaped

Inverted-U-shaped

The most common way of including a quadratic term into a linear regression model is to

• construct a new variable – a “squared term” that is the square of the original variable X ➔ X^2

• include the squared term in the model

• determine if the term “adds” to the model

• t-test of “b” or F-test of R^2-change

• +b means a U-shaped quadratic component

• -b means an inverted-U-shaped quadratic component

For example, if we wanted to test for a non-linear relationship between practice and performance…

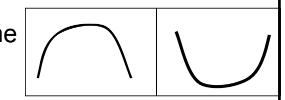
compute practsq = pract * pract

dep = perform / enter pract paractsq

However, this common method is not doing quite what we had in mind …

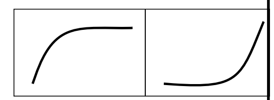
The practsq term really combines the linear and the quadratic trends into a single term

•a “quadratic only” trend will have the same DV values at each end of the predictor continuum, with a hump in between



• however, squaring makes initially larger numbers “more larger” than initially smaller numbers (2*2=4   5*5=25)

• so, the squaring process alone will give you a “curved” shape (linear + quadratic), but won’t give you a “humped” or “pure quadratic” shape



There are two consequences of working with this “combined” term instead of a “quadratic only” term…

• the bivariate analysis using this constructed variable won’t be testing what you had intended (a problem similar to using a single dummy or effect code from a k-group coding in a simple correlation)

• the multivariate analysis will have increased collinearity - because both the linear and the “quadratic” term have a linear component

• though this also informs us that in the multiple regression, the b for the quadratic term will be “controlled for” the linear term, and so will provide a test of the “pure” quadratic trend

Is there a way to get a “pure” quadratic term ???

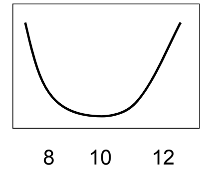
Sure there is…

- first center the predictor (subtract the mean of the predictor from each person’s score) X ➔ (X – M)
- square this centered score $(X - M)^2$

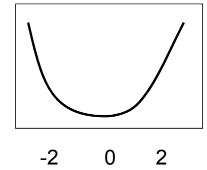
How does that work ???

- Consider two persons with scores equally far above and below the mean (mean = 10, p1 = 8 p2 = 12)
- After centering, they are equally far from the mean, but with opposite signs (p1 = -2 p2 = 2)
- When the centered scores are squared, they are now equally far from the mean & iin the same direction (p1 = p2 = 4)
- If there is a “pure” quadratic term, these squared centered scores are now linearly related to the criterion

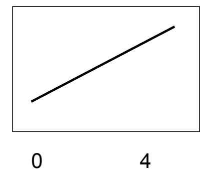
Original scores show a “pure” quad trend w/ criterion



Centered scores show the same trend w/ criterion



Squared centered scores show a linear trend with criterion



Thus, a quadratic trend between the criterion and the predictor is revealed as a linear trend between the centered & squared predictor and the criterion.

But wait – there’s more!!!

How do we interpret the quadratic term regression weight ?

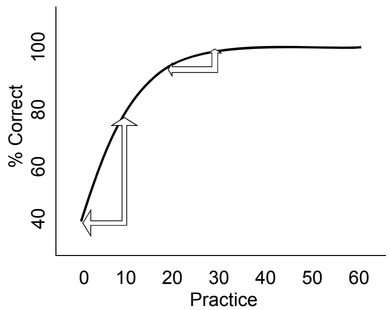
Some background….
To answer that, you have to realize that a quadratic term is “a kind of interaction” – HUH??

- the quadratic term tells how the slope of the linear y-x relationship changes as x changes. So, it is a conditional statement… So, it is an interaction!

- +b indicates that the y-x linear slope is getting more positive as x increases
- -b indicates that the y-x linear slope is getting less positive (more negative) as x increases

Not sure that makes sense???

Take a look at these data…



There is a definite +linear trend! On average, more practice is better! (a main effect)

However, there is also a quadratic trend! How much the next practice will help depends on how much practices you've already done. (an interaction)

10 practices will help someone who has done 0 practices MORE THAN 10 practices will help someone who has already done 20 !

So, how do we interpret the quadratic term regression weight ?

Well, there's a bit of a problem, which can be expressed 2 ways:

First, there's no way to fix the linear term and allow the quadratic term to vary! Said differently, if X changes X^2 changes….

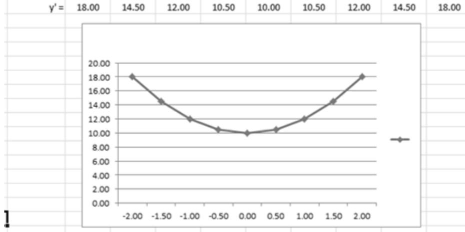
Second, since it is a kind of interaction term, the change in Y for a 1-unit change in X depends on the starting value of X !!!

Here's an example…

We're going to start with a model that has no linear trend, but a quadratic trend (making it easier to track the quadratic!)

$$Y' = 0*(X-M) + 2*(X-M)^2 + 10$$

So, how do we interpret the quadratic term regression weight ?



x-raw	-2.00	-1.50	-1.00	-0.50	0.00	0.50	1.00	1.50	2.00
y' =	18.00	14.50	12.00	10.50	10.00	10.50	12.00	14.50	18.00

For X=0 ➔ X=1
Y'Δ 10.00 ➔ 12.00 which is the quadratic coefficient = 2

For X=1 ➔ X=2
Y'Δ 12.00 ➔ 18.00 6 ?????

$$Y' = 0*(X-M) + 2*(X-M)^2 + 10$$

We're going to think about it as an interaction term…

In what direction & by how much does the slope of the Y-X regression line as X increases by 1 ???

b-linear tells Y-X slope when x = 0

b-quad tells how Y-X slope changes as X changes

- As X increases, the Y-X slope becomes more positive
- As X decreases, the Y-X slope becomes less positive