Complex Regression Models with Coded, Centered & Quadratic Terms

We decided to continue our study of the relationships among amount and difficulty of exam practice with exam performance in the first graduate research methods/data analysis course by including the program Psychology graduate students were in (1=experimental and 2=clinical programs), their future employment intentions (1=quantitative, 2=research, 3=teaching), the number of stats courses they had taken before the current one, and a measure of academic performance motivation.

Based on pilot data and our reading of related performance literatures, we wanted to explore both the linear and nonlinear relationships of the three quantitative variables (practice, prior stats courses, and motivation) with exam performance.

The univaraite stats for our quantitative predictors is shown at the right.

Notice the range of the DV – this will be important to remember later…

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
perfperc	143	50.00	98.00	72.5594	8.11157
prac	143	1.00	10.00	5.8182	2.23807
pristats	143	.00	5.00	2.3986	1.04234
motv	143	24.00	81.00	51.0629	12.10530
Valid N (listwise)	143				

Before performing the multiple regression, categorical variables were dummy-coded with the highest coded group as the comparison group, quantitative variables were mean-centered and quadratic terms were computed…

if (prac1e2s = 1) pract1e0s = 1.
if (prac1e2s = 2) pract1e0s = 0.

🡸 dummy code for practice difficulty
🡸 same = 0 & easier = 1

if (prog1exp_2clin = 1) prog1exp_0clin = 1.
if (prog1exp_2clin = 2) prog1exp_0clin = 0.

🡸 dummy code for grad program
🡸 clinical = 0 & experimental = 1

if (interst_1qnt_2rsh_3tch = 1) int_1qnt_0rsh_0tch = 1.
if (interst_1qnt_2rsh_3tch = 2) int_1qnt_0rsh_0tch = 0.
if (interst_1qnt_2rsh_3tch = 3) int_1qnt_0rsh_0tch = 0.

🡸 first dummy code for intention compares quant with teaching

if (interst_1qnt_2rsh_3tch = 1) int_0qnt_1rsh_0tch = 0.
if (interst_1qnt_2rsh_3tch = 2) int_0qnt_1rsh_0tch = 1.
if (interst_1qnt_2rsh_3tch = 3) int_0qnt_1rsh_0tch = 0.

🡸 second dummy code for intention compares research with teaching

compute prac_mcen = prac - 5.8182. 🡸 mean-centering practice

compute pristats_mcen = pristats - 2.3986. 🡸 mean-centering # prior stats classes

compute motv_mcen = motv - 51.0629. 🡸 mean-centering motivation

compute prac_cen_quad = prac_mcen ** 2. 🡸 quadratic term for # practices

compute pristats_cquad = pristats_mcen ** 2. 🡸 quadratic term for # prior stats clasxses

compute motv_cquad = motv_mcen ** 2. 🡸 quadratic term for motivation

exe.

```
REGRESSION
  /STATISTICS COEFF OUTS R ANOVA
  /DEPENDENT perfperc
  /METHOD=ENTER int_1qnt_0rsh_0tch  int_0qnt_1rsh_0tch  prac1e0s  prog1exp_0clin
                        prac_mcen  prac_cen_quad  pristats_mcen  pristats_cquad  motv_mcen  motv_cquad.
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.785[a]	.616	.587	5.21032

a. Predictors: (Constant), motv_cquad, pristats_mcen, motv_mcen, prac_mcen, prog1exp_0clin, prac_cen_quad, prac1e0s, pristats_cquad, int_0qnt_1rsh_0tch, int_1qnt_0rsh_0tch

The model accounts for nearly 62% of the variance of exam performance, which is statistically significant.

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	5759.785	10	575.978	21.217	.000[b]
	Residual	3583.460	132	27.147		
	Total	9343.245	142			

a. Dependent Variable: perfperc

b. Predictors: (Constant), motv_cquad, pristats_mcen, motv_mcen, prac_mcen, prog1exp_0clin, prac_cen_quad, prac1e0s, pristats_cquad, int_0qnt_1rsh_0tch, int_1qnt_0rsh_0tch

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	70.730	1.259		56.158	.000
	int_1qnt_0rsh_0tch	9.968	1.342	.545	7.427	.000
	int_0qnt_1rsh_0tch	1.327	1.091	.081	1.216	.226
	prac1e0s	-3.926	.996	-.240	-3.942	.000
	prog1exp_0clin	-.131	.907	-.008	-.144	.885
	prac_mcen	-.063	.203	-.017	-.310	.757
	prac_cen_quad	-.169	.078	-.121	-2.162	.032
	pristats_mcen	-.371	.435	-.048	-.852	.396
	pristats_cquad	.268	.336	.045	.799	.426
	motv_mcen	.153	.038	.229	4.016	.000
	motv_cquad	.006	.003	.143	2.485	.014

a. Dependent Variable: perfperc

Interpreting the multiple regression weights

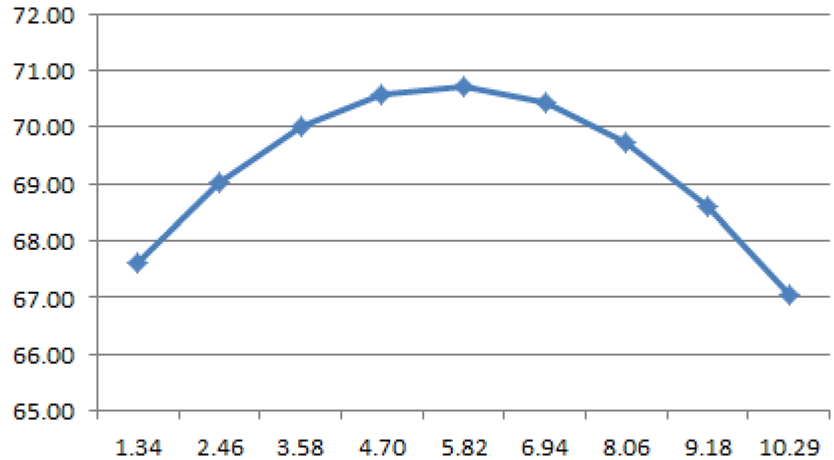
int_1qnt_0rsh_0tch int	Performance of those intending a quantitative career is 9.968 % higher than those intending a teaching career, after controlling for the other variables in the model.
0tch int_0qnt_1rsh_0tch	Performance of those intending a quantitative career and those intending a teaching career are not significantly different, after controlling for the other variables in the model.
prac1e0s	Performance of those completing easier practices is 3.936% lower than those completing similarly difficult practices, after controlling for the other variables in the model.
prog1exp_0clin	Performance of those in the experimental and clinical programs is not significantly different, after controlling for the other variables in the model.
prac_mcen	There is no relationship between practice and performance, after controlling for the other variables in the model.
prac_cen_quad	There is an inverted u-shaped quadratic relationship between practice and performance, after controlling for the other variables in the model. Please see plot below.
pristats_mcen	There is no linear relationship between the number of prior stats courses and performance, after controlling for the other variables in the model.
pristats_cquad	There is no quadratic relationship between the number of prior stats courses and performance, after controlling for the other variables in the model.
motv_mcen	Performance is expected to increase by 0.15% for each 1-unit increase in motivation, after controlling for the other variables in the model.
motv_cquad	There is a u-shaped quadratic relationship between motivation and performance, after controlling for the other variables in the model.

Especially for the quadratic terms, the verbal description can be augmented by showing a plot of the model.

For example, if we wanted to show the shape of the relationship between practice and performance (controlling for the other variables in the model) we can use the xls plotting computator.

Using the “q nonlinear” tab, we would enter the weights for the linear and quadratic components of practice.

height	constant	70.73
slope	b(x)	-0.063
curve	$b(x^2)$	-0.169
	x(mean)	5.8182
	x(std)	2.23807



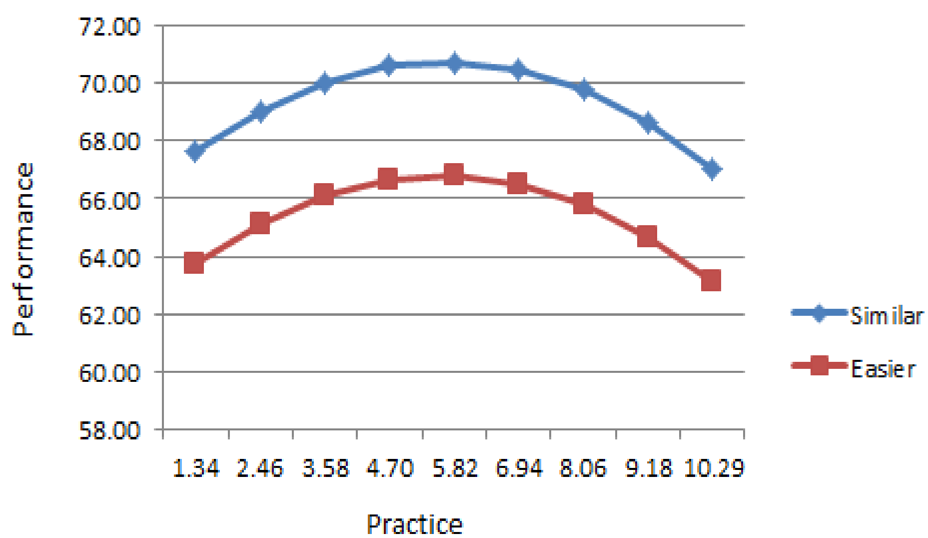
This certainly looks like a relationship with a non-significant negative linear relationship (notice the right side of the plotted model is slightly lower than the left side) and a strong inverted u-shaped quadratic relationship! However, the range of the performance scores seems odd – only about 67-71.

Why? Remember that this is the linear & quadratic relationship between practice and performance, controlling for the other variables in the model. Specifically, this is the shape of this relationship for those intending to teach (coded 0), who practiced with similar difficulty problems (coded 0), were in the clinical program (coded 0), had the sample average motivation (51.0629 re-centered to 0), and who had the sample average number of prior stats courses (2.3986 re-centered to 0).

We can plot more than one variable at a time. The other primary variable in this study was the difficulty of the practice problems (similar difficulty to the test=0, easier =1). We can show the plot of the portion of the model involving these two variables together.

Using the “2xQ nonlinear” tab we would enter the weights for practice and for the difficulty dummy code. We would enter “0” for the interaction weights, because we used a main effects model without any interaction terms.

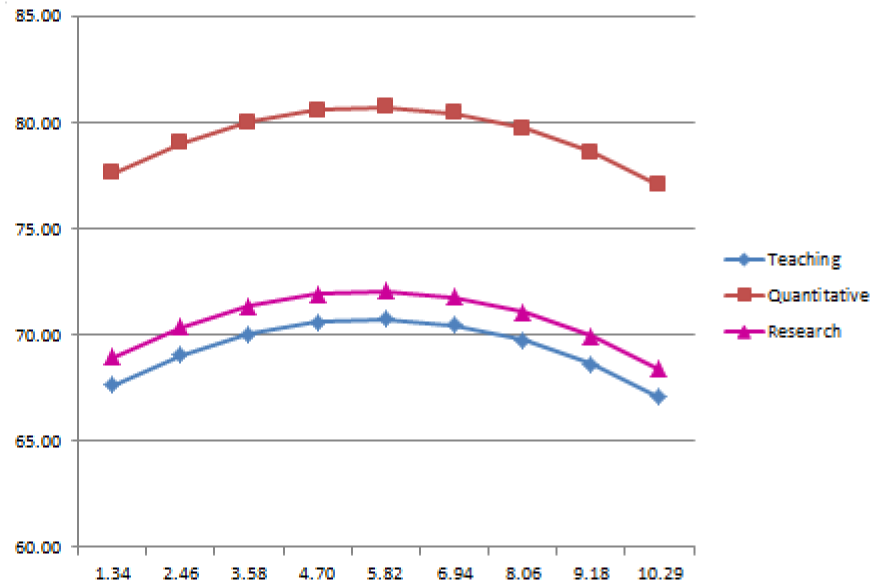
height z=0	constant	70.73
slope z=0	b(x)	-0.063
curve z=0	$b(x^2)$	-0.169
height dif z=1	b(z)	-3.926
slope dif z=1	0	0
curve dif z=1	0	0
	x(mean)	5.8182
	x(std)	2.23807



One way to show a greater range of the criterion variable is to add another variable to the plot of the model. We found a fairly large career intention effect. So, let's include that variable in our plot.

Using the "3XQ nonlinear" tab, we would enter the weights for the linear and quadratic components of practice and the dummy codes for career intention. We would enter "0" for the interaction weights, because we used a main effects model without any interaction terms.

height z1=0 z2=0	constant	70.73
slope z1=0 z2=0	b(x)	-0.063
curve z1=0 z2=0	b(x^2)	-0.169
height dif z1=1 z2=0	b(z1)	9.968
slope dif z1=1 z2=0	b(xz1)	0
curve dif z1=1 z2=0	b(x^2z1)	0
height dif z1=0 z2=1	b(z2)	1.327
slope dif z1=0 z2=1	b(xz2)	0
curve dif z1=0 z2=1	b(x^2z2)	0
	x(mean)	5.8182
	x(std)	2.23807



*

This plot shows two of the model effects, the quadratic relationship of practice with performance, and the career intention group difference, for those who practiced with similar difficulty problems (coded 0), were in the clinical program (coded 0), had the sample average motivation (51.0629 re-centered to 0), -3.and who had the sample average number of prior stats courses (2.3986 re-centered to 0).

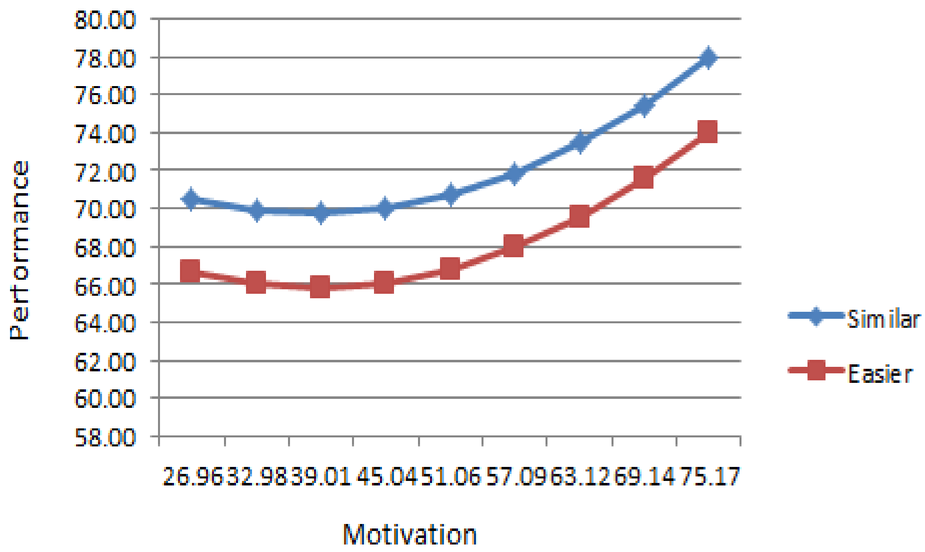
This is an interesting finding! Those with "quantitative intent" out-performed the teachers and researcher – even after controlling for motivation and prior stats (among others). It would not have been surprising to find that a bivariate relationship between career intention and performance on a stats test would "wash out" after control for motivation and prior stats courses. Why? One might expect that those intending a quant career would have had more prior stats classes and be more motivated in a stats class. So, one might expect that when you control for these "confounds" the career intention effect would go away!

One should not get excited or give an interpretation to the fact that the shape of the relationship between practice and performance is the same for these career intention groups? Remember that we didn't include interaction terms in the model, so these three groups "must" have the same slope and curve or the relationship between practice and performance! This would be an interesting set of interaction terms to add to this model later!

Another interesting part of the model is the linear & quadratic relationship of motivation with performance. Let's plot that and include the effect of similarly difficulty versus easier practices.

Using the "2XQ nonlinear" tab we would enter the weights for the linear and quadratic components of motivation and the dummy code for practice difficulty. We would enter "0" for the interaction weights, because we used a main effects model without any interaction terms.

height z=0	constant	70.73
slope z=0	b(x)	0.153
curve z=0	$b(x^2)$	0.006
height dif z=1	b(z)	-3.926
slope dif z=1	0	0
curve dif z=1	0	0
	x(mean)	51.0629
	x(std)	12.053



This is a very interesting result! Notice that the most motivated students who completed the easier practices outperform the lest motivated students completed the similar difficulty practices. This tells us that if a student is sufficiently motivated, that motivational effect can offset the disadvantage of completing the easier exam practices! These sorts of "off-setting effects" are interesting to find, especially if you intend to apply a model. In this case, changing the practice item difficulty is fairly easy, but this also suggests we might want to improve student's motivation, or the work to change the exam prep will

SPSS GLM Analysis

We could obtain the same model, and a bit more info about it, using GLM! The important difference between running this model in multiple regression and in GLM is that we used dummy-coded categorical variables in multiple regression, but we will use the original categorical variables in the GLM and SPSS will do the coding for us. We will, however, still do the mean centering and compute the quadratic terms.

compute prac_mcen = prac - 5.8182.	← mean-centering practice
compute pristats_mcen = pristats - 2.3986.	← mean-centering # prior stats classes
compute motv_mcen = motv - 51.0629.	← mean-centering motivation
compute prac_cen_quad = prac_mcen ** 2.	← quadratic term for # practices
compute pristats_cquad = pristats_mcen ** 2.	← quadratic term for # prior stats clasxses
compute motv_cquad = motv_mcen ** 2.	← quadratic term for motivation
exe.	

UNIANOVA perfperc	← list the DV
BY interst_1qnt_2rsh_3tch prac1e2s prog1exp_2clin	← list the categorical variables – SPSS will code these with the highest valued group as the comparison group
WITH prac_mcen prac_cen_quad pristats_mcen pristats_cquad motv_mcen motv_cquad	← list the mean-centered quant variables and the quad terms
/METHOD=SSTYPE(3)	← asks for unique effects model (same as multiple regression)
/PRINT=PARAMETER	← gets the regression weights
/EMMEANS=TABLES (interst_1qnt_2rsh_3tch) COMPARE (interst_1qnt_2rsh_3tch)	← gets the corrected/expected means and pairwise comparisons among the career intention groups
/EMMEANS=TABLES (prac1e2s) COMPARE (prac1e2s)	← gets the corrected/expected means and comparison between the practice difficulty groups
/EMMEANS=TABLES (prog1exp_2clin) COMPARE (prog1exp_2clin)	← gets the corrected/expected means and comparison between the training programs
/DESIGN=interst_1qnt_2rsh_3tch prac1e2s prog1exp_2clin prac_mcen prac_cen_quad pristats_mcen pristats_cquad motv_mcen motv_cquad.	← specifies the model – notice that there are no interactions in the model – this is a “regular multiple regression” with only main effects included

Hang on – this output goes on for a bit…

Tests of Between-Subjects Effects

Dependent Variable: perfperc

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	5759.785[a]	10	575.978	21.217	.000
Intercept	227259.796	1	227259.796	8371.321	.000
interst_1qnt_2rsh_3tch	1741.776	2	870.888	32.080	.000
prac1e2s	421.864	1	421.864	15.540	.000
prog1exp_2clin	.566	1	.566	.021	.885
prac_mcen	2.604	1	2.604	.096	.757
prac_cen_quad	126.855	1	126.855	4.673	.032
pristats_mcen	19.685	1	19.685	.725	.396
pristats_cquad	17.337	1	17.337	.639	.426
motv_mcen	437.944	1	437.944	16.132	.000
motv_cquad	167.663	1	167.663	6.176	.014
Error	3583.460	132	27.147		
Total	762220.000	143			
Corrected Total	9343.245	142			

a. R Squared = .616 (Adjusted R Squared = .587)

The F-tests in the ANOVA table parallel the t-tests of the regression weights, except for the career interest variable, which is expressed as a 3-group comparison in the F-tests and dummy code-pairwise comparisons in the t-tests.

Parameter Estimates

Dependent Variable: perfperc

Parameter	B	Std. Error	t	Sig.
Intercept	70.730	1.259	56.158	.000
[interst_1qnt_2rsh_3tch=1.00]	9.968	1.342	7.427	.000
[interst_1qnt_2rsh_3tch=2.00]	1.327	1.091	1.216	.226
[interst_1qnt_2rsh_3tch=3.00]	0[a]	.	.	.
[prac1e2s=1.00]	-3.926	.996	-3.942	.000
[prac1e2s=2.00]	0[a]	.	.	.
[prog1exp_2clin=1.00]	-.131	.907	-.144	.885
[prog1exp_2clin=2.00]	0[a]	.	.	.
prac_mcen	-.063	.203	-.310	.757
prac_cen_quad	-.169	.078	-2.162	.032
pristats_mcen	-.371	.435	-.852	.396
pristats_cquad	.268	.336	.799	.426
motv_mcen	.153	.038	4.016	.000
motv_cquad	.006	.003	2.485	.014

a. This parameter is set to zero because it is redundant.

The regression weights are the same values and interpretations as were obtained from the multiple regression model earlier.

As there are no interactions in the model, each of these is an expression of a "unique main effect" – the relationship between variable (or pairwise comparison) and the criterion variable, with all the other variables held constant at zero.

One advantage of using GLM is that it give more complete information about the categorical variables than does he multiple regression, especially for the multiple-category variables (for which it give all possible pairwise comparisons, rather than just the k-1 pairwise comparisons expressed in the dummy code regression weights).

Estimated Marginal Means

1. interst_1qnt_2rsh_3tch

Estimates

Dependent Variable: perfperc

interst_1qnt_2rsh_3tch	Mean	Std. Error
1.00	79.056[a]	.988
2.00	70.415[a]	.707
3.00	69.088[a]	.814

a. Covariates appearing in the model are evaluated at the following values: prac_mcen = .0000, prac_cen_quad = 4.9739, pristats_mcen = .0000, pristats_cquad = 1.0789, motv_mcen = .0000, motv_cquad = 145.5135.

For each categorical variable, the output includes the estimated or corrected mean for each group.

Each mean is the estimated criterion variable score for the members of that group, holding constant all of the other predictors at their mean. Because we mean-centered each quantitative predictor, those means are each zero.

These means are also controlled for the categorical predictors in the model. Specifically, these represent the expected value of the criterion for each group, for the comparison group (coded = 0).

Univariate Tests

Dependent Variable: perfperc

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	1741.776	2	870.888	32.080	.000
Error	3583.460	132	27.147		

The F tests the effect of interst_1qnt_2rsh_3tch. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

The F-value is the same as given in the ANOVA table above.

The pairwise mean differences represent the same information as the multiple regression weights from the dummy codes.

The mean difference between group 1 (quant) and 3 (teaching) of 9.968 matches the multiple regression weight for the dummy code representing this comparison above. The significance test is from the same t-value (9.968 / 1.342 = 7.427).

Pairwise Comparisons

Dependent Variable: perfperc

(I) interst_1qnt_2rsh_3tch	(J) interst_1qnt_2rsh_3tch	Mean Difference (I-J)	Std. Error	Sig.[b]
1.00	2.00	8.641*	1.223	.000
	3.00	9.968*	1.342	.000
2.00	1.00	-8.641*	1.223	.000
	3.00	1.327	1.091	.226
3.00	1.00	-9.968*	1.342	.000
	2.00	-1.327	1.091	.226

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

The mean difference of group 2 (research) and 3 (teaching) of 1.327 matches the multiple regression weight for the regression dummy code for this comparison (1327 / 1.091 = 1.216).

The "bonus" of doing the GLM is the comparison of groups 1 & 2, which is not directly available from the multiple regression weights.

2. prac1e2s

Estimates

Dependent Variable: perfperc

prac1e2s	Mean	Std. Error
1.00	70.890[a]	.747
2.00	74.816[a]	.606

a. Covariates appearing in the model are evaluated at the following values: prac_mcen = .0000, prac_cen_quad = 4.9739, pristats_mcen = .0000, pristats_cquad = 1.0789, motv_mcen = .0000, motv_cquad = 145.5135.

Pairwise Comparisons

Dependent Variable: perfperc

(I) prac1e2s	(J) prac1e2s	Mean Difference (I-J)	Std. Error	Sig.[b]
1.00	2.00	-3.926*	.996	.000
2.00	1.00	3.926*	.996	.000

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

For 2-group variables, the GLM output is completely parallel to the information available from the multiple regression weight, except that it does give you the estimated group means, which is convenient!

Univariate Tests

Dependent Variable: perfperc

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	421.864	1	421.864	15.540	.000
Error	3583.460	132	27.147		

The F tests the effect of prac1e2s. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Again, the mean difference is the same as the multiple regression weight for the corresponding dummy code.

For a 2-group variable, the t-test is parallel to the F-test $(3.926 / .996)^2 = 15.540$

3. prog1exp_2clin

Estimates

Dependent Variable: perfperc

prog1exp_2clin	Mean	Std. Error
1.00	72.788[a]	.586
2.00	72.919[a]	.705

a. Covariates appearing in the model are evaluated at the following values: prac_mcen = .0000, prac_cen_quad = 4.9739, pristats_mcen = .0000, pristats_cquad = 1.0789, motv_mcen = .0000, motv_cquad = 145.5135.

Pairwise Comparisons

Dependent Variable: perfperc

(I) prog1exp_2clin	(J) prog1exp_2clin	Mean Difference (I-J)	Std. Error	Sig.[a]
1.00	2.00	-.131	.907	.885
2.00	1.00	.131	.907	.885

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Again, the mean difference is the same as the multiple regression weight for the corresponding dummy code, and the F-test and t-test information is equivalent.

Univariate Tests

Dependent Variable: perfperc

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	.566	1	.566	.021	.885
Error	3583.460	132	27.147		

The F tests the effect of prog1exp_2clin. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.