1-Sample t-test -- Analysis of the mean value for a single sample/population

Application: To test whether the mean of a sample is equivalent to the hypothesized mean of a particular population. Most folks are familiar with the t-test for comparing the means of two independent groups and the t-testfor comparing the means of two dependent groups. There is also a third type of t-test, called a Single-Sample t-test. The Single Sample version is used when you want to test whether the mean of a single sample is different from some specific "hypothesized" or "population" mean. This t-test is very useful to help determine if your sample represents the population of interest, by computing one or more single sample t-tests to see if the demographics of your sample are consistent with what you would expect of a representative sample.

H0: The mean of the sample equals the hypothesized mean of the population -- symbolically expressed as **H0: X = ▄▄**

To reject H0 : is to say that the sample mean is different from (either larger or smaller) than the hypothesized population mean.

The data: You have collected the age of each participant in your study.

Research Hypothesis : You intend the sample to represent a population with a mean age of 20 years.

⮞ **The RH: and H0: are often the same for this analysis !**

H0: for this analysis : The mean age of the sample is equivalent to (not significantly different from) 20 years.

The steps for calculating a Single-Sample t-test "from scratch" are:

1. Determine the "hypothesized" or "population" mean $\mu = 20$

2. Compute the sample mean $\overline{X}$ = $\sum X / n$ =   19+18+17+17+19+18+20+19+20 / 9   =   18.56

3. Compute $\sum X^2$ $\sum X^2$ = $19^2 + 18^2 + 17^2 + 17^2 + 19^2 + 18^2 + 20^2 + 19^2 + 20^2$ =   3109

4. Compute SS (Sums of Squares)

$$SS = \sum X^2 - \frac{(\sum X)^2}{n} = 3109 - \frac{(167)^2}{9} = 10.22$$

5. Compute s^2 (variance) s^2 =   SS / (n-1)   =   10.22 / 8 = 1.28

6. Compute SEM (Standard Error of the Mean)   SEM   = $\sqrt{s^2 / n}$ = $\sqrt{1.28 / 9}$ =   .38

7. Compute t   t = $(\mu - X) / SEM$ =   (20 - 18.6) / .38   =   3.68

8. Determine degrees of freedom for t   df = n - 1   =   9 - 1   =   8

9. Determine critical value for t with that degrees of freedom and = .05   t(8) = 2.31

10. Compare the ABSOLUTE VALUE of your obtained t (3.68) to the critical value.

    a. If the absolute value of your obtained t is larger than the tabled value, then the sample is significantly different from the "hypothesized" or "population" mean.
    b. If the absolute value of your obtained t is smaller than the tabled value, then the sample is not significantly different from the "hypothesized" or "population" mean.

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Our outcome conforms to Step 10(a), so we would conclude that our sample is younger
than the population it was to represent.  Thus, we should NOT use the results from this
sample to represent the college population.
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Reporting the Results of the single-sample t-test includes two parts, the verbal part and the report of the t-value, degrees of freedom (parenthetically) and the associated p-value. For these data that would be:

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      Statistical analysis revealed that the sample was younger than intended, with a
mean age of 18.56 years,rather than the intended mean of 20 years (t(9) = 3.68, p<.05).
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Other ways to calculate single-sample t-tests

Often, when you want to perform a 1-sample t-test, you will not have access to the raw data. For example, a researcher reports their interest in sampling from a population with a typical salary of $25,000, and that the mean salary of their sample was $17,500 with a standard error of the mean of $1,500. You might want to do the 1-sample t-test to see if their "wants" and "gots" agreed. But, you don't have the raw data, only the null mean ($25,000), sample mean ($17,500) and SEM ($1,500).

The "basic" formula for a 1-sample t-test (as shown above) is:
This is the one you would use, given the information you have from the story.

$$t = \frac{\mu - \bar{X}}{SEM}$$

Here are two other formulas for the single-sample t-test -- notice that they differ only in terms of the measure of variability (standard deviation "std" OR variance "var").

$$t = \frac{\mu - \bar{X}}{std / \sqrt{n}} \qquad\qquad t = \frac{\mu - \bar{X}}{\sqrt{var / n}}$$

So, there you have different 1-sample t-test formulas, depending upon whether you start with raw data, mean & SEM, mean & standard deviation, or mean & variance.

t-Table   Critical values of t for a= .05 & a = .01

df	α = .05	α = .01		df	α = .05	α = .01
1	12.71	63.66		18	2.10	2.88
2	4.30	9.92		19	2.09	2.86
3	3.18	5.84		20	2.09	2.84
4	2.78	4.60		21	2.08	2.83
5	2.57	4.03		22	2.07	2.82
6	2.45	3.71		23	2.07	2.81
7	2.36	3.50		24	2.06	2.80
8	2.31	3.36		25	2.06	2.79
9	2.26	3.25		26	2.06	2.78
10	2.23	3 17		27	2.05	2.77
11	2.20	3.11		28	2.05	2.76
12	2.18	3.06		29	2.04	2.76
13	2.16	3.01		30	2.04	2.75
14	2.14	2.98		40	2.02	2.70
15	2.13	2.95		60	2.00	2.66
16	2.12	2.92		120	1.98	2.62
17	2.11	2.90		Infinity	1.96	2.58