Dependent Groups Least Significant Difference (LSD) -- Pairwise Comparisons for
k-Within-Group Designs

Application: To perform pairwise comparisons of means of a quantitative variable obtained from 3 or more dependent groups (repeated measures or matched groups) -- usually used as a follow-up after rejecting H0: from a dependent groups ANOVA.

The LSD procedure for Dependent Groups designs is just like the one you completed earlier for Between Groups designs. The procedure has two steps. First, compute the smallest difference between condition means (or the means of the different quantitative variables) that will allow you to reject the H0: that the pair of conditions represent populations with the same mean scores. Second, compare that minimum mean difference with each of the pairwise mean differences to decide which pairs of conditions have means that are "significantly different." The Least Significant Difference approach is roughly equivalent to performing all pairwise t-test (or F-tests), comparing each to a critical value based on p = .05 (except that the error term of the LSD is based on data from all the conditions, rather than being recomputed for each pair). Remember, this procedure emphasizes sensitivity (minimizing Type II errors), but at the possible cost of increasing Type I errors (claiming that conditions have different means when they really don't).

The data for this example are taken from the Dependent Groups ANOVA example above.

Step 1 Arrange the conditions so that the means are in descending order.

		Fish	Mammals	Reptiles
		23.92	21.50	9.25
Fish	23.92			
Mammals	21.50			
Reptiles	9.25			

Step 2 Compute the pairwise mean differences between conditions. We need fill-in only the bottom triangle, because the upper diagonal would show the same information
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		Fish	Mammals	Reptiles
		23.92	21.50	9.25
Fish	23.92			
Mammals	21.50	2.42		
Reptiles	9.25	14.67	12.25	

Step 3 Obtain the df_{Error} from the dependent groups omnibus-F analysis -- given in Step 13 of that procedure.

From the example data, df_{Error} = 22

Step 4 Use Table t to determine the critical value of t for α = .05 with df_{WG}. Sometimes, with larger df, the table doesn't include the df you are looking for. When this happens, just use the next smaller df that is included on the table. For example, if you had df = 33, you would use the t for df = 30.

t (22, α = .05) = 2.07

Step 5 Obtain the MS_{Error} from the dependent groups omnibus-F analysis -- given in Step 14 of that procedure.

From the example data, MS_{Error} = 33.39

Step 6 Obtain n, the number of participants in the design.

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                  From the example data,  n = 12   in each of the conditions
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Step 7 Compute d_{LSD} -- the minimum significant pairwise mean difference, based on the LSD procedure

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           t  *  √ (2 * MS_Error)      2.07 * √ (2 * 33.39)   2.07 * √66.78
  d_LSD = ----------------------- = --------------------- = --------------
                   √ n                       √ 12                3.46

        2.07 * 8.17        16.92
      = ------------- =  -------- =  4.89
            3.46    3.46
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Step 8 Compare each of the pairwise mean differences to the d_{LSD}, and determine whether to reject or retain the null hypothesis for that pairwise comparison. Remember to do this for each of the multiple pairwise comparisons.

-- if the mean difference is less than the d_{LSD}, then retain the null hypothesis -- conclude that the populations represented by those two conditions have the same mean score on the quantitative variable

-- if the mean difference is greater than the d_{LSD} then reject the null hypothesis -- conclude that the populations represented by those two conditions have different mean scores on the quantitative variable.

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     For these data, we would conclude that there is not a significant difference
between the mean number of fish and mammals displayed (2.42 < 4.89); there is a
significant difference between the mean number of fish and reptiles (14.67 > 4.89), and
also a significant difference between the mean number of reptiles and mammals displayed
(12.25 > 4.89).
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Step 9 IF you reject the null hypothesis, determine whether the pattern of the mean differences completely supports, partially supports, or does not support the research hypothesis. You must consider the research hypothesis carefully!
Sometimes researcher hypotheses that a pair of conditions will have different means (predict rejecting the null for that pairwise comparison); sometimes they will predict that a pair of conditions will have similar means (predict retaining the null for that pairwise comparison).

-- IF EVERY pairwise comparison has results consistent with the research hypothesis (including the DIRECTION of the mean difference if you reject the null for that pairwise comparison), then the research hypothesis is completely supported.

-- IF SOME BUT NOT ALL of the pairwise comparisons have results consistent with the research hypothesis, then the research hypothesis is partially supported.

There are three ways a pairwise comparison would not support the research hypothesis:
  -- the research hypothesis predicts a mean difference and you retain the null for that comparison
  -- the research hypothesis predicts a mean difference in the opposite direction of the one found in that comparison
  -- the research hypothesis predicts the two conditions will have similar means, but you rejected the null hypothesis for that comparison

-- IF NO pairwise comparison has results consistent with the research hypothesis, then the research hypothesis is not supported by the data.

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     The researcher hypothesized that stores would tend to display more fish than
other types of animals, fewer reptiles, and an intermediate number of mammals. This
"translates" into an expected pairwise difference between all pairs of store types fish
> mammals > reptiles (with the implication that fish > reptiles).

     Based on the results of the LSD analysis, we would conclude there is only PARTIAL
SUPPORT for the research hypothesis.  As hypothesized, stores displayed significantly
more fish than reptiles, and also displayed significantly more mammals than reptiles.
However, contrary to the research hypothesis, there was no significant difference
between the number of fish and mammals displayed.
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Step 10 Describing the results of a dependent groups ANOVA with pairwise comparisons using the LSD procedure -- be sure to include the following

- Name the quantitative variables and tell mean and standard deviation for each
- The F-value, df (in parentheses) and p-value ($p < .05$ or $p > .05$).
- Tell that an LSD was used and report the minimum mean difference.
- Describe the pattern of the data (which group(s) has the larger mean, if there is a significant difference)
- Whether or not the results support the research hypothesis

      You should notice that the important difference between this report and that given above following the ANOVA is that here the description of differences among the means is attributed to the results of the LSD analysis, rather than "inspection" of the means. The example also shows a common way of reporting the results of the pairwise LSD comparisons – using letters to indicate which means were significantly different.

      There was a significant difference among the distributions of the three types of animals, $\underline{F}(2,22) = 22.22$, $\underline{p} <$ .05, $\underline{MSE} = 33.39$. Pairwise comparison of the means using the $\underline{LSD}$ procedure (minimum mean difference = 4.89) revealed that,consistent with the research hypothesis, more fish ($\underline{M} = 23.92$, $\underline{SD} = 9.61$) than reptiles ($\underline{M} = 9.25$, $\underline{SD} = 4.27$) were displayed on average and also more mammals ($\underline{M} = 21.50$, $\underline{SD} = 12.87$) than reptiles were displayed on average. However, contrary to the research hypothesis, there was not much of a difference between the average number of fish and mammals displayed.

Here is the same thing using a Table to present the univariate statistics.

      Table 1 summarizes the data for the numbers of animals displayed at the stores. There was a significant difference among the means of the three types of animals, $\underline{F}(2,22) = 22.22$, $\underline{p} < .05$, $\underline{MSE} = 33.39$. Pairwise comparison of the means using the $\underline{LSD}$ procedure (minimum mean difference = 4.89) revealed that, consistent with the research hypothesis, more fish than reptiles were displayed on average and also more mammals than reptiles were displayed on average. However, contrary to the research hypothesis, there was not a significant difference between the average number of fish and mammals displayed.

Table 1
<u>Number of animals of each type displayed in the pet stores.</u>

	Type of Animal		
Number of Animals Displayed	Fish	Mammals	Reptiles
<u>M</u>	23.92[a]	21.50[a]	9.25[b]
<u>SD</u>	9.61	12.87	4.27

Note: Means that were significantly different based on LSD analysis are labeled with   different letters.

Table t: t-tests   Critical values of t for a = .05 & a = .01

df	α = .05	α = .01
1	12.71	63.66
2	4.30	9.92
3	3.18	5.84
4	2.78	4.60
5	2.57	4.03
6	2.45	3.71
7	2.36	3.50
8	2.31	3.36
9	2.26	3.25
10	2.23	3 17
11	2.20	3.11
12	2.18	3.06
13	2.16	3.01
14	2.14	2.98
15	2.13	2.95
16	2.12	2.92
17	2.11	2.90
18	2.10	2.88
19	2.09	2.86
20	2.09	2.84
21	2.08	2.83
22	2.07	2.82
23	2.07	2.81
24	2.06	2.80
25	2.06	2.79
26	2.06	2.78
27	2.05	2.77
28	2.05	2.76
29	2.04	2.76
30	2.04	2.75
40	2.02	2.70
60	2.00	2.66
120	1.98	2.62
∞	1.96	2.58