Dependent Groups Honestly Significant Difference (HSD) -- Pairwise Comparisons for k-Within-Group Designs

Application: To perform pairwise comparisons of means of a quantitative variable obtained from 3 or more dependent groups (repeated measures or matched groups) -- usually used as a follow-up after rejecting H0: from a dependent groups ANOVA.

The procedure has the same two steps as the LSD, however the minimum significant mean difference is computed differently. Remember, the HSD procedure is more conservative than the LSD procedure (minimizing Type I errors), but at the possible cost of increasing Type II errors. Thus, for a given set of data, the minimum mean difference using the HSD procedure will always be larger than the minimum mean difference based on the LSD procedure.

The data for this example are taken from the Dependent Groups ANOVA example above.

Step 1 Arrange the conditions so that the means are in descending order.

		Fish	Mammals	Reptiles
		23.92	21.50	9.25
Fish	23.92			
Mammals	21.50			
Reptiles	9.25			

Step 2 Compute the pairwise mean differences between conditions. We need fill-in only the bottom triangle, because the upper diagonal would provide the same information.

		Fish	Mammals	Reptiles
		23.92	21.50	9.25
Fish	23.92			
Mammals	21.50	2.42		
Reptiles	9.25	14.67	12.25	

Step 3 Obtain the MS_{Error} from the between groups omnibus-F analysis -- given in Step 14 of that procedure.

From the example data, $MS_{Error} = 33.39$

Step 4 Obtain n, the number of subjects in the design.

From the example data, n = 12 in each of the conditions

Step 5 Obtain k, the number of conditions or means involved in the design

From the example data, k = 3 conditions or means

Step 6 Obtain the df_{Error} from the between groups omnibus-F analysis -- given in Step 13 of that procedure.

From the example data, $df_{Error} = 22$

Step 7 Use Table Q to find the Studentized range statistic. To use Table Q you must know three values, error given by df_{Error}, k (the number of conditions), and the p-value (use $\alpha = .05$). Sometimes, with larger df, the table doesn't include the df you are looking for. When this happens, just use the next smaller df that is included on the table. For example, if you had df = 33, you would use the Q for df = 30.

For the example data, $Q(df_{Error} = 22, k = 3, p = .05) = 3.58$

Step 8 Compute d_{HSD} -- the minimum significant pairwise mean difference, based on the HSD procedure

$$d_{HSD} = Q * \frac{\sqrt{MS_{Error}}}{\sqrt{n}} = 3.58 * \frac{\sqrt{33.39}}{\sqrt{12}} = 4.04 * \frac{5.78}{3.46} = 3.58 * 1.67 = 5.98$$

Step 9 Compare each of the pairwise mean differences to the d_{HSD}, and determine whether to reject or retain the null hypothesis for each pairwise comparison. Remember to examine each of the pairwise comparisons.

-- if the mean difference is less than the d_{HSD}, then retain the null hypothesis -- conclude that the populations represented by those two conditions have the same mean score on the quantitative variable

-- if the mean difference is greater than the d_{HSD} then reject the null hypothesis -- conclude that the populations represented by those two conditions have different mean scores on the quantitative variable

For these data, we would conclude that there is not a significant difference between the mean number of fish and mammals displayed (2.42 < 5.98); there is a significant difference between the mean number of fish and reptiles (14.67 > 5.98), and also a significant difference between the mean number of reptiles and mammals displayed (12.25 > 5.98).

Step 10 IF you reject the null hypothesis, determine whether the pattern of the mean differences completely supports, partially supports, or does not support the research hypothesis. You must consider the research hypothesis carefully! Sometimes researchers hypothesize that a pair of conditions will have different means (predict rejecting the null). Sometimes they will predict that a pair of conditions will have similar means (predict retaining the null).

-- IF EVERY pairwise comparison has results that agree exactly with the research hypothesis (including the DIRECTION of the mean difference if you reject the null for that pairwise comparison) agrees exactly with the research hypothesis, then the research hypothesis is completely supported.

-- IF SOME BUT NOT ALL of the pairwise comparisons have results consistent with the research hypothesis, then the research hypothesis is partially supported

There are three ways a pairwise comparison would not support the research hypothesis:
-- the research hypothesis predicts a mean difference and you retain the null for that comparison
-- the research hypothesis predicts a mean difference in the opposite direction of the one found in that comparison
-- the research hypothesis predicts the two conditions will have similar means, but you rejected the null hypothesis for that comparison

-- IF NO pairwise comparison has results consistent with the research hypothesis, then the research hypothesis is not supported by the data.

The researcher hypothesized that stores would tend to display more fish than other types of animals, fewer reptiles, and an intermediate number of mammals. This "translates" into an expected pairwise difference between all pairs of store types fish > mammals > reptiles (with the implication that fish > reptiles).

Based on the results of the HSD analysis, we would conclude there is only PARTIAL SUPPORT for the research hypothesis. As hypothesized, stores displayed significantly more fish than either reptiles, and also displayed significantly more mammals than reptiles. However, contrary to the research hypothesis, there was no significant difference between the number of fish and mammals displayed.

The data: In this analysis (which corresponds to the second application described above) the quantitative variable is the number of each of three types of animals (fish, reptiles, or mammals).

- Name the quantitative variables and tell mean and standard deviation for each
- The F-value, df (in parentheses) and p-value ($p < .05$ or $p > .05$).
- Tell that an LSD was used and report the minimum mean difference.
- Describe the pattern of the data (which group(s) has the larger mean, if there is a significant difference)
- Whether or not the results support the research hypothesis

You should notice that the important difference between this report and that given above following the ANOVA is that here the description of differences among the means is attributed to the results of the HSD analysis, rather than "inspection" of the means. The example also shows a common way of reporting the results of the pairwise HSD comparisons – using letters to indicate which means were significantly different.

There was a significant difference among the distributions of the three types of animals, $F(2,22) = 22.22$, $p < .05$, $MSE = 33.39$. Pairwise comparison of the means using the HSD procedure (minimum mean difference = 5.98) revealed that, consistent with the research hypothesis, more fish ($M = 23.92$, $S = 9.61$) than reptiles ($M = 9.25$, $S = 4.27$) were displayed on average and also more mammals ($M = 21.50$, $S = 12.87$) than reptiles were displayed on average. However, contrary to the research hypothesis, there was not much of a difference between the average number of fish and mammals

Here is the same thing using a Table to present the univariate statistics.

Table 1 summarizes the data for the numbers of animals displayed at the stores. There was a significant difference among the means of the three types of animals, $F(2,22) = 22.22$, $p < .05$, $MSE = 33.39$. Pairwise comparison of the means using the HSD procedure (minimum mean difference = 5.98) revealed that, consistent with the research hypothesis, more fish than reptiles were displayed on average and also more mammals than reptiles were displayed on average. However, contrary to the research hypothesis, there was not a significant difference between the average number of fish and mammals displayed.

Table 1
Number of animals of each type displayed in the pet stores.

	Type of Animal		
Number of Animals Displayed	Fish	Mammals	Reptiles
M	23.92^a	21.50^a	9.25^b
S	9.61	12.87	4.27

Note: Means that were significantly different based on HSD analysis are labeled with different letters.

By the Way: Sometimes LSD and HSD analyses will produce different results for one or more of the pairwise comparisons. If so, the difference will always be that for the comparison in question, you have rejected H0: based on the LSD test (the more sensitive test) and retained H0: based on the HSD test (the more conservative test). When this happens you should consider the following: 1) There is a general trend among statisticians (and journal editors) towards "statistical conservatism". (This leads into a complicated but very important discussion of whether we should be more fearful of "claiming effects that are not really there," or of "failing to identify effect that are really there."); 2) More importantly, you should remember rejecting the null for a particular analysis is not a guarantee that the effect is "really there" (no matter what p-value is used). Replication (finding the effect in several different studies) is a much better (but still not perfect) indicator of the "reality" of an effect.

Table Q: Studentized Range Statistic Values of Q (for HSD computations) for a = .05 & a = .01

Denominator df	α	k = number of means 2	3	4	5	6
5	.05	3.64	4.60	5.22	5.67	6.03
	.01	5.70	6.98	7.80	8.42	8.91
6	.05	3.46	4.34	4.90	5.30	5.63
	.01	5.24	6.33	7.03	7.56	7.97
7	.05	3.34	4.16	4.68	5.06	5.36
	.01	4.95	5.92	6.54	7.01	7.37
8	.05	3.26	4.04	4.53	4.89	5.17
	.01	4.75	5.64	6.20	6.62	6.96
9	.05	3.20	3.95	4.41	4.76	5.02
	.01	4.60	5.43	5.96	6.35	6.66
10	.05	3.15	3.88	4.33	4.65	4.91
	.01	4.48	5.27	5.77	6.14	6.43
11	.05	3.11	3.82	4.26	4.57	4.82
	.01	4.39	5.15	5.62	5.97	6.25
12	.05	3.08	3.77	4.20	4.51	4.75
	.01	4.32	5.05	5.50	5.84	6.10
13	.05	3.06	3.73	4.15	4.45	4.69
	.01	4.26	4.96	5.40	5.73	5.98
14	.05	3.03	3.70	4.11	4.41	4.64
	.01	4.21	4.89	5.32	5.63	5.88
15	.05	3.01	3.67	4.08	4.37	4.59
	.01	4.17	4.84	5.25	5.56	5.80
16	.05	3.00	3.65	4.05	4.33	4.56
	.01	4.13	4.79	5.19	5.49	5.72
17	.05	2.98	3.63	4.02	4.30	4.52
	.01	4.10	4.74	5.14	5.43	5.66
18	.05	2.97	3.61	4.00	4.28	4.49
	.01	4.07	4.70	5.09	5.38	5.60
19	.05	2.96	3.59	3.98	4.25	4.47
	.01	4.05	4.67	5.05	5.33	5.55
20	.05	2.95	3.58	3.96~	4.23	4.45
	.01	4.02	4.64	5.02	5.29	5.51
30	.05	2.89	3.49	3.85	4.10	4.30
	.01	3.89	4.45	4.80	5.05	5.24
40	.05	2.86	3.44	3.79	4.04	4.23
	.01	3.82	4.37	4.70	4.93	5.11
60	.05	2.83	3.40	3.74	3.98	4.16
	.01	3.?6	4.28	4.59	4.82	4.99
120	.05	2.80	3.36	3.68	3.92	4.10
	.01	3.70	4.20	4.50	4.71	4.87
∞	.05	2.77	3.31	3.63	3.86	4.03
	.01	3.64	4.12	4.40	4.60	4.76