

The ANOVA for Independent Groups -- Analysis of k-Between-Group Data with a Quantitative Response Variable

Application: To compare means of a quantitative response variable obtained from 2 or more independent groups. Thus, ANOVA can be thought of as being very much like a t-test, except that it can be applied to either 2 or more groups.

About the terms used: Descriptions of ANOVA are often in terms of the IV and the DV. However, this can be confusing when working with non-experimental data. In addition, different terminology is often used for between groups ANOVA than for within-groups ANOVA, adding to the confusion. The terms **Condition** and **Error** will be used below for two reasons. First, it will allow you to see that the between groups ANOVA and within-groups ANOVA do basically the same thing -- comparing size of the mean differences (variability associated with which condition the score came from) to the variability around those means (an index of error). Second, it avoids the use of "causal words".

H0: The populations represented by the conditions of the qualitative variable have the same mean on the quantitative response variable.

To reject H0: is to say that the populations represented by the conditions of the qualitative variable have different means on the quantitative response variable.

The data: This analysis involves the grouping variable **chain** (1 = chain store, 2 = privately owned store, 3 = coop owned store) and the response variable **fishnum** (number of fish on display). Below are the scores for the 12 stores (chain, fishnum).

3,32 3,41 3,31 3,38 1,21 1,13 2,17 1,22 2,24 1,11 2,17 1,20

Research Hypothesis: The researcher hypothesized that Coop stores would display the most fish, Chain stores would display the least, and Private pet stores would display an intermediate amount.

H0: for this analysis : The three different types of pet shops display the same mean number of fish displayed.

Step 1 Assemble the data for analysis. Rearrange the data so that the subjects in each condition are in the same column . There should be as many columns as there are conditions. Label the quantitative variable as X and each condition of the grouping variable as a different value of k, to simplify the presentation and use of the formulas below.

chain store (k1)		private store (k2)		coop store (k3)	
X		X		X	
21		17		32	
13		24		41	
22		17		31	
11				38	
20					

Step 2 Compute the square of each score and place it in an adjacent column.

chain store (k1)		private store (k2)		coop store (k3)	
X	X ²	X	X ²	X	X ²
21	441	17	289	32	1024
13	169	24	576	41	1681
22	484	17	289	31	961
11	121			38	1444
20	400				

Step 3 Compute $\sum X$, $\sum X^2$ for each group, also determine the sample size for each group.

	chain store (k1)	private store (k2)	coop store (k3)
$\sum X$	87	58	142
$\sum X^2$	1615	1154	5110
$n_1 =$	5	$n_2 =$ 3	$n_3 =$ 4

Step 4 Compute SS_{Total} , a measure of the total variability among all the scores (ignoring that the scores come from different groups).

$$\begin{aligned}
 SS_{Total} &= \frac{(\sum X_{k1} + \sum X_{k2} + \sum X_{k3})^2}{N} - \frac{(\sum X_{k1}^2 + \sum X_{k2}^2 + \sum X_{k3}^2)}{N} \\
 &= \frac{(87 + 58 + 142)^2}{12} - \frac{(1615 + 1154 + 5110)}{12} \\
 &= 7879 - \frac{(287)^2}{12} = 7879 - 6864.08 = 1014.92
 \end{aligned}$$

Step 5 Compute SS_{Cond} (sometimes referred to as SS_{BG} for between groups of SS_{IV} if the data are from an experiment), a measure of the variability (difference) among the means from the different groups. (Notice that the "right side" of this step is the same as for Step 4 -- saves some work).

$$\begin{aligned}
 SS_{Cond} &= \left[\frac{(\sum X_{k1})^2}{n_1} + \frac{(\sum X_{k2})^2}{n_2} + \frac{(\sum X_{k3})^2}{n_3} \right] - \frac{(\sum X_{k1} + \sum X_{k2} + \sum X_{k3})^2}{N} \\
 &= \left[\frac{87^2}{5} + \frac{58^2}{3} + \frac{142^2}{4} \right] - 6864.08 = 7676.13 - 6864.08 = 812.05
 \end{aligned}$$

Step 6 Compute SS_{Error} (also called SS_{WG} or SS_{Within}), a measure of the variability among the scores within each condition. (Notice that the "right side" of this equation is the same as the "left side" of Step 5, also the "left side" of this step is the same as for Step 4-- saves some computation).

$$\begin{aligned}
 SS_{Error} &= (\sum X_{k1}^2 + \sum X_{k2}^2 + \sum X_{k3}^2) - \left[\frac{(\sum X_{k1})^2}{n_1} + \frac{(\sum X_{k2})^2}{n_2} + \frac{(\sum X_{k3})^2}{n_3} \right] \\
 &= 7879 - 7676.13 = 202.87
 \end{aligned}$$

Step 7 Perform a computational check to be sure that $SS_{Total} = SS_{Cond} + SS_{Error}$.

$$1014.92 = 812.05 + 202.87$$

Step 8 Compute df_{Cond} (also called df_{BG})

$$df_{\text{Cond}} = k - 1 = 3 - 1 = 2$$

Step 9 Compute MS_{Cond}

$$MS_{\text{Cond}} = \frac{SS_{\text{Cond}}}{df_{\text{Cond}}} = \frac{812.05}{2} = 406.03$$

Step 10 Compute df_{Error} (also called df_{WG} or df_{Within})

$$df_{\text{Error}} = N - k = 12 - 3 = 9$$

Step 11 Compute MS_{Error} (Also called the MSE for mean squared error)

$$MS_{\text{Error}} = \frac{SS_{\text{Error}}}{df_{\text{Error}}} = \frac{202.87}{9} = 22.54$$

Step 12 Compute the omnibus F (also called the obtained F)

$$F = \frac{MS_{\text{Cond}}}{MS_{\text{Error}}} = \frac{406.03}{22.54} = 18.01$$

Step 13 Use Table F to determine the critical value of F for $\alpha = .05$ and the appropriate degrees of freedom
numerator degrees of freedom = $df_{\text{IV}} = 2$ and denominator degrees of freedom = $df_{\text{Error}} = 9$.

$$F(2, 9, \alpha = .05) = 4.26$$

Step 14 Compare the obtained F and critical F, and determine whether to reject or retain the null hypothesis.

- if the obtained F is less than the critical F, then retain the null hypothesis -- conclude that the populations represented by the different conditions of the qualitative grouping variable have the same mean score on the quantitative variable
- if the obtained F is greater than the critical F, then reject the null hypothesis -- conclude that the populations represented by the different conditions of the qualitative grouping variable have different mean scores on the quantitative variable

For the example data, we would decide to reject the null hypothesis, because the obtained F value of 18.01 is larger than the critical F value of 4.26.

Step 15 IF you reject the null hypothesis, determine whether the pattern of the mean differences supports, partially supports, or does not support the research hypothesis.

- IF you reject the null hypothesis, AND if the pattern of means for the conditions agrees exactly with the research hypothesis, then the research hypothesis is completely supported.
- IF you reject the null hypothesis, AND if part of the pattern of means for the conditions agrees part, but not all of the research hypothesis, then the research hypothesis is partially supported.
- IF you retain the null hypothesis, OR if you reject the null BUT no part of the pattern of data in the contingency table agrees with the research hypothesis, then the research hypothesis is not at all supported.

By the Way: To properly determine if the hypothesized pattern of mean differences was found, one should perform pairwise comparisons (using one of the procedures described in the next section); the report of the results given below are based on informal "inspection" of the means.

By the Way: Usually the researcher hypothesizes that there is a difference between the conditions. Sometimes, however, the research hypothesis is that there is NO difference between the conditions. If so, the research hypothesis and H_0 are the same! When this is the case, retaining H_0 provides support for the research hypothesis, whereas rejecting H_0 provides evidence that research hypothesis is incorrect.

The researcher hypothesized that Coop stores would have the most fish on display, Chain stores would display the least, and Private pet stores would display an intermediate amount. This "translates" into an expected pairwise difference between all pairs of store types $\text{Coop} > \text{Private} > \text{Chain}$ (with the implication that $\text{Coop} > \text{Chain}$).

Based on inspection of the means, we would conclude there is only PARTIAL SUPPORT for the research hypothesis. Coop stores displayed more fish than did Private stores, and also Coop stores displayed more fish than did Chain stores. However, contrary to the research hypothesis, there was not a large difference between the mean number of fish displayed by Private and Chain stores.

Step 16 Preparing a Summary Table (also called a Source Table)

As in the other statistics (Correlation and Chi-square) you must be able to summarize and present the results from an ANOVA analysis. There are two different ways that this is done. The first is called a "Source Table" or "Summary Table" and is a presentation of the intermediate results from the calculation of the F. The values come from the ANOVA shown above -- the table on the left shows which steps in the analysis produced each value, while the one on the right shows the actual values.

df = degrees of freedom, SS = sum of squares, and MS = mean squares

Summary Table Showing
Computational Steps

Source	df	SS	MS	F	p
Cond	8	5	9	12	14
Error	10	6	11		
Total	*	4			

Summary Table Showing
Values from Example Data

Source	df	SS	MS	F	p
Cond	2	812.05	406.03	18.01	<.05
Error	9	202.87	22.54		
Total	11	1014.92			

* the Total df is calculated as the sum of df(Cond) and df(WG) or as $N - 1$.

Step 17 Describing the results of the Independent Groups Analysis of Variance -- be sure to include the following

- Tell the conditions of the qualitative or grouping variable
- Name quantitative variable and tell its mean and standard deviation for each condition of the qualitative variable
- The F-value, df (in parentheses) and p-value ($p < .05$ or $p > .05$).

- Describe the pattern of the data (which group(s) has the larger mean, if there is a significant difference)
- Whether or not the results support the research hypothesis

Please remember, this description of the data is based upon inspection of numerical differences among the means and not the formal "follow-up" analyses you will learn about later.

There were significant differences among the mean numbers of fish displayed by the three types of stores, $F(2,9) = 18.01$, $MSE = 22.54$, $p < .05$. Inspection of the means revealed that, consistent with the research hypothesis, Coop stores displayed more fish ($M = 35.50$, $S = 4.80$) than either Private ($M = 19.33$, $S = 4.04$) or Chain stores ($M = 17.40$, $S = 5.03$). However, contrary to the research hypothesis, there was no difference between the average number of fish displayed by Chain and Private pet stores.

Here is the same write-up using a Table to present the univariate statistics. Tables reduce the amount of parenthetical information that can clutter a write-up, especially when there are multiple conditions.

The number of fish displayed at each type of store is summarized in Table 1. There were significant differences among the mean numbers of fish displayed by the three types of stores, $F(2,9) = 18.01$, $MSE = 22.54$, $p < .05$. Inspection of the means revealed that, consistent with the research hypothesis, Coop stores displayed more fish than either Private or Chain stores. However, contrary to the research hypothesis, there was no difference between the average number of fish displayed by Chain and Private pet stores.

Table 1
Number of Fish Displayed at each Store Type

	Store Type		
	Chain Number of Fish Displayed	Privately Store	Coop Owned Store
<u>M</u>	17.40	19.33	35.50
<u>SD</u>	5.03	4.04	4.80

Table F: ANOVA (F-tests) Critical values of F for $\alpha = .05$ & $\alpha = .01$

Denominator		Numerator df					
df	α	1	2	3	4	5	6
1	.05	161	200	216	225	230	234
2	.05	18.5	19.0	19.2	19.2	19.3	19.3
	.01	98.5	99.0	99.2	99.2	99.3	99.3
3	.05	10.1	9.55	9.28	9.12	9.01	8.94
	.01	34.1	30.8	29.5	28.7	28.2	27.9
4	.05	7.71	6.94	6.59	6.39	6.26	6.16
	.01	21.2	18.0	16.7	16.0	15.5	15.2
5	.05	6.61	5.79	5.41	5.19	5.05	4.95
	.01	16.3	13.3	12.1	11.4	11.0	10.7
6	.05	5.99	5.14	4.76	4.53	4.39	4.28
	.01	13.7	10.9	9.78	9.15	8.75	8.47
7	.05	5.59	4.74	4.35	4.12	3.97	3.87
	.01	12.2	9.55	8.45	7.85	7.46	7.19
8	.05	5.32	4.46	4.07	3.84	3.69	3.58
	.01	11.3	8.65	7.59	7.01	6.63	6.37
9	.05	5.12	4.26	3.86	3.63	3.48	3.37
	.01	10.6	8.02	6.99	6.42	6.06	5.80
10	.05	4.96	4.10	3.71	3.48	3.33	3.22
	.01	10.0	7.56	6.55	5.99	5.64	5.39
11	.05	4.84	3.98	3.59	3.36	3.20	3.09
	.01	9.65	7.21	6.22	5.67	5.32	5.07
12	.05	4.75	3.89	3.49	3.26	3.11	3.00
	.01	9.33	6.93	5.95	5.41	5.06	4.82
13	.05	4.67	3.81	3.41	3.18	3.03	2.92
	.01	9.07	6.70	5.74	5.21	4.86	4.62
14	.05	4.60	3.74	3.34	3.11	2.96	2.85
	.01	8.86	6.51	5.56	5.04	4.69	4.46
15	.05	4.54	3.68	3.29	3.06	2.90	2.79
	.01	8.68	6.36	5.42	4.89	4.56	4.32
16	.05	4.49	3.63	3.24	3.01	2.85	2.74
	.01	8.53	6.23	5.29	4.77	4.44	4.20
17	.05	4.45	3.59	3.20	2.96	2.81	2.70
	.01	8.40	6.11	5.18	4.67	4.34	4.10
18	.05	4.41	3.55	3.16	2.93	2.77	2.66
	.01	8.29	6.01	5.09	4.58	4.25	4.01

Numerator df Denominator								
df	α	1	2	3	4	5	6	
19	.05	4.38	3.52	3.13	2.90	2.74	2.63	
	.01	8.18	5.93	5.01	4.50	4.17	3.94	
20	.05	4.35	3.49	3.10	2.87	2.71	2.60	
	.01	8.10	5.85	4.94	4.43	4.10	3.87	
22	.05	4.30	3.44	3.05	2.82	2.66	2.55	
	.01	7.95	5.72	4.82	4.31	3.99	3.76	
24	.05	4.26	3.40	3.01	2.78	2.62	2.51	
	.01	7.82	5.61	4.72	4.22	3.90	3.67	
26	.05	4.23	3.37	2.98	2.74	2.59	2.47	
	.01	7.72	5.53	4.64	4.14	3.82	3.59	
28	.05	4.20	3.34	2.95	2.71	2.56	2.45	
	.01	7.64	5.45	4.57	4.07	3.75	3.53	
30	.05	4.17	3.32	2.92	2.69	2.53	2.42	
	.01	7.56	5.39	4.51	4.02	3.70	3.47	
40	.05	4.08	3.23	2.84	2.61	2.45	2.34	
	.01	7.31	5.18	4.31	3.83	3.51	3.29	
60	.05	4.00	3.15	2.76	2.53	2.37	2.25	
	.01	7.08	4.98	4.13	3.65	3.34	3.12	
120	.05	3.92	3.07	2.68	2.45	2.29	2.17	
	.01	6.85	4.79	3.95	3.48	3.17	2.96	
200	.05	3.89	3.04	2.65	2.42	2.26	2.14	
	.01	6.76	4.71	3.88	3.41	3.11	2.89	
∞	.05	3.84	3.00	2.60	2.37	2.21	2.10	
	.01	6.65	4.61	3.78	3.32	3.02	2.80	