

Independent Groups Honestly Significant Difference (HSD) -- Pairwise Comparisons for k-Between Group Designs

Application: To perform pairwise comparisons of means of a quantitative variable obtained from 3 or more independent groups -- this analysis is usually used as a follow-up to a rejecting H_0 : from an Independent groups ANOVA.

The procedure has the same two steps as the LSD, however the minimum significant mean difference is computed differently. Remember, the HSD procedure is more conservative than the LSD procedure (minimizing Type I errors), but at the possible cost of increasing Type II errors. Thus, for a given set of data, the minimum mean difference using the HSD procedure will always be larger than the minimum mean difference based on the LSD procedure.

The data for this example are taken from the Independent Groups ANOVA example above.

Step 1 Arrange the conditions so that the means are in descending order.

	Coop Store	Private Store	Chain Store
	35.50	19.33	17.40
Coop Store	35.50		
Private Store	19.33		
Chain Store	17.40		

Step 2 Compute the pairwise mean differences between conditions. We need fill-in only the bottom triangle, because the upper diagonal would show the same information.

	Coop Store	Private Store	Chain Store
	35.50	19.33	17.40
Coop Store	35.50		
Private Store	19.33	16.17	
Chain Store	17.40	18.10	1.93

Step 3 Obtain the MS_{Error} from the independent groups omnibus-F analysis -- given in Step 11 of that procedure.

From the example data, $MS_{\text{Error}} = 22.54$

Step 4 Obtain n , the number of participants in each condition of the independent groups design. If there is an unequal number of participants in the different conditions, use the average number in the conditions.

From the example data (with unequal n), the average $n = 12 / 3 = 4.00$

Step 5 Obtain k , the number of conditions or means involved in the design

From the example data, $k = 3$ conditions or means

Step 6 Obtain the df_{Error} from the independent groups omnibus-F analysis -- given in Step 10 of that procedure.

From the example data, $df_{\text{Error}} = 9$

Step 7 Use Table Q to find the Studentized range statistic. To use Table Q you must know three values, error df (given by df_{Error} , k (the number of conditions), and the p-value (use $p=.05$). Sometimes, with larger df, the table doesn't include the df you are looking for. When this happens, just use the next smaller df that is included on the table. For example, if you had $df = 33$, you would use the Q for $df = 30$.

For the example data, $Q(df_{\text{Error}} = 9, k = 3, p = .05) = 3.95$

Step 8 Compute d_{HSD} -- the minimum significant pairwise mean difference, based on the HSD procedure

$$d_{\text{HSD}} = Q * \frac{\sqrt{MS_{\text{Error}}}}{\sqrt{n}} = 3.95 * \frac{\sqrt{22.54}}{\sqrt{4}} = 3.95 * \frac{4.75}{2.00} = 3.95 * 2.37 = 9.36$$

Step 9 Compare each of the pairwise mean difference to the d_{HSD} , and determine whether to reject or retain the null hypothesis for each pairwise comparison. Remember to examine each of the pairwise comparisons.

- if the mean differences is less than the d_{HSD} , then retain the null hypothesis -- conclude that the populations represented by those two conditions have the same mean score on the quantitative variable
- if the mean difference is greater than the d_{HSD} then reject the null hypothesis -- conclude that the populations represented by those two conditions have different mean scores on the quantitative variable

For these data, we would conclude that there is a significant difference between the mean number of fish displayed by Coop and Private stores ($16.17 > 9.36$), and a significant difference between the mean number of fish displayed by Coop and Chain stores ($18.10 > 9.36$). There is not a significant difference between the mean number of fish displayed by Private and Chain stores, however ($1.93 < 9.36$).

Step 10 IF you reject the null hypothesis, determine whether the pattern of the mean differences supports, partially supports, or does not support the research hypothesis. You must consider the research hypothesis carefully! Sometimes researchers hypothesize that a pair of conditions will have different means (predict rejecting the null), sometimes they will predict that a pair of conditions will have similar means (predict retaining the null).

- IF EVERY pairwise comparison has results consistent with the research hypothesis (including the DIRECTION of the mean difference if you reject the null for that pairwise comparison) agrees exactly with the research hypothesis, then the research hypothesis is completely supported.
- IF SOME BUT NOT ALL of the pairwise comparisons have results consistent with the research hypothesis, then the research hypothesis is partially supported
There are three ways a pairwise comparison would not support the research hypothesis:
 - the research hypothesis predicts a mean difference and you retain the null for that comparison
 - the research hypothesis predicts a mean difference in the opposite direction of the one found in that comparison
 - the research hypothesis predicts the two conditions will have similar means, but you rejected the null hypothesis for that comparison
- IF NO pairwise comparison has results consistent with the research hypothesis, then the research hypothesis is not supported by the data.

The researcher hypothesized that Coop stores would have the most fish on display, Chain stores would display the least, and Private pet stores would display an intermediate amount. This "translates" into an expected pairwise difference between all pairs of store types $\text{Coop} > \text{Private} > \text{Chain}$ (with the implication that $\text{Coop} > \text{Chain}$).

Based on the results of the HSD analysis, we would conclude there is only PARTIAL SUPPORT for the research hypothesis. Consistent with the research hypothesis, Coop stores displayed significantly more fish than did Private stores, and also Coop stores displayed significantly more fish than did Chain stores. However, contrary to the research hypothesis, there was no significant difference between the average number of fish displayed by Private and Chain stores.

By the Way: Sometimes LSD and HSD analyses will produce different results for one or more of the pairwise comparisons. If so, the difference will always be that you have rejected H_0 : based on the LSD test (the more sensitive test) and retained H_0 : based on the HSD test (the more conservative test). When this happens you should consider the following:

1) There is a general trend among statisticians (and journal editors) towards "statistical conservatism". (This leads into a complicated but very important discussion of whether we should be more fearful of "claiming effects that are not really there," or of "failing to identify effect that are really there." 2). More importantly, you should remember that rejecting the null for a particular analysis is not a guarantee that the effect is "really there" (no matter what p-value is used). Replication (finding the effect in several different studies) is a much better (but still not perfect) indicator of the "reality" of an effect.

Step 11 Describing the results of a between groups ANOVA with pairwise comparisons using the HSD procedure-- be sure to include the following

- Tell the conditions of the qualitative or grouping variable
- Name quantitative variable and tell its mean and standard deviation for each condition of the qualitative variable
- The F-value, df (in parentheses) and p-value ($p < .05$ or $p > .05$).
- Tell that an HSD was used and report the minimum mean difference.
- Describe the pattern of the data (which group(s) has the larger mean, if there is a significant difference)
- Whether or not the results support the research hypothesis

You should notice that the important difference between this report and that given above following the ANOVA is that here the description of differences among the means is attributed to the results of the HSD analysis, rather than "inspection" of the means. The example also shows a common way of reporting the results of the pairwise HSD comparisons – using letters to indicate which means were significantly different.

There were significant differences among the mean numbers of fish displayed by the three types of stores, $F(2,9) = 18.01$, $MSE = 22.54$, $p < .05$. Pairwise comparisons using HSD (minimum mean difference = 9.36) revealed that, consistent with the research hypothesis, Coop stores displayed more fish ($\bar{M} = 35.50$, $S = 4.80$) than either Private ($\bar{M} = 19.33$, $S = 4.04$) or Chain stores ($\bar{M} = 17.40$, $S = 5.03$). However, contrary to the research hypothesis, there was no difference between the average number of fish displayed by Chain and Private pet stores.

Table Q: Studentized Range Statistic Values of Q (for HSD computations) for $\alpha = .05$ & $\alpha = .01$ Denominator
df α 2 3 4 5 6 k = number of means

5	.05	3.64	4.60	5.22	5.67	6.03
	.01	5.70	6.98	7.80	8.42	8.91
6	.05	3.46	4.34	4.90	5.30	5.63
	.01	5.24	6.33	7.03	7.56	7.97
7	.05	3.34	4.16	4.68	5.06	5.36
	.01	4.95	5.92	6.54	7.01	7.37
8	.05	3.26	4.04	4.53	4.89	5.17
	.01	4.75	5.64	6.20	6.62	6.96
9	.05	3.20	3.95	4.41	4.76	5.02
	.01	4.60	5.43	5.96	6.35	6.66
10	.05	3.15	3.88	4.33	4.65	4.91
	.01	4.48	5.27	5.77	6.14	6.43
11	.05	3.11	3.82	4.26	4.57	4.82
	.01	4.39	5.15	5.62	5.97	6.25
12	.05	3.08	3.77	4.20	4.51	4.75
	.01	4.32	5.05	5.50	5.84	6.10
13	.05	3.06	3.73	4.15	4.45	4.69
	.01	4.26	4.96	5.40	5.73	5.98
14	.05	3.03	3.70	4.11	4.41	4.64
	.01	4.21	4.89	5.32	5.63	5.88
15	.05	3.01	3.67	4.08	4.37	4.59
	.01	4.17	4.84	5.25	5.56	5.80
16	.05	3.00	3.65	4.05	4.33	4.56
	.01	4.13	4.79	5.19	5.49	5.72
17	.05	2.98	3.63	4.02	4.30	4.52
	.01	4.10	4.74	5.14	5.43	5.66
18	.05	2.97	3.61	4.00	4.28	4.49
	.01	4.07	4.70	5.09	5.38	5.60
19	.05	2.96	3.59	3.98	4.25	4.47
	.01	4.05	4.67	5.05	5.33	5.55
20	.05	2.95	3.58	3.96~	4.23	4.45
	.01	4.02	4.64	5.02	5.29	5.51
30	.05	2.89	3.49	3.85	4.10	4.30
	.01	3.89	4.45	4.80	5.05	5.24
40	.05	2.86	3.44	3.79	4.04	4.23
	.01	3.82	4.37	4.70	4.93	5.11
60	.05	2.83	3.40	3.74	3.98	4.16
	.01	3.76	4.28	4.59	4.82	4.99
120	.05	2.80	3.36	3.68	3.92	4.10
	.01	3.70	4.20	4.50	4.71	4.87
∞	.05	2.77	3.31	3.63	3.86	4.03
	.01	3.64	4.12	4.40	4.60	4.76