The ANOVA for2 Dependent Groups -- Analysis of 2-Within (or Matched)-Group Data with a Quantitative Response Variable

Application: This statistic has two applications that can appear very different, but are really just two variations of the same statistical question. In one application the same quantitative variable is measured under two different conditions from the same sample (or from two or more samples that have been matched on some important variable). In the other application, two or more comparable quantitative variables are measured from the same sample (usually at the same time). In both applications, the dependent ANOVA is used to compare the means of the quantitative variables.

There are two specific versions of the H0:, depending upon whether one characterizes the data as representing a single population under two or more different conditions (e.g., comparing treated vs. not treated or comparing different treatments -- some consider this a representation of two or more different populations) or as representing comparable variables measured from a single population (as in the example below). Here are versions of the H0: statement for each of these characterizations.

H0: The population represented by the sample has the same mean on the quantitative variable under the "k"conditions.

To reject H0: is to say that the population represented by the sample has different means under the "k" conditions.

H0: The population represented by the sample has the same mean on the "k" different quantitative measures.

To reject H0: is to say that the means of the "k" variables are different in the population represented by the sample.

The data: In this analysis (which corresponds to the second application described above) the two quantitative variables are the quality ratings of two different types of animals. From our database, we use the variables **reptgood** (reptile quality rated on a 1-10 scale) and **fishgood** (fish quality rated on a 1-10 scale). These scores are shown for the 12 stores below (fishgood, reptgood).

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2,6  8,5   9,3   7,3  4,7  7,9   4,9   4,8   5,6   9,9   7,7   2,8
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Research Hypothesis: The researcher hypothesized that a store's fish would be of higher quality than its reptiles, because of the greater difficulty obtaining and maintaining healthy reptiles.

H0: for this analysis: The quality ratings of reptiles and fish displayed by pet stores have the same means.

Note: **k** = the # of conditions **n** = # of data points in a condition **N** = total # of data points

Assemble the data for analysis. Rearrange the data so that scores from each participant are in the appropriate columns, one for each condition.

reptgood (k1)	fishgood (k2)
X	X
2	6
8	5
9	3
7	3
4	7
7	9
4	9
4	8
5	6
9	9
7	7
2	8

Compute the square of each score and place each in an adjacent column.

reptgood (k1)		fishgood (k2)	
X	X^2	X	X^2
2	4	6	36
8	64	5	25
9	81	3	9
7	49	3	9
4	16	7	49
7	49	9	81
4	16	9	81
4	16	8	64
5	25	6	36
9	81	9	81
7	49	7	49
2	4	8	64

Compute S **X and** S **X²** for each condition or variable, the S **S²** (the sum of the squared Subject totals) and determine the **sample size** for each.

repgood (k_1)		fishgood (k_2)	
ΣX_{k1}	68	ΣX_{k2}	80
ΣX_{k1}^2	454	ΣX_{k2}^2	584
n_{k1}	12	n_{k2}	12

Compute the mean and std for each condition or variable.

Example using data from the repgood:

$$\text{Mean} = \frac{\Sigma X_{k1}}{n_{k1}} = \frac{68}{12} = 5.67$$

$$\text{Standard deviation} = \sqrt{\left(\frac{\Sigma X_{k1}^2 - [(\Sigma X_{k1})^2/n_{k1}]}{n_{k1}-1}\right)} = \sqrt{\left(\frac{454 - [(68)^2/12]}{12-1}\right)} = 2.50$$

Compute the subject total and the **squared subject total for each participant**

reptgood (k1)	fishgood (k2)	Subject Total	Squared Subject Total
X	X	S	S^2
2	6	8	64
8	5	13	169
9	3	12	144
7	3	10	100
4	7	11	121
7	9	16	256
4	9	13	169
4	8	12	144
5	6	11	121

9	9	18	324
7	7	14	196
2	8	10	100

Compute the sum of squared subject totals -- ΣS^2

$$\Sigma S^2 = 64 + 169 + 144 + 100 + 121 + 256 + 169 + 144 + 121 + 324 + 196 + 100 = 1908$$

Compute SS_{Total} -- Please note: N = the total number of data values (not the number of participants)

$$SS_{Total} = (\Sigma X_{k1}^2 + \Sigma X_{k2}^2) - \frac{(\Sigma X_{k1} + \Sigma X_{k2})^2}{N}$$

$$= (454 + 584) - \frac{(68 + 80)^2}{24} = 1038 - \frac{148^2}{24}$$

$$= 1038 - \frac{21904}{24} = 1038 - 912.67 = 125.33$$

Compute SS_{Cond} (also called SS_{Effect} or SS_{IV} for experimental data) -- Please note: n = the number of participants (data values in each condition). Also, you should notice that the "right side" of this step is the same as for Step 5.

$$SS_{Cond} = \frac{(\Sigma X_{k1})^2 + (\Sigma X_{k2})^2}{n} - \frac{(\Sigma X_{k1} + \Sigma X_{k2})^2}{N}$$

$$= \frac{68^2 + 80^2}{12} - \frac{(68 + 80)^2}{24} =$$

$$= \frac{4624 + 6400}{12} - 912.67 = \frac{11024}{12} - 912.67 =$$

$$= 918.67 - 912.67 = 6.00$$

Compute $SS_{subject}$ You should notice that the "right side" of this step is the same as in Steps 5 & 6.

$$SS_{subject} = \frac{\Sigma S^2}{k} - \frac{(\Sigma X_{k1} + \Sigma X_{k2})^2}{N} = \frac{1908}{2} - 912.67$$

$$= 954 - 912.67 = 41.33$$

Compute SS_{Error} (also called $SS_{Residual}$)

$$SS_{Error} = SS_{Total} - SS_{Cond} - SS_{Subject} = 125.33 - 6.00 - 41.33 = 78.0$$

Compute df_{Cond} (also called df_a)

$$df_{Cond} = k - 1 = 2 - 1 = 1$$

Compute MS_{Cond}

$$MS_{Cond} = \frac{SS_{cond}}{df_{cond}} = \frac{6.00}{1} = 6.00$$

Compute df_{Total} Remember, N = total number of data values.

$$df_{Total} = N - 1 = 24 - 1 = 23$$

Compute $df_{Subjects}$

$$df_{Subjects} = n - 1 = 12 - 1 = 11$$

Compute df_{Error} (also called $df_{Residual}$)

$$df_{Error} = df_{Total} - df_{Cond} - df_{Subjects} = 23 - 1 - 11 = 11$$

Compute MS_{Error} (also called $MS_{Residual}$)

$$MS_{Error} = \frac{SS_{Error}}{df_{Error}} = \frac{78.0}{11} = 7.09$$

Compute F (also called the F-obtained, F-computed, or F-omnibus)

$$F = \frac{MS_{Cond}}{MS_{Error}} = \frac{6.00}{7.09} = 0.846$$

Prepare the Summary Table (Also called a Source Table)

The Summary Table for a dependent groups design looks a bit different than the one for independent groups ANOVA that you learned about last time, because we have to add a row for the "subjects" source of variance. The values come from the ANOVA example. Again, the total degrees of freedom is computed as the sum of the condition, subject, and error degrees of freedom. df = degrees of freedom, SS = sum of squares, and MS = mean squares

Source	df	SS	MS	F	p
Cond	1	6.00	6.00	0.846	> .05
Subj	11	41.33			
Error	11	78.0	7.09		
Total	23	125.33			

Look up the F-critical for α = .05 and the appropriate degrees of freedom using the F-table.
Use numerator degrees of freedom = df_{IV} = 1 and denominator degrees of freedom = df_{Error} = 11.

$$F(1, 11, \alpha = .05) = 4.84$$

Determine whether to retain or reject H0:

- if the obtained F is less than the critical F, then retain the null hypothesis -- conclude that the populations represented by the different conditions o the qualitative grouping variable have the same mean score on the quantitative variable
- if the obtained F is greater than the critical F, then reject the null hypothesis -- conclude that the populations represented by the different conditions of the qualitative grouping variable have different mean scores on the quantitative variable

For the example data, we would decide to retain the null hypothesis, because the obtained F value of 0.846 is less than the critical F value of 4.84.

Determine whether the pattern of the mean differences support or does not support the research hypothesis.

- Usually the researcher hypothesizes that there is a difference between the conditions. If so, then to support the research hypothesis will require:
  - Reject H0: that there is no mean difference
  - The mean difference must be in the same direction as that specified in the research hypothesis

- Sometimes, however, the research hypothesis is that there is **no** difference between the conditions. If so, the research hypothesis and H0: are the same!
  - When this is the case, retaining H0: provides support for the research hypothesis, whereas rejecting H0: provides evidence that research hypothesis is incorrect.

- **Please note:** When you have decided to retain H0: (because F < F-critical), then don't talk about one group having a mean that is larger or smaller than the other -- retaining H0: is saying that the means are the same (and any apparent difference is probably due to sampling variation, chance, etc.)

For the example data, since we retained the H0: (deciding that there is not a mean difference between the average reptile quality and average fish quality) we would conclude that there is no support for the research hypothesis that stores will have higher quality fish than reptiles

Describe the results of the Dependent Groups Analysis of Variance-- be sure to include the following

- Name the conditions or quantitative variables and tell the mean and standard deviation for each
- Tell the F-value, df (in parentheses) and p-value (p < .05 or p > .05).
- If you reject H0:, tell which condition/variable has the larger mean
  - If you retain H0:, tell that the condition/variable means aren't significantly different
- Tell whether or not the results support the research hypothesis

Please note: Reporting ANOVA results is not a form of "creative writing". The idea is to be succinct, clear, and follow the prescribed format -- it is really a lot like completing a fill-in-the-blanks sentence. After you write and read enough of these you'll develop some "style", but for now just follow the format.

Here are two write-ups of these results that say the same thing. The 1st reports the univariates and then the significance test. The 2nd combines them into a single sentence -- either is fine.

The mean rating for fish quality was 5.67 (S = 2.50) and the mean rating for reptile quality was 6.67 (S = 2.15). Contrary to the research hypothesis there was not a significant difference between these means, F(1,11) = 0.846, p > .05, MSe = 7.09.

Contrary to the hypothesis there was not a significant difference between the mean rating for fish quality (M =5.67, S = 2.50) and the mean rating for reptile quality (M = 6.67, S = 2.15), F(1,11) = 0.846, p > .05, MSe = 7.09.

You should review the section in the Between Groups ANOVA called "Write-up Examples for Every Occasion" for examples of write-ups for other combinations of RH:, mean patterns and H0: decisions.

Here is a version of the write-up using a Table to present the univariate statistics.

Table 1 summarizes the data for the quality ratings of animals at each of the stores. Contrary to the hypothesis there was not a significant difference between the mean rating for fish quality (M =5.67, S = 2.50) and the mean rating for reptile quality (M = 6.67, S = 2.15), $F(1,11) = 0.846$, $p > .05$.

Table 1
Number of animals of each type displayed in the pet stores.

	Type of Animal	
Quality of Animals Displayed	Fish	Reptiles
M	5.67	6.67
SD	2.50	2.15
n	12	12

Finally, here is what a Figure representing these data would look like. The height of the bars represents the mean of each condition and the "whiskers" show one std above and below the mean of each condition.

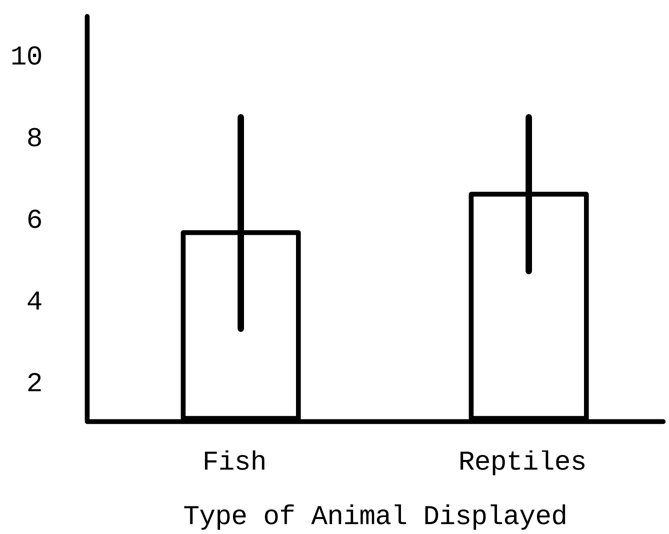

Figure 1.
Mean quality for each type of animal (+/- 1 std shown)