Independent Groups Least Significant Difference (LSD) -- Pairwise Comparisons for k-Between Group Designs

Application: To perform pairwise comparisons of means of a quantitative variable obtained from 3 or more independent groups -- usually used as a follow-up to a rejecting H0: from an Independent groups ANOVA.

The procedure has two steps. First, compute the smallest difference between condition means that will allow you to reject the H0: that a pair of conditions represent populations with the same mean on the quantitative variable. Second, compare that minimum mean difference with each of the pairwise mean differences to decide which pairs of conditions have means that are "significantly different." The Least Significant Difference approach is roughly equivalent to performing all pairwise t-test (or F-tests), comparing each to a critical value based on p = .05 (except that the error term of the LSD is based on data from all the conditions, rather than being recomputed for each pair). Remember, this procedure emphasizes sensitivity (minimizing Type II errors), but at the possible cost of increasing Type I errors (claiming that conditions have different means when they really don't).

The data for this example are taken from the Independent Groups ANOVA example above.

Step 1 Arrange the conditions so that the means are in descending order.

		Coop Store	Private Store	Chain Store
		35.50	19.33	17.40
Coop Store	35.50			
Pritate Store	19.33			
Chain Store		17.40		

Step 2 Compute the pairwise mean differences between conditions. We need fill-in only the bottom triangle, because the upper diagonal would show the same information.

		Coop Store	Private Store	Chain Store
		35.50	19.33	17.40
Coop Store	35.50			
Private Store	19.33	16.17		
Chain Store		17.40	18.10	1.93

Step 3 Obtain the df_{Error} from the independent groups omnibus-F analysis -- given in Step 10 of that procedure.

From the example data, df_{Error} = 9

Step 4 Use Table t to determine the critical value of t for α = .05 with df_{Error}. Sometimes, with larger df, the table doesn't include the df you are looking for. When this happens, just use the next smaller df that is included on the table. For example, if you had df = 33, you would use the t for df = 30.

t (9, α = .05) = 2.26

Step 5 Obtain the MS_{Error} from the between groups omnibus-F analysis -- given in Step 11 of that procedure.

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         From the example data,  MS_Error  =   22.54
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Step 6 Obtain n, the number of participants ts in each condition of the independent groups design. If there is an unequal number of participants in the different conditions, use the average number in the conditions.

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    From the example data (with unequal n), the average n = 12 / 3  = 4.00
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Step 7 Compute d_{LSD} -- the minimum significant pairwise mean difference, based on the LSD procedure

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         t * √ (2 * MS_Error)    2.26  *  √ (2 * 22.54)
         d_LSD = ----------------- = ----------------------- =
                    √  n                    √ 4.00


          2.26 * √ 45.08        2.26 * 6.71          15.16
         ----------------  =    ------------  =    -------  =  7.58
              2.00                  2.00              2.00
```

Step 8 Compare each of the pairwise mean differences to the d_{LSD}, and determine whether to reject or retain the null hypothesis for **each** pairwise comparison.

   -- if the mean difference is less than the d_{LSD}, then retain the null hypothesis -- conclude that the populations represented by those two conditions have the same mean score on the quantitative variable

   -- if the mean difference is greater than the d_{LSD} then reject the null hypothesis -- conclude that the populations represented by those two conditions have different mean scores on the quantitative variable

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       For these data, we would conclude that there is a significant difference between
  the mean number of fish displayed by Coop and Private stores (16.17 > 7.58), a
  significant difference between the mean number of fish displayed by Coop and Chain
  stores (18.10 > 7.58), but not a significant difference between the mean number of fish
  displayed by Private and Chain stores (1.93 < 7.58).
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Step 9   IF you reject the null hypothesis, determine whether the pattern of the mean differences completely supports, partially supports, or does not support the research hypothesis. You must consider the research hypothesis carefully! Sometimes researchers hypothesize that a pair of conditions will have different means (predict rejecting the null for that pairwise comparison) sometimes they will predict that a pair of conditions will have similar means (predict retaining the null for   that pairwise comparisons).

   -- IF EVERY pairwise comparison has results consistent with the research hypothesis (including the DIRECTION of the mean difference if you reject the null for that pairwise comparison), then the research hypothesis is completely supported.

   -- IF SOME BUT NOT ALL of the pairwise comparisons have results consistent with the research hypothesis,   then the research hypothesis is partially supported. There are three ways a pairwise comparison would not support the research hypothesis:
   -- the research hypothesis predicts a mean difference and you retain the null for that comparison
   -- the research hypothesis predicts a mean difference in the opposite direction of the one found in that comparison
   -- the research hypothesis predicts the two conditions will have similar means, but you rejected the null hypothesis for that comparison

   -- IF NO pairwise comparison has results consistent with the research hypothesis, then the research hypothesis is not supported by the data.

         The researcher hypothesized that Coop stores would have the most fish on display, Chain stores would display the least, and Private pet stores would display an intermediate amount. This "translates" into an expected pairwise difference between all pairs of store types Coop > Private > Chain (with the implication that Coop > Chain).

         Based on the results of the LSD analysis, we would conclude there is only PARTIAL SUPPORT for the research hypothesis. Consistent with the research hypothesis, Coop stores displayed significantly more fish than did Private stores, and also Coop stores displayed significantly more fish than did Chain stores. However, contrary to the research hypothesis, there was no significant difference between the average number of fish displayed by Private and Chain stores.

Step 10 Describing the results of a independent groups ANOVA with pairwise comparisons using the LSD procedure
-- be sure to include the following

- Tell the conditions of the qualitative or grouping variable
- Name the quantitative variable and tell its mean and standard deviation for each condition of the grouping variable
- The F-value, df (in parentheses) and p-value ($p < .05$ or $p > .05$).
- Tell that an LSD was used and report the minimum mean difference.
- Describe the pattern of the data (which group(s) has the larger mean, if there is a significant difference)
- Whether or not the results support the research hypothesis

         You should notice that the important difference between this report and he one given following the ANOVA (above) is that here the description of differences among the means is attributed to the results of the LSD analysis, rather than "inspection" of the means. The example also shows a common way of reporting the results of the pairwise LSD comparisons – using letters to indicate which means were significantly different.

         There were significant differences among the mean numbers of fish displayed by the three types of stores, $F(2,9) = 18.01$, MSE = 22.54, $p < .05$. Pairwise comparisons using LSD (minimum mean difference = 7.58) revealed that, consistent with the research hypothesis, Coop stores displayed more fish (M = 35.50, S = 4.80) than either Private (M = 19.33, S = 4.04) or Chain stores(M = 17.40, S = 5.03). However, contrary to the research hypothesis, there was no difference between the average number of fish displayed by Chain and Private pet stores.

Here is the same write-up using a Table to present the univariate statistics.

The number of fish displayed at each type of store is summarized in Table 1. There were significantly difference among the mean number of fish displayed by the three types of stores, $\underline{F}(2,9) = 18.01$, $\underline{MSE} = 22.54$, p <.05. Pairwise comparisons using LSD (minimum mean difference = 7.58) revealed that, consistent with the research hypothesis, Coop stores displayed significantly more fish than either Private or Chain stores. However, contrary to the research hypothesis, there was no significant difference between the average number of fish displayed by Chain and Private pet stores.

Table 1
Number of Fish Displayed at each Store Type

	Store Type		
Number of Fish Displayed	Chain Privately Store	Coop Owned	Store
$\underline{M}$	17.40^a	19.33^a	35.50^b
$\underline{S}$	5.03	4.04	4.80

Note: Means that were significantly different based on LSD analysis are labeled with different letters.

Table t: t-tests   Critical values of t for a = .05 & a = .01

df	α = .05	α = .01
1	12.71	63.66
2	4.30	9.92
3	3.18	5.84
4	2.78	4.60
5	2.57	4.03
6	2.45	3.71
7	2.36	3.50
8	2.31	3.36
9	2.26	3.25
10	2.23	3 17
11	2.20	3.11
12	2.18	3.06
13	2.16	3.01
14	2.14	2.98
15	2.13	2.95
16	2.12	2.92
17	2.11	2.90
18	2.10	2.88
19	2.09	2.86
20	2.09	2.84
21	2.08	2.83
22	2.07	2.82
23	2.07	2.81
24	2.06	2.80
25	2.06	2.79
26	2.06	2.78
27	2.05	2.77
28	2.05	2.76
29	2.04	2.76
30	2.04	2.75
40	2.02	2.70
60	2.00	2.66
120	1.98	2.62
∞	1.96	2.58