Pearson's 2x2 Chi-Square Test of Independence -- Analysis of the Relationship between two Qualitative (Binary) Variables

or **Analysis of 2-Between-Group Data with a Qualitative (Binary) Response Variable**

Application: This statistic has two applications that can appear very different, but are really just two variations of the same question. The first application is to compare the distributions of scores on a qualitative response variable) obtained from 2 or more groups that represent different populations. Thus, it is applied in the same data situation as an ANOVA for independent samples, except that it is used when the response variable is a qualitative variable. The other application is to test for a pattern of relationship between two qualitative variables -- in this respect, it is something like a correlation (though looking for a *pattern of* relationship between qualitative variables, rather than a *linear* relationship between quantitative variables).

There are two versions of the H0:, depending upon whether one characterizes the analysis as a test of whether the populations represented by one of the variables differ in their patterns of response to the response variable (the other variable -- which corresponds to the first application described above) or as a test of whether there is a relationship between the two variables in the single population represented by the sample as a whole (which corresponds to the second application described above). The H0: for each is given below.

H0: The populations represented by the two conditions have the same pattern of responses across the categories of the response variable.

To reject H0: is to say that the two populations differ in their response patterns to the categories of the response variable.

H0: The variables have no pattern of relationship between the variables within the population represented by the sample.

To reject H0: is to say that there is a pattern of relationship between the variables in the population.

The data: The analysis involves the variables **reptdept** (1 = not separate reptile department, 2 = separate reptile department) and **fishdept** (1 = only freshwater fish available, 2 = fresh and saltwater fish available).

1,1 1,1 1,2 1,1 2,1 2,2 2,2 2,2 2,2 1,1 1,1 2,2

Researcher Hypothesis: The researcher hypothesized that stores without a separate reptile department would be more likely to display only freshwater fish, whereas those stores with a separate reptile department would be more likely to display both freshwater and saltwater fish,

H0: for this analysis: There is no pattern of relationship between whether or not pet stores have separate reptile departments and whether they display only freshwater fish or both freshwater and saltwater fish.

Organize the scores into a contingency table. Since both of these categorical variables have two categories, the contingency table will be a 2x2, for a total of 4 cells, as shown below.

Each store's data will be collated into one of the four cells. For example, a store that did not have a separate reptile department and that displayed only freshwater fish would be tallied into the cell in the upper left; a store that had a separate reptile department and displayed both fresh- and saltwater fish would be tallied into the cell in the lower right. Below is the contingency table filled with the responses from the 12 stores. These values are the obtained frequency (**of**) for each of the cells.

	Fishdept	
Reptdebt	freshwater	fresh- & saltwater
Not separate	5	1
Separate	1	5

Compute the row totals (sum across the values on the same row) and **column totals** (sum across the values on the same column) and the **overall total** of the observed frequencies. As a computational check, be sure that the row totals and the column totals sum to the same value for the overall total.

Reptdebt	Fishdept: freshwater	Fishdept: fresh- & saltwater	row totals
Not separate	5	1	6
Separate	1	5	6
column totals	6	6	overall total 12

Compute the expected frequency (ef) of each cell. This expected frequency is computed as the product of the row total and the column total for that cell, divided by the overall total. For example, the upper left-hand cell (stores not having a separate reptile department that display only freshwater fish) has an expected frequency of:

$$ef = \frac{\text{row total} * \text{col total}}{\text{overall total}} = \frac{6 * 6}{12} = 3.00$$

Reptdebt	Fishdept: freshwater of	freshwater ef	fresh- & saltwater of	fresh- & saltwater ef	row totals
Not separate	5	3	1	3	6
Separate	1	3	5	3	6
column totals	6		6		overall total 12

Compute Chi-Square

$$X^2 = \sum \frac{(of - ef)^2}{ef} = \frac{(5 - 3)^2}{3} + \frac{(1 - 3)^2}{3} + \frac{(5 - 3)^2}{3} + \frac{(1 - 3)^2}{3} = 5.33$$

Compute the degrees of freedom df

df = (number of columns - 1) * (number of rows - 1) = (2-1) * (2-1) = 1 * 1 = 1

Look up the X^2-critical for α = .05 and the appropriate degrees of freedom using the X^2-table.

$$X^2(df=1, p=.05) = 3.84$$

Determine whether to retain or reject H0:

-- if the obtained X^2 is less than the critical X^2, then retain the null hypothesis -- conclude that there is no relationship between values on one categorical variable and values on the other categorical variable, in the population represented by the sample

-- if the obtained X^2 is greater than the critical X^2, then reject the null hypothesis -- conclude that there is a relationship between the values on one categorical variable and values on the other categorical variable, in the population represented in the sample.

`For the example data, we would decide to reject the null hypothesis, because the obtained Chi-square value of 29.99 is larger than the critical Chi-square value of 5.991.`

By the way: This test should only be applied when at least 80% of the cells have expected frequencies (**ef**) of five or larger. Applying the test when there are fewer cells with this minimum expected frequency can lead to inaccurate results.

Determine whether or not the results support the research hypohtesis.

- Usually the researcher hypothesizes that there is a pattern of relationship between the variables. If so, then to support the research hypothesis will require:
- Reject H0: that there is no pattern of relationship between the variables
- The pattern of the relationship must be the same as that specified in the research hypothesis
  - If you reject the null hypothesis, and if the pattern of data in the contingency table **agrees exactly** with the research hypothesis, then the research hypothesis is **completely supported**.
  - If you reject the null hypothesis, and if **part** of the pattern of data in the contingency table agrees with the research hypothesis, but part of the pattern of data does not, then the research hypothesis **partially supported**.
  - If you retain the null hypothesis, or you reject the null but **no part** of the pattern of data in the contingency table agrees with the research hypothesis, then the research hypothesis is **not at all supported**.

- Sometimes, however, the research hypothesis is that there is **no** pattern of relationship between the variables. If so, the research hypothesis and H0: are the same!
  - When this is the case, retaining H0: provides support for the research hypothesis, whereas rejecting H0: provides evidence that research hypothesis is incorrect.

- **Please note:** When you have decided to retain H0: (because $X^2 < X^2$-critical), then don't talk about there being a pattern of relationship between the variables -- retaining H0: is saying that there is no pattern of relationship between the variables in the population (and any apparent pattern of relationship is probably due to sampling variation, chance, etc.)

`Consistent with the research hypothesis, those stores without separate reptile departments tended to display only freshwater fish whereas those stores that had separate reptile departments tended to display both freshwater and saltwater fish.`

Describe the results of the correlation analysis -- be sure to include the following

- Name each variable and tell the univariate statistics (the frequencies in the conditions of each qualitative variable)
- The X^2-value, df (in parentheses) and p-value ($p < .05$ or $p > .05$).
- If you reject H0:, then describe the pattern of the relationship between the variables
  - If your retain the H0:, then say that there is no significant pattern of relationship between the variables
- Whether or not the results support the research hypothesis

Please note: Reporting correlation results is not a form of "creative writing". The idea is to be succinct, clear, and follow the prescribed format -- it is really a lot like completing a fill-in-the-blanks sentence. After you write and read enough of these you'll develop some "style", but for now just follow the format.

Please note: Describing the results of a X^2 analysis -- the pattern of a relationship between two qualitative variables -- requires more precision and "more words" than describing the results of a correlation or a mean comparison. Take your time and give a complete description!

X^2 write-ups almost always include a table showing the cell and marginal counts. Also, notice how the write-up tells the univariates (the frequencies of the conditions for each qualitative variable) and then the significance test.

Table 1 shows the 2x2 table of these variables. The sample of stores was evenly divided between the two types of reptile departments and also evenly divided between the two types of fish departments. As hypothesized, those stores with separate reptile departments tended to have both fresh- and saltwater fish, whereas, those stores without separate reptile departments tended to have only freshwater fish, $X^2(1) = 5.33$, $p = .021$.

Table 1
Relationship between Type of Reptile Department and Type of Fish Available

	Type of Reptile Department		
Type of Fish Available	Not separate Department	Separate Department	total
Freshwater Fish Only	5	1	6
Fresh- and Saltwater Fish	1	5	6
total	6	6	12

"X^2 Write-up Examples for Other Occasions"

Here's are examples (not all using the same data as the computational example above) of what we would write for different combinations of RH:, contingency table results & significance test results.

For ..

- RH: that stores with separate reptile departments would tend to have fresh/saltwater fish, where those without separate reptile departments would tend to have only freshwater fish
- Rejected H0:
- Found a pattern of relationship between the variables opposite that of the RH:

	Fishdept		
Reptdep	fresh	fresh/salt	
not sep	1	5	6
separate	5	1	6
	6	6	

The sample of stores was evenly divided between the two types of reptile departments and also evenly divided between the two types of fish departments. Contrary to the research hypothesis, those stores with separate reptile departments tended to have only freshwater fish, whereas, those stores without separate reptile departments tended to have both fresh- and saltwater fish, $X^2(1) = 5.33$, $p = .021$.

For …

- RH: that stores with separate reptile departments would tend to have fresh/saltwater fish, where those without separate reptile departments would tend to have only freshwater fish
- Rejected H0:
- Found a pattern of relationship between the variables partially supporting the RH:

	Fishdept		
Reptdep	fresh	fresh/salt	
not sep	6	6	12
separate	1	11	12
	7	17	

The sample of stores was evenly divided between the two types of reptile departments and 71% displayed both fresh- and saltwater fish. Consistent with the research hypothesis, those stores with separate reptile departments tended to have both fresh- and saltwater fish, however, contrary to the research hypothesis those stores without separate reptile departments tended to be evenly divided between having only freshwater fish and having both fresh- and salt-water fish, $X^2(1) = 5.04$, $p = .025$.

For ..

- RH: that stores with separate reptile departments would tend to have fresh/saltwater fish, where those without separate reptile departments would tend to have only freshwater fish
- Retained H0:

The sample of stores was evenly divided between the two types of reptile departments and also about evenly divided between the two types of fish departments. Contrary to the research hypothesis, there was no significant pattern of relationship between these two variables, $\underline{X}^2(1) = .343$, $\underline{p} = .558$.

	Fishdept		
Reptdep	fresh	fresh/salt	
not sep	3	3	6
separate	4	2	6
	7	5	