

Dependent t-test -- Analysis of 2-Within (or Matched)-Group Data with a Quantitative Response Variable

Application: This statistic has two applications that can appear very different, but are really just two variations of the same statistical question. In one application, the same quantitative variable is measured under two different conditions from the same sample (or from two samples that have been matched on some important variable). In the other application, two comparable quantitative variables are measured from the same sample (usually at the same time). In both applications, the dependent t-test is used to compare the means of the two quantitative variables.

There are two specific versions of the H_0 ;, depending upon whether one characterizes the data as representing a single population under two different conditions (e.g., comparing treated vs. not treated or comparing two different treatments -- some consider this a representation of two different populations) or as representing comparable variables measured from a single population (as in the example below). Here are versions of the H_0 : statement for each of these characterizations.

H_0 : The population represented by the sample has the same mean on the quantitative variable under the two conditions.

To reject H_0 : is to say that the population represented by the sample has different means under the two conditions.

H_0 : The population represented by the sample has the same mean on the two different quantitative measures.

To reject H_0 : is to say that the means of the two variables are different in the population represented by the sample.

The data: In this analysis (which corresponds to the second application described above) the two quantitative variables are the quality ratings of two different types of animals. From our database, we use the variables **reptgood** (reptile quality rated on a 1-10 scale) and **fishgood** (fish quality rated on a 1-10 scale). These scores are shown for the 12 stores below (fishgood, reptgood).

2,6 8,5 9,3 7,3 4,7 7,9 4,9 4,8 5,6 9,9 7,7 2,8

Research Hypothesis: The researcher hypothesized that a store's fish would be of higher quality than its reptiles, because of the greater difficulty obtaining and maintaining healthy reptiles.

H_0 : for this analysis: The quality ratings of reptiles and fish displayed by pet stores have the same means.

Rearrange the data so that scores from each participant are in the appropriate columns, one for each quantitative variable

reptgood	fishgood
2	6
8	5
9	3
7	3
4	7
7	9
4	9
4	8
5	6
9	9
7	7
2	8

Get the square of each value, and compute sums of original scores ($\sum X$) and sums of the squared scores ($\sum X^2$)

Reptgood (k1)		fishgood (k2)	
X	X ²	X	X ²
2	4	6	36
8	64	5	25
9	81	3	9
7	49	3	9
4	16	7	49
7	49	9	81
4	16	9	81
4	16	8	64
5	25	6	36
9	81	9	81
7	49	7	49
2	4	8	64
$\sum X_{k1} = 68$	$\sum X_{k1}^2 = 454$	$\sum X_{k2} = 80$	$\sum X_{k2}^2 = 584$

Compute the mean and std for each condition or variable.

Example using data from the reptgood:

$$\text{Mean} = \frac{\sum X_{k1}}{n_{k1}} = \frac{68}{12} = 5.67$$

$$SS = \sum X_{k1}^2 - \frac{(\sum X_{k1})^2}{n_{k1}} = 454 - \frac{68^2}{12} = 68.67$$

$$\text{Standard deviation} = \sqrt{\left[\frac{SS}{n_{k1}-1} \right]} = \sqrt{\left[\frac{68.67}{11} \right]} = 2.50$$

Compute a difference score (D) for each participant. Always compute the difference in the same direction for each participant. Sometimes the signs (+ or -) of the difference scores will be different, depending upon for which variable the participant had the higher score.

reptgood	fishgood	D
2	6	-4
8	5	3
9	3	6
7	3	4
4	7	-3
7	9	-2
4	9	-5
4	8	-4
5	6	-1
9	9	0
7	7	0
2	8	-6

Compute the square of each difference score and place it in an adjacent column (D^2).

reptgood	fishgood	D	D^2
2	6	-4	16
8	5	3	9
9	3	6	36
7	3	4	16
4	7	-3	9
7	9	-2	4
4	9	-5	25
4	8	-4	16
5	6	-1	1
9	9	0	0
7	7	0	0
2	8	-6	36

Compute $\sum D$, $\sum D^2$, the mean and sum of squares for D (following the formulas shown in the Descriptive Statistics section earlier) and complete the following table.

$\sum D$	-12	
$\sum D^2$	168	$SS_D = \sum D^2 - \frac{(\sum D)^2}{n} = 168 - \frac{-12^2}{12} = 156$
\bar{D}	-1	

Compute the variance of D

$$S_D^2 = \frac{SS}{N - 1} = \frac{156}{12 - 1} = \frac{156}{11} = 14.18$$

Compute the standard error of the direct difference between dependent means (Standard Error of Mean Differences - SEMD)

$$SEMD = \sqrt{\frac{S_D^2}{N}} = \sqrt{\frac{14.18}{12}} = \sqrt{1.18} = 1.09$$

Compute t

$$t = \frac{\bar{D}}{SEMD} = \frac{-1.00}{1.03} = -.97$$

Determine the degrees of freedom for the t-test

$$df = N - 1 = 12 - 1 = 11$$

Use Table t to determine the critical value of t, for a two-tailed test with $\alpha = .05$ and the appropriate degrees of freedom. Sometimes, with larger df, the table doesn't include the df you are looking for. When this happens, just use next smaller df that is included on the table. For example, if you had $df = 33$, you would use the t for $df = 30$.

$$t(11, \alpha = .05) = 2.20$$

Compare the obtained t and critical t values and determine whether to retain or reject the null hypothesis (that there is no mean difference between quantitative variables). Remember that t values can be positive or negative, and so we will compare the absolute value of the obtained t to the critical t

- if the absolute value of the obtained t is less than the critical t , then retain the null hypothesis and conclude that there is no mean difference between the two quantitative variables.
- if the absolute value of the obtained t is greater than the critical t , then reject the null hypothesis and conclude that there is a mean difference between the quantitative variables.

For the example data, we would decide to retain the null hypothesis, because the absolute value of the obtained t (.97) is smaller than the critical t (2.20).

Please Note: Usually the researcher hypothesizes that there is a mean difference between the two quantitative variables. Sometimes, however, the research hypothesis is that there is NO mean difference between the variables. If so, the research hypothesis and H_0 are the same! When this is the case, retaining H_0 provides support for the research hypothesis, whereas rejecting H_0 provides evidence that research hypothesis is incorrect.

IF you reject the null hypothesis, determine whether the data support or do not support the research hypothesis.

- IF you reject the null hypothesis AND the direction of the mean difference between the two quantitative variables agrees with the direction of the research hypothesis, then the research hypothesis is supported.
- IF you retain the null hypothesis OR if you reject the null BUT the direction of the mean difference disagrees with the direction of the research hypothesis, then the research hypothesis is not supported.

For the example data, we would decide that the research hypothesis is not supported, because we retained the null hypothesis.

Reporting the results of the dependent samples t -test -- be sure to include the following

- Name each quantitative variable and tell its mean and standard deviation
- The t -value, df (in parentheses) and p -value ($p < .05$ or $p > .05$).
- Describe the pattern of the data (which variable has the larger mean, if there is a significant difference)
- Whether or not the results support the research hypothesis

Contrary to the research hypothesis, there was no difference between the mean quality ratings given to fish ($\bar{M} = 5.67$, $\bar{S} = 2.50$) and to reptiles ($\bar{M} = 6.67$, $\bar{S} = 2.15$) in these stores, $t(11) = -.97$, $p > .05$.

Check the section about 2 WG ANOVA for descriptions of how to portray the results using a Table or Figure.