

## CFA Example Using Forgiveness of Situations

The Forgiveness of Situation Subscale includes 6 items, 3 of which are reverse-coded, on a 7-point scale:

1. When things go wrong for reasons that can't be controlled, I get stuck in negative thoughts about it. **(R)**
2. With time I can be understanding of bad circumstances in my life.
3. If I am disappointed by uncontrollable circumstances in my life, I continue to think negatively about them. **(R)**
4. I eventually make peace with bad situations in my life.
5. It's really hard for me to accept negative situations that aren't anybody's fault. **(R)**
6. Eventually I let go of negative thoughts about bad circumstances that are beyond anyone's control.

Response Anchors:

- 1 = Almost Always False of Me
- 3 = More Often False of Me
- 5 = More Often True of Me
- 7 = Almost Always True of Me

**N = 1103**

| Observed           |              |              |              |              |              |              |
|--------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Correlation Matrix | R1           | 2            | R3           | 4            | R5           | 6            |
| R1                 | <b>1.000</b> |              |              |              |              |              |
| 2                  | 0.240        | <b>1.000</b> |              |              |              |              |
| R3                 | <b>0.647</b> | 0.317        | <b>1.000</b> |              |              |              |
| 4                  | 0.300        | <b>0.570</b> | 0.369        | <b>1.000</b> |              |              |
| R5                 | <b>0.453</b> | 0.255        | <b>0.482</b> | 0.289        | <b>1.000</b> |              |
| 6                  | 0.297        | <b>0.457</b> | 0.356        | <b>0.448</b> | 0.304        | <b>1.000</b> |
| <b>Means</b>       | 4.547        | 5.289        | 4.896        | 5.359        | 4.860        | 5.321        |
| <b>Variances</b>   | 3.049        | 1.903        | 2.543        | 1.967        | 2.945        | 2.341        |

| Observed          |              |              |              |              |              |              |
|-------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Covariance Matrix | R1           | 2            | R3           | 4            | R5           | 6            |
| R1                | <b>3.049</b> |              |              |              |              |              |
| 2                 | 0.577        | <b>1.903</b> |              |              |              |              |
| R3                | 1.802        | 0.697        | <b>2.543</b> |              |              |              |
| 4                 | 0.734        | 1.103        | 0.824        | <b>1.967</b> |              |              |
| R5                | 1.358        | 0.604        | 1.319        | 0.695        | <b>2.945</b> |              |
| 6                 | 0.795        | 0.965        | 0.868        | 0.962        | 0.798        | <b>2.341</b> |

To do CFA analysis, you only need means, variances, and either correlations or covariances among items:

$$\text{Covariance} = \text{Correlation} * \text{SD}(X) * \text{SD}(Y)$$

$$\text{Correlation} = \text{Covariance} / \text{SD}(X) * \text{SD}(Y)$$

**Mplus Code to Read in Data:**

```

TITLE:      CFA of Situation Factor
DATA:      FILE IS Study2.dat;           ! Don't need path if in same directory
              FORMAT IS free;             ! Default
              TYPE IS INDIVIDUAL;         ! Default

VARIABLE:  NAMES ARE Self1 Self2r Self3 Self4r Self5 Self6r
              Other1r Other2 Other3r Other4 Other5r Other6
              Sit1r Sit2 Sit3r Sit4 Sit5r Sit6
              Selfsub Othsub Sitsub HFSsum;           ! Every variable in DATASET

              USEVARIABLES ARE Sit1r Sit2 Sit3r Sit4 Sit5r Sit6;       ! Every variable in MODEL
              MISSING ARE ALL (99999);

ANALYSIS:  TYPE IS GENERAL;             ! Default
              ESTIMATOR IS ML;             ! Default if TYPE=GENERAL and all continuous items

MODEL:      (model syntax goes here, to be changed for each model)
OUTPUT:    SAMPSTAT                    ! Sample descriptives to verify data
              MODINDICES                  ! Voodoo suggestions to improve the model
              STDYX                       ! Requests fully standardized solution
              RESIDUAL;                   ! Requests normalized residuals

```

**Model 1. Fully Z-Scored Factor Model Identification  
(Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)**

The following code refers to EVERY model parameter for completeness:

```

!Model 1 - Fully Z-Scored Factor Identification Approach

! Item factor loadings --> @=fixed, *=free
  Sit BY Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

! Item intercepts --> [ ] indicates means or intercepts
  [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];

! Item error variances --> just list item by itself
  Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

! Factor variance --> just list factor by itself
  Sit@1;

! Factor mean --> [ ] indicates means or intercepts
  [Sit@0];

```

In reality, all you'd need to write to define this model is:

```

! Item factor loadings --> @=fixed, *=free
  Sit BY Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

! Factor variance --> just list factor by itself
  Sit@1;

```

All intercepts are estimated and the factor mean is fixed at 0 by default.

All residual variances for the items are estimated by default, too.

Factor variances and covariances are estimated by default unless specified otherwise.

## Model 1. Fully Z-Scored Factor Model Identification (Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)

### UNSTANDARDIZED MODEL RESULTS

|   |       | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|---|-------|----------|-------|-----------|-----------------------|
| <b>FACTOR LOADINGS (regression slopes of item response on factor)</b> |       |          |       |           |                       |
| SIT   | BY    |          |       |           |                       |
|   | SIT1R | 1.234    | 0.053 | 23.145    | 0.000                 |
|   | SIT2  | 0.702    | 0.047 | 14.882    | 0.000                 |
|   | SIT3R | 1.241    | 0.048 | 25.939    | 0.000                 |
|   | SIT4  | 0.784    | 0.047 | 16.765    | 0.000                 |
|   | SIT5R | 1.023    | 0.052 | 19.667    | 0.000                 |
|   | SIT6  | 0.819    | 0.050 | 16.317    | 0.000                 |

### Means (of Factor)

999 = "cannot be computed" - here, because the parameter is fixed to 1 already

| SIT | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|-----|----------|-------|-----------|-----------------------|
| SIT | 0.000    | 0.000 | 999.000   | 999.000               |

### Intercepts (of Items) - HERE, ARE ACTUAL ITEM MEANS

| SIT   | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|-------|----------|-------|-----------|-----------------------|
| SIT1R | 4.547    | 0.053 | 86.474    | 0.000                 |
| SIT2  | 5.289    | 0.042 | 127.347   | 0.000                 |
| SIT3R | 4.896    | 0.048 | 101.959   | 0.000                 |
| SIT4  | 5.359    | 0.042 | 126.896   | 0.000                 |
| SIT5R | 4.860    | 0.052 | 94.060    | 0.000                 |
| SIT6  | 5.321    | 0.046 | 115.493   | 0.000                 |

### Variances (of Factor)

| SIT | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|-----|----------|-------|-----------|-----------------------|
| SIT | 1.000    | 0.000 | 999.000   | 999.000               |

### Residual Variances (variance of e's)

| SIT   | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|-------|----------|-------|-----------|-----------------------|
| SIT1R | 1.526    | 0.094 | 16.165    | 0.000                 |
| SIT2  | 1.409    | 0.071 | 19.859    | 0.000                 |
| SIT3R | 1.004    | 0.078 | 12.945    | 0.000                 |
| SIT4  | 1.352    | 0.071 | 19.118    | 0.000                 |
| SIT5R | 1.899    | 0.093 | 20.388    | 0.000                 |
| SIT6  | 1.671    | 0.083 | 20.054    | 0.000                 |

### Making use of the unstandardized model estimates:

#### Writing out the model – individual predicted values:

$$Y_1 = \mu_1 + \lambda_1 F + e_1$$

$$Y_1 = 4.547 + 1.234F + e_1$$

#### Writing out the model – predicted item variances and covariances:

$$\text{Var}(Y_1) = (\lambda_1^2) \text{Var}(F) + \text{Var}(e_1)$$

$$\text{Var}(Y_1) = (1.234^2) * (1) + 1.526 = 3.049 \text{ (= original item variance)}$$

$$\text{Cov}(Y_1, Y_2) = \lambda_1 * \text{Var}(F) * \lambda_2$$

$$\text{Cov}(Y_1, Y_2) = (1.234) * (1) * (.702) = .866 \text{ (actual = .577, so over-predicted)}$$

**STDYX STANDARDIZED MODEL RESULTS (FULLY STANDARDIZED WITH RESPECT TO X & Y)**

|   | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |         |
|---|----------|-------|-----------|-----------------------|---------|
| <b>FACTOR LOADINGS (correlations of item response with factor)</b>                                  |          |       |           |                       |         |
| Square these to get reliability (proportion "true variance") per item                               |          |       |           |                       |         |
| SIT   | BY       |       |           |                       |         |
| SIT1R   |          | 0.707 | 0.022     | 32.280                | 0.000   |
| SIT2  |          | 0.509 | 0.030     | 17.174                | 0.000   |
| SIT3R   |          | 0.778 | 0.020     | 38.487                | 0.000   |
| SIT4  |          | 0.559 | 0.028     | 20.074                | 0.000   |
| SIT5R   |          | 0.596 | 0.023     | 25.419                | 0.000   |
| SIT6  |          | 0.535 | 0.028     | 19.405                | 0.000   |
| <b>Means (of Factor)</b>  |          |       |           |                       |         |
| SIT   |          | 0.000 | 0.000     | 999.000               | 999.000 |
| <b>Intercepts (of Items) → is intercept / SD(Y) → not usually reported</b>                          |          |       |           |                       |         |
| SIT1R   |          | 2.604 | 0.063     | 41.273                | 0.000   |
| SIT2  |          | 3.834 | 0.087     | 44.067                | 0.000   |
| SIT3R   |          | 3.070 | 0.072     | 42.659                | 0.000   |
| SIT4  |          | 3.821 | 0.087     | 44.048                | 0.000   |
| SIT5R   |          | 2.832 | 0.067     | 42.021                | 0.000   |
| SIT6  |          | 3.477 | 0.080     | 43.508                | 0.000   |
| <b>Variances (of Factor) → will always be 1 in a standardized solution</b>                          |          |       |           |                       |         |
| SIT   |          | 1.000 | 0.000     | 999.000               | 999.000 |
| <b>Residual Variances (variance of e's)</b>   |          |       |           |                       |         |
| SIT1R   |          | 0.500 | 0.031     | 16.168                | 0.000   |
| SIT2  |          | 0.741 | 0.030     | 24.521                | 0.000   |
| SIT3R   |          | 0.395 | 0.031     | 12.549                | 0.000   |
| SIT4  |          | 0.687 | 0.031     | 22.047                | 0.000   |
| SIT5R   |          | 0.645 | 0.028     | 23.056                | 0.000   |
| SIT6  |          | 0.714 | 0.030     | 24.167                | 0.000   |
| <b>R-SQUARE (equals 1-residual variance OR standardized loading squared in standardized metric)</b> |          |       |           |                       |         |
| SIT1R   |          | 0.500 | 0.031     | 16.140                | 0.000   |
| SIT2  |          | 0.259 | 0.030     | 8.587                 | 0.000   |
| SIT3R   |          | 0.605 | 0.031     | 19.244                | 0.000   |
| SIT4  |          | 0.313 | 0.031     | 10.037                | 0.000   |
| SIT5R   |          | 0.355 | 0.028     | 12.709                | 0.000   |
| SIT6  |          | 0.286 | 0.030     | 9.702                 | 0.000   |

The standardized solution will look identical across methods of model identification with respect to the factor loadings, error variances, and R-square values for the items. The standardized intercepts will change because they depend on the unstandardized intercepts (but nobody reports them anyway).

**Making use of the standardized model estimates:**Writing out the model – predicted item correlations:

$$\text{Corr}(Y_1, Y_2) = \lambda_1 * \text{Var}(F) * \lambda_2$$

$$\text{Corr}(Y_1, Y_2) = (.707) * (1) * (.509) = .360 \text{ (actual} = .240, \text{ so over-predicted)}$$

Calculating Omega Model-Based Reliability:

$$\text{Omega} = (\text{sum of loadings})^2 / [(\text{sum of loadings})^2 + \text{sum residual variances}]$$

$$\text{Sum of loadings} = (.707 + .509 + .778 + .559 + .596 + .535) = 3.684$$

$$\text{Sum of residual variances} = 3.682$$

$$\text{Omega} = 3.684^2 / (3.684^2 + 3.682) = .787$$

**Now let's see the model parameters when using the marker item approach to model identification...  
Model 2. Marker Item Loading = 1, Factor Mean = 0 (Factor Variance, All Intercepts Estimated)**

```
! Model 2 -- Marker Item Loading with Factor Mean = 0
  Sit BY Sit1r@1 Sit2* Sit3r* Sit4* Sit5r* Sit6*;      ! Loadings (#1 fixed=1)
  [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];          ! Intercepts (all free)
  Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;           ! Residual variances (all free)
  Sit*;                                               ! Factor variance (free)
  [Sit@0];                                             ! Factor mean (fixed=0)
```

**UNSTANDARDIZED MODEL RESULTS**

|   | Estimate | S.E.  | Est./S.E. | Two-Tailed P-Value |
|---|----------|-------|-----------|--------------------|
| <b>FACTOR LOADINGS (regression slopes of item response on factor)</b> |          |       |           |                    |
| Here, loading for SIT1R is not tested because it is fixed=1           |          |       |           |                    |
| SIT   | BY       |       |           |                    |
| SIT1R   |          | 1.000 | 0.000     | 999.000            |
| SIT2  |          | 0.569 | 0.046     | 12.293             |
| SIT3R   |          | 1.005 | 0.044     | 22.627             |
| SIT4  |          | 0.636 | 0.047     | 13.512             |
| SIT5R   |          | 0.829 | 0.049     | 16.890             |
| SIT6  |          | 0.664 | 0.050     | 13.402             |
| <b>Means (of Factor)</b>  |          |       |           |                    |
| SIT   |          | 0.000 | 0.000     | 999.000            |
| <b>Intercepts (of Items) - EXPECTED Y WHEN FACTOR = 0</b>             |          |       |           |                    |
| HERE, WHICH IS WHEN ITEM 1 = MEAN                                     |          |       |           |                    |
| SIT1R   |          | 4.547 | 0.053     | 86.474             |
| SIT2  |          | 5.289 | 0.042     | 127.346            |
| SIT3R   |          | 4.896 | 0.048     | 101.959            |
| SIT4  |          | 5.359 | 0.042     | 126.896            |
| SIT5R   |          | 4.860 | 0.052     | 94.060             |
| SIT6  |          | 5.321 | 0.046     | 115.493            |
| <b>Variances (of Factor)</b>  |          |       |           |                    |
| SIT   |          | 1.523 | 0.132     | 11.573             |
| <b>Residual Variances (variances of e's)</b>                          |          |       |           |                    |
| SIT1R   |          | 1.526 | 0.094     | 16.165             |
| SIT2  |          | 1.409 | 0.071     | 19.859             |
| SIT3R   |          | 1.004 | 0.078     | 12.945             |
| SIT4  |          | 1.352 | 0.071     | 19.118             |
| SIT5R   |          | 1.899 | 0.093     | 20.388             |
| SIT6  |          | 1.671 | 0.083     | 20.054             |

**Model 3. Marker Item Loading = 1 and Intercept = 0 (Factor Variance and Mean Estimated)**

```
! Model 3 -- Marker Item Loading and Intercept
  Sit BY Sit1r@1 Sit2* Sit3r* Sit4* Sit5r* Sit6*;      ! Loadings (1 fixed=1)
  [Sit1r@0 Sit2* Sit3r* Sit4* Sit5r* Sit6*];          ! Intercepts (1 fixed=0)
  Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;           ! Residual variances (all free)
  Sit*;                                               ! Factor variance (free)
  [Sit*];                                             ! Factor mean (free)
```

**Means (of Factor) → Note is mean of marker item**

|     |       |       |        |       |
|-----|-------|-------|--------|-------|
| SIT | 4.547 | 0.053 | 86.474 | 0.000 |
|-----|-------|-------|--------|-------|

**Intercepts (of Items) - EXPECTED Y WHEN FACTOR = 0**  
HERE, WHICH IS WHEN ITEM 1 = 0 → beyond scale of item, so values are very low

|       |       |       |         |         |
|-------|-------|-------|---------|---------|
| SIT1R | 0.000 | 0.000 | 999.000 | 999.000 |
| SIT2  | 2.701 | 0.215 | 12.588  | 0.000   |
| SIT3R | 0.325 | 0.208 | 1.567   | 0.117   |
| SIT4  | 2.469 | 0.218 | 11.325  | 0.000   |
| SIT5R | 1.092 | 0.229 | 4.767   | 0.000   |
| SIT6  | 2.304 | 0.230 | 10.026  | 0.000   |

**Calculating model degrees of freedom:**

$$\text{Total df} = [p(p+1) / 2] + p = 27$$

$$\text{Spent by model} = 18$$

$$\text{Leftover df} = 9$$

**Model fit information for a single-factor model (same regardless of identification method):****Chi-Square Test of Model Fit (Significance is bad here)**

|                    |   |
|--------------------|---|
| Value              | 427.937                                 |
| Degrees of Freedom | 9 → leftover after estimating our model |
| P-Value            | 0.0000                                  |

**Chi-Square Test of Model Fit for the Baseline Model (fit if no covariance modeled at all)**

|                    |  |
|--------------------|--|
| Value              | 1981.034   |
| Degrees of Freedom | 15 → leftover if just means and variances of items |
| P-Value            | 0.0000   |

**CFI/TLI (want close to 1)**

|     |       |
|-----|-------|
| CFI | 0.787 |
| TLI | 0.645 |

**Loglikelihood (in SAS, is given as -2LL instead)**

|          |            |
|----------|------------|
| H0 Value | -11536.404 |
| H1 Value | -11322.435 |

**Information Criteria (smaller is better)**

|                           |                                    |
|---------------------------|------------------------------------|
| Number of Free Parameters | 18 → actually df spent, not "free" |
| Akaike (AIC)              | 23108.808                          |
| Bayesian (BIC)            | 23198.912                          |
| Sample-Size Adjusted BIC  | 23141.739                          |
| (n* = (n + 2) / 24)       |                                    |

**RMSEA (Root Mean Square Error Of Approximation) (want close to 0)**

|                          |             |
|--------------------------|-------------|
| Estimate                 | 0.205       |
| 90 Percent C.I.          | 0.189 0.222 |
| Probability RMSEA <= .05 | 0.000       |

**SRMR (Standardized Root Mean Square Residual) (want close to 0)**

|       |       |
|-------|-------|
| Value | 0.086 |
|-------|-------|

**Not so good... what do the voo-doo modification indices suggest we do to fix it?****MODEL MODIFICATION INDICES**

Minimum M.I. value for printing the modification index 10.000

EPC = EXPECTED PARAMETER CHANGE

|  | M.I. | E.P.C. | Std E.P.C. | StdYX E.P.C. |
|--|------|--------|------------|--------------|
|--|------|--------|------------|--------------|

**WITH Statements (SUGGESTED ERROR COVARIANCES)**

|              |                   |                |              |              |              |
|--------------|-------------------|----------------|--------------|--------------|--------------|
| SIT2         | WITH SIT1R        | 68.990         | -0.464       | -0.464       | -0.316       |
| <b>SIT3R</b> | <b>WITH SIT1R</b> | <b>199.696</b> | <b>1.023</b> | <b>1.023</b> | <b>0.827</b> |
| SIT3R        | WITH SIT2         | 48.510         | -0.357       | -0.357       | -0.300       |
| SIT4         | WITH SIT1R        | 50.443         | -0.403       | -0.403       | -0.280       |
| <b>SIT4</b>  | <b>WITH SIT2</b>  | <b>224.292</b> | <b>0.702</b> | <b>0.702</b> | <b>0.509</b> |
| SIT4         | WITH SIT3R        | 40.618         | -0.336       | -0.336       | -0.288       |
| SIT6         | WITH SIT1R        | 33.475         | -0.358       | -0.358       | -0.224       |
| <b>SIT6</b>  | <b>WITH SIT2</b>  | <b>88.831</b>  | <b>0.486</b> | <b>0.486</b> | <b>0.317</b> |
| SIT6         | WITH SIT3R        | 31.132         | -0.319       | -0.319       | -0.246       |
| SIT6         | WITH SIT4         | 64.705         | 0.415        | 0.415        | 0.276        |

**Another approach – how about we examine local fit and see where the problems seem to be?**

The means and variances of the items will be perfectly reproduced, so that's not an issue...

misfit boils down to the difference between the observed and model-predicted covariances.

Although Mplus gives us the residual (defined as observed – predicted) covariance matrix, it is scale dependent and thus not so helpful. We can calculate the residual correlation matrix instead:

| <b>Residual Correlation Matrix</b> | R1     | 2            | R3     | 4            | R5     | 6 |
|------------------------------------|--------|--------------|--------|--------------|--------|---|
| <b>R1</b>                          |        |              |        |              |        |   |
| <b>2</b>                           | -0.120 |              |        |              |        |   |
| <b>R3</b>                          | 0.097  | -0.079       |        |              |        |   |
| <b>4</b>                           | -0.095 | <b>0.285</b> | -0.066 |              |        |   |
| <b>R5</b>                          | 0.032  | -0.048       | 0.018  | -0.044       |        |   |
| <b>6</b>                           | -0.081 | <b>0.185</b> | -0.060 | <b>0.149</b> | -0.015 |   |

Mplus also gives us “normalized” residuals, which can be thought of as z-scores for how large the residual is in absolute terms. Because the denominator decreases with sample size, however, these values may be inflated in large samples.

**“Normalized” Residuals = (obs – exp) / SD(obs)**

**Normalized Residuals for Covariances/Correlations/Residual Correlations**

|       | SIT1R  | SIT2         | SIT3R  | SIT4         | SIT5R  | SIT6  |
|-------|--------|--------------|--------|--------------|--------|-------|
| SIT1R | 0.000  |              |        |              |        |       |
| SIT2  | -3.887 | 0.000        |        |              |        |       |
| SIT3R | 2.713  | -2.506       | 0.000  |              |        |       |
| SIT4  | -3.043 | <b>8.229</b> | -2.075 | 0.000        |        |       |
| SIT5R | 0.966  | -1.555       | 0.545  | -1.429       | 0.000  |       |
| SIT6  | -2.575 | <b>5.579</b> | -1.904 | <b>4.510</b> | -0.479 | 0.000 |

**NEGATIVE** NORMALIZED RESIDUAL → Less related than you predicted

**POSITIVE** NORMALIZED RESIDUAL → More related than you predicted

Why might the normalized residuals (residual correlations) for the positive-worded items be larger than for the negatively-worded items?

These results suggest that wording valence is playing a larger role than what the one-factor model predicts. Rather than adding voo-doo error covariances, how about a two-factor model instead?

#### Model 4. Fully Z-Scored, 2-Factor Model

```
! Model 4 -- Fully Z-Scored 2-Factor Model
  SitP BY Sit2* Sit4* Sit6*;           ! SitP loadings (all free)
  SitN BY Sit1r* Sit3r* Sit5r*;       ! SitN loadings (all free)
  [Sit2* Sit4* Sit6];                 ! SitP intercepts (all free)
  [Sit1r* Sit3r* Sit5r*];            ! SitN intercepts (all free)
  Sit2* Sit4* Sit6;                   ! SitP residual variances (all free)
  Sit1r* Sit3r* Sit5r*;               ! SitN residual variances (all free)
  SitP@1; SitN@1;                     ! Factor variances (fixed=1)
  SitP WITH SitN*;                     ! Factor covariance (free)
  [SitP@0 SitN@0];                     ! Factor means (fixed=0)
```

#### Chi-Square Test of Model Fit

|                    |        |
|--------------------|--------|
| Value              | 35.410 |
| Degrees of Freedom | 8      |
| P-Value            | 0.0000 |

#### CFI/TLI

|     |       |
|-----|-------|
| CFI | 0.986 |
| TLI | 0.974 |

#### Information Criteria

|                           |           |
|---------------------------|-----------|
| Number of Free Parameters | 19        |
| Akaike (AIC)              | 22718.281 |
| Bayesian (BIC)            | 22813.391 |
| Sample-Size Adjusted BIC  | 22753.042 |
| (n* = (n + 2) / 24)       |           |

#### RMSEA (Root Mean Square Error Of Approximation)

|                          |             |
|--------------------------|-------------|
| Estimate                 | 0.056       |
| 90 Percent C.I.          | 0.038 0.075 |
| Probability RMSEA <= .05 | 0.277       |

#### SRMR (Standardized Root Mean Square Residual)

|       |       |
|-------|-------|
| Value | 0.029 |
|-------|-------|

**Is the 2-factor model better than the 1-factor model?  
How do we know?**

**UNSTANDARDIZED RESULTS**

|   |           | Estimate     | S.E.         | Est./S.E.     | Two-Tailed<br>P-Value |
|---|-----------|--------------|--------------|---------------|-----------------------|
| <b>SITP</b>   | <b>BY</b> |              |              |               |                       |
|   | SIT2      | 1.007        | 0.042        | 24.075        | 0.000                 |
|   | SIT4      | 1.064        | 0.043        | 24.971        | 0.000                 |
|   | SIT6      | 0.956        | 0.048        | 19.931        | 0.000                 |
| <b>SITN</b>   | <b>BY</b> |              |              |               |                       |
|   | SIT1R     | 1.325        | 0.051        | 26.124        | 0.000                 |
|   | SIT3R     | 1.349        | 0.046        | 29.386        | 0.000                 |
|   | SIT5R     | 1.009        | 0.052        | 19.280        | 0.000                 |
| <b>SITP WITH SITN = Covariance between factors (= correlation if variances=1)</b> |           |              |              |               |                       |
|   |           | <b>0.564</b> | <b>0.030</b> | <b>18.976</b> | <b>0.000</b>          |
| <b>Means</b>  |           |              |              |               |                       |
|   | SITP      | 0.000        | 0.000        | 999.000       | 999.000               |
|   | SITN      | 0.000        | 0.000        | 999.000       | 999.000               |
| <b>Intercepts</b>   |           |              |              |               |                       |
|   | SIT1R     | 4.547        | 0.053        | 86.474        | 0.000                 |
|   | SIT2      | 5.289        | 0.042        | 127.346       | 0.000                 |
|   | SIT3R     | 4.896        | 0.048        | 101.959       | 0.000                 |
|   | SIT4      | 5.359        | 0.042        | 126.896       | 0.000                 |
|   | SIT5R     | 4.860        | 0.052        | 94.060        | 0.000                 |
|   | SIT6      | 5.321        | 0.046        | 115.493       | 0.000                 |
| <b>Variances</b>  |           |              |              |               |                       |
|   | SITP      | 1.000        | 0.000        | 999.000       | 999.000               |
|   | SITN      | 1.000        | 0.000        | 999.000       | 999.000               |
| <b>Residual Variances</b>   |           |              |              |               |                       |
|   | SIT1R     | 1.294        | 0.085        | 15.187        | 0.000                 |
|   | SIT2      | 0.888        | 0.058        | 15.231        | 0.000                 |
|   | SIT3R     | 0.724        | 0.074        | 9.766         | 0.000                 |
|   | SIT4      | 0.835        | 0.061        | 13.669        | 0.000                 |
|   | SIT5R     | 1.926        | 0.094        | 20.419        | 0.000                 |
|   | SIT6      | 1.428        | 0.077        | 18.660        | 0.000                 |

**STDYX STANDARDIZED RESULTS**

|                           |           | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|---------------------------|-----------|----------|-------|-----------|-----------------------|
| <b>SITP</b>               | <b>BY</b> |          |       |           |                       |
|                           | SIT2      | 0.730    | 0.021 | 34.325    | 0.000                 |
|                           | SIT4      | 0.759    | 0.021 | 36.144    | 0.000                 |
|                           | SIT6      | 0.625    | 0.024 | 25.724    | 0.000                 |
| <b>SITN</b>               | <b>BY</b> |          |       |           |                       |
|                           | SIT1R     | 0.759    | 0.019 | 39.934    | 0.000                 |
|                           | SIT3R     | 0.846    | 0.018 | 47.325    | 0.000                 |
|                           | SIT5R     | 0.588    | 0.024 | 24.663    | 0.000                 |
| <b>Residual Variances</b> |           |          |       |           |                       |
|                           | SIT1R     | 0.425    | 0.029 | 14.729    | 0.000                 |
|                           | SIT2      | 0.467    | 0.031 | 15.023    | 0.000                 |
|                           | SIT3R     | 0.285    | 0.030 | 9.421     | 0.000                 |
|                           | SIT4      | 0.425    | 0.032 | 13.333    | 0.000                 |
|                           | SIT5R     | 0.654    | 0.028 | 23.313    | 0.000                 |
|                           | SIT6      | 0.610    | 0.030 | 20.107    | 0.000                 |
| <b>R-SQUARE</b>           |           |          |       |           |                       |
|                           | SIT1R     | 0.575    | 0.029 | 19.967    | 0.000                 |
|                           | SIT2      | 0.533    | 0.031 | 17.162    | 0.000                 |
|                           | SIT3R     | 0.715    | 0.030 | 23.662    | 0.000                 |
|                           | SIT4      | 0.575    | 0.032 | 18.072    | 0.000                 |
|                           | SIT5R     | 0.346    | 0.028 | 12.332    | 0.000                 |
|                           | SIT6      | 0.390    | 0.030 | 12.862    | 0.000                 |

|                                 |
|---------------------------------|
| Omega for Positive Factor = .58 |
|---------------------------------|

|                                 |
|---------------------------------|
| Omega for Negative Factor = .57 |
|---------------------------------|

**Any more local fit problems? Maybe...****Residuals of covariance matrix (so unstandardized):**

|       | SIT1R  | SIT2   | SIT3R  | SIT4   | SIT5R | SIT6  |
|-------|--------|--------|--------|--------|-------|-------|
| SIT1R | 0.000  |        |        |        |       |       |
| SIT2  | -0.176 | 0.000  |        |        |       |       |
| SIT3R | 0.016  | -0.069 | 0.000  |        |       |       |
| SIT4  | -0.062 | 0.031  | 0.015  | 0.000  |       |       |
| SIT5R | 0.021  | 0.030  | -0.042 | 0.089  | 0.000 |       |
| SIT6  | 0.080  | 0.003  | 0.140  | -0.055 | 0.254 | 0.000 |

**“Normalized” residuals**

|       | SIT1R         | SIT2   | SIT3R  | SIT4   | SIT5R        | SIT6  |
|-------|---------------|--------|--------|--------|--------------|-------|
| SIT1R | 0.000         |        |        |        |              |       |
| SIT2  | <b>-2.358</b> | 0.000  |        |        |              |       |
| SIT3R | 0.157         | -0.997 | 0.000  |        |              |       |
| SIT4  | -0.799        | 0.464  | 0.203  | 0.000  |              |       |
| SIT5R | 0.214         | 0.414  | -0.462 | 1.175  | 0.000        |       |
| SIT6  | 0.955         | 0.036  | 1.797  | -0.777 | <b>3.071</b> | 0.000 |

**Any suggested voo-doo?****MODEL MODIFICATION INDICES**

Minimum M.I. value for printing the modification index 10.000

M.I. E.P.C. Std E.P.C. StdYX E.P.C.

**BY Statements - these are cross-loadings**

|      |         |        |        |        |        |
|------|---------|--------|--------|--------|--------|
| SITN | BY SIT2 | 13.888 | -0.224 | -0.224 | -0.162 |
| SITN | BY SIT6 | 15.383 | 0.245  | 0.245  | 0.160  |

**WITH Statements - these are error covariances**

|      |           |        |        |        |        |
|------|-----------|--------|--------|--------|--------|
| SIT4 | WITH SIT2 | 15.386 | 0.332  | 0.332  | 0.386  |
| SIT6 | WITH SIT4 | 13.884 | -0.273 | -0.273 | -0.250 |

**Because we have no real reason to fit any of these suggested parameters, we will not add any new parameters. This will be about as good as it gets.**

## One last example: Formal Tests of CTT Assumptions

We first test the CTT assumption of tau-equivalence (equal factor loadings):

```
! Model 5 -- Tau-Equivalent Items 2-Factor Model
  SitP BY Sit2* Sit4* Sit6* (1);      ! SitP loadings (all held equal)
  SitN BY Sit1r* Sit3r* Sit5r* (2);   ! SitN loadings (all held equal)
  [Sit2* Sit4* Sit6];                 ! SitP intercepts (all free)
  [Sit1r* Sit3r* Sit5r*];             ! SitN intercepts (all free)
  Sit2* Sit4* Sit6;                   ! SitP residual variances (all free)
  Sit1r* Sit3r* Sit5r*;               ! SitN residual variances (all free)
  SitP@1; SitN@1;                     ! Factor variances (fixed=1)
  SitP WITH SitN*;                    ! Factor covariance (free)
  [SitP@0 SitN@0];                    ! Factor means (fixed=0)
```

### UNSTANDARDIZED MODEL RESULTS

|                    | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|--------------------|----------|-------|-----------|-----------------------|
| SITP BY            |          |       |           |                       |
| SIT2               | 1.014    | 0.029 | 34.721    | 0.000                 |
| SIT4               | 1.014    | 0.029 | 34.721    | 0.000                 |
| SIT6               | 1.014    | 0.029 | 34.721    | 0.000                 |
| SITN BY            |          |       |           |                       |
| SIT1R              | 1.254    | 0.035 | 36.261    | 0.000                 |
| SIT3R              | 1.254    | 0.035 | 36.261    | 0.000                 |
| SIT5R              | 1.254    | 0.035 | 36.261    | 0.000                 |
| Residual Variances |          |       |           |                       |
| SIT1R              | 1.335    | 0.075 | 17.866    | 0.000                 |
| SIT2               | 0.882    | 0.052 | 16.867    | 0.000                 |
| SIT3R              | 0.857    | 0.059 | 14.582    | 0.000                 |
| SIT4               | 0.887    | 0.052 | 16.973    | 0.000                 |
| SIT5R              | 1.805    | 0.094 | 19.148    | 0.000                 |
| SIT6               | 1.382    | 0.071 | 19.358    | 0.000                 |

### STANDARDIZED STYDX MODEL RESULTS

|                    | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|--------------------|----------|-------|-----------|-----------------------|
| SITP BY            |          |       |           |                       |
| SIT2               | 0.734    | 0.015 | 49.347    | 0.000                 |
| SIT4               | 0.733    | 0.015 | 49.041    | 0.000                 |
| SIT6               | 0.653    | 0.015 | 44.691    | 0.000                 |
| SITN BY            |          |       |           |                       |
| SIT1R              | 0.735    | 0.014 | 52.318    | 0.000                 |
| SIT3R              | 0.804    | 0.014 | 58.783    | 0.000                 |
| SIT5R              | 0.682    | 0.013 | 51.054    | 0.000                 |
| Residual Variances |          |       |           |                       |
| SIT1R              | 0.459    | 0.021 | 22.218    | 0.000                 |
| SIT2               | 0.462    | 0.022 | 21.166    | 0.000                 |
| SIT3R              | 0.353    | 0.022 | 16.025    | 0.000                 |
| SIT4               | 0.463    | 0.022 | 21.158    | 0.000                 |
| SIT5R              | 0.534    | 0.018 | 29.308    | 0.000                 |
| SIT6               | 0.573    | 0.019 | 30.044    | 0.000                 |

Why are the standardized loadings not held equal like the unstandardized loadings are?

| Fit of previous 2-factor model:     |                          |       |        | Fit of tau-equivalent items 2-factor model: |                          |       |        |
|-------------------------------------|--------------------------|-------|--------|---|--------------------------|-------|--------|
| <b>Chi-Square Test of Model Fit</b> |                          |       |        | <b>Chi-Square Test of Model Fit</b>         |                          |       |        |
|                                     | Value                    |       | 35.410 |   | Value                    |       | 73.500 |
|                                     | Degrees of Freedom       |       | 8      |   | Degrees of Freedom       |       | 12     |
|                                     | P-Value                  |       | 0.0000 |   | P-Value                  |       | 0.0000 |
| <b>CFI/TLI</b>                      |                          |       |        | <b>CFI/TLI</b>                              |                          |       |        |
|                                     | CFI                      |       | 0.986  |   | CFI                      |       | 0.969  |
|                                     | TLI                      |       | 0.974  |   | TLI                      |       | 0.961  |
| <b>RMSEA</b>                        |                          |       |        | <b>RMSEA</b>                                |                          |       |        |
|                                     | Estimate                 |       | 0.056  |   | Estimate                 |       | 0.068  |
|                                     | 90 Percent C.I.          | 0.038 | 0.075  |   | 90 Percent C.I.          | 0.054 | 0.084  |
|                                     | Probability RMSEA <= .05 |       | 0.277  |   | Probability RMSEA <= .05 |       | 0.021  |

**Does the assumption of tau-equivalence hold? How do we know?**

**The previous model tests of the assumption of tau-equivalence for both factors at once. It is possible, however, that it might hold for one factor and not the other, in which case we would be misled. Given the pattern of loadings, let's test tau-equivalence only for the positive factor instead:**

```
! Model 6 -- Tau-Equivalent Items on Positive Only 2-Factor Model
  SitP BY Sit2* Sit4* Sit6* (1);      ! SitP loadings (all held equal)
  SitN BY Sit1r* Sit3r* Sit5r*      ;      ! SitN loadings (all free now)
  (rest same)
```

|                              |                          |       |        |
|------------------------------|--------------------------|-------|--------|
| Chi-Square Test of Model Fit |                          |       |        |
|                              | Value                    |       | 38.676 |
|                              | Degrees of Freedom       |       | 10     |
|                              | P-Value                  |       | 0.0000 |
| CFI/TLI                      |                          |       |        |
|                              | CFI                      |       | 0.985  |
|                              | TLI                      |       | 0.978  |
| RMSEA                        |                          |       |        |
|                              | Estimate                 |       | 0.051  |
|                              | 90 Percent C.I.          | 0.035 | 0.068  |
|                              | Probability RMSEA <= .05 |       | 0.429  |

**What do we conclude?**

#### UNSTANDARDIZED MODEL RESULTS

|                    |       | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|--------------------|-------|----------|-------|-----------|-----------------------|
| SITP               | BY    |          |       |           |                       |
|                    | SIT2  | 1.014    | 0.029 | 34.724    | 0.000                 |
|                    | SIT4  | 1.014    | 0.029 | 34.724    | 0.000                 |
|                    | SIT6  | 1.014    | 0.029 | 34.724    | 0.000                 |
| SITN               | BY    |          |       |           |                       |
|                    | SIT1R | 1.325    | 0.051 | 26.132    | 0.000                 |
|                    | SIT3R | 1.349    | 0.046 | 29.397    | 0.000                 |
|                    | SIT5R | 1.010    | 0.052 | 19.289    | 0.000                 |
| Residual Variances |       |          |       |           |                       |
|                    | SIT1R | 1.295    | 0.085 | 15.206    | 0.000                 |
|                    | SIT2  | 0.881    | 0.052 | 16.855    | 0.000                 |
|                    | SIT3R | 0.725    | 0.074 | 9.784     | 0.000                 |
|                    | SIT4  | 0.886    | 0.052 | 16.963    | 0.000                 |
|                    | SIT5R | 1.925    | 0.094 | 20.415    | 0.000                 |
|                    | SIT6  | 1.384    | 0.072 | 19.359    | 0.000                 |

## STANDARDIZED STDYX MODEL RESULTS

|                    |       | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|--------------------|-------|----------|-------|-----------|-----------------------|
| SITP               | BY    |          |       |           |                       |
|                    | SIT2  | 0.734    | 0.015 | 49.359    | 0.000                 |
|                    | SIT4  | 0.733    | 0.015 | 49.051    | 0.000                 |
|                    | SIT6  | 0.653    | 0.015 | 44.678    | 0.000                 |
| SITN               | BY    |          |       |           |                       |
|                    | SIT1R | 0.759    | 0.019 | 39.966    | 0.000                 |
|                    | SIT3R | 0.846    | 0.018 | 47.375    | 0.000                 |
|                    | SIT5R | 0.588    | 0.024 | 24.680    | 0.000                 |
| Residual Variances |       |          |       |           |                       |
|                    | SIT1R | 0.425    | 0.029 | 14.747    | 0.000                 |
|                    | SIT2  | 0.461    | 0.022 | 21.149    | 0.000                 |
|                    | SIT3R | 0.285    | 0.030 | 9.438     | 0.000                 |
|                    | SIT4  | 0.463    | 0.022 | 21.143    | 0.000                 |
|                    | SIT5R | 0.654    | 0.028 | 23.301    | 0.000                 |
|                    | SIT6  | 0.574    | 0.019 | 30.071    | 0.000                 |

**Given that tau-equivalence held for the positive factor, we can also test the assumption of parallel items as equal residual variances:**

**! Model 7 -- Parallel Items on Positive Only 2-Factor Model**

```

SitP BY Sit2* Sit4* Sit6* (1);      ! SitP loadings (all held equal)
SitN BY Sit1r* Sit3r* Sit5r*      ;      ! SitN loadings (all free)
Sit2* Sit4* Sit6 (3);              ! SitP residual variances (all held equal)
Sit1r* Sit3r* Sit5r*              ;      ! SitN residual variances (all free)
(rest same)

```

## Chi-Square Test of Model Fit

|                    |        |
|--------------------|--------|
| Value              | 79.049 |
| Degrees of Freedom | 12     |
| P-Value            | 0.0000 |

## CFI/TLI

|     |       |
|-----|-------|
| CFI | 0.966 |
| TLI | 0.957 |

## RMSEA

|                          |             |
|--------------------------|-------------|
| Estimate                 | 0.071       |
| 90 Percent C.I.          | 0.057 0.086 |
| Probability RMSEA <= .05 | 0.009       |

**What do we conclude?**

## UNSTANDARDIZED MODEL RESULTS

|                    |      | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|--------------------|------|----------|-------|-----------|-----------------------|
| SITP               | BY   |          |       |           |                       |
|                    | SIT2 | 1.005    | 0.029 | 34.221    | 0.000                 |
|                    | SIT4 | 1.005    | 0.029 | 34.221    | 0.000                 |
|                    | SIT6 | 1.005    | 0.029 | 34.221    | 0.000                 |
| Residual Variances |      |          |       |           |                       |
|                    | SIT2 | 1.060    | 0.032 | 33.212    | 0.000                 |
|                    | SIT4 | 1.060    | 0.032 | 33.212    | 0.000                 |
|                    | SIT6 | 1.060    | 0.032 | 33.212    | 0.000                 |

## STANDARDIZED STDYX MODEL RESULTS

|                    |      | Estimate | S.E.  | Est./S.E. | Two-Tailed<br>P-Value |
|--------------------|------|----------|-------|-----------|-----------------------|
| SITP               | BY   |          |       |           |                       |
|                    | SIT2 | 0.698    | 0.013 | 55.463    | 0.000                 |
|                    | SIT4 | 0.698    | 0.013 | 55.463    | 0.000                 |
|                    | SIT6 | 0.698    | 0.013 | 55.463    | 0.000                 |
| Residual Variances |      |          |       |           |                       |
|                    | SIT2 | 0.512    | 0.018 | 29.117    | 0.000                 |
|                    | SIT4 | 0.512    | 0.018 | 29.117    | 0.000                 |
|                    | SIT6 | 0.512    | 0.018 | 29.117    | 0.000                 |