Multiple Regression Models with Binary & Quantitative Predictors (& their Interaction)

These data are from a treatment outcome study. Folks who were diagnosed as depressed were given the opportunity to begin treatment immediately, or to wait until they "feel it would be a good time to begin therapy". Treatment was either a "peer support group" (code = 1) or "traditional cognitive-behavioral therapy" (code = 2). The researcher also recorded the time between when the diagnosis was made and when the assigned treatment was started, in days (delay). After three months of treatment, each participant was assessed by a panel of clinicians to obtain the outcome variable score (dep -- a depression measure, so higher scores are "poorer").

Here are the group means for Depression and Time.

There is a small group difference for Depression.

There is also a small group difference for Time

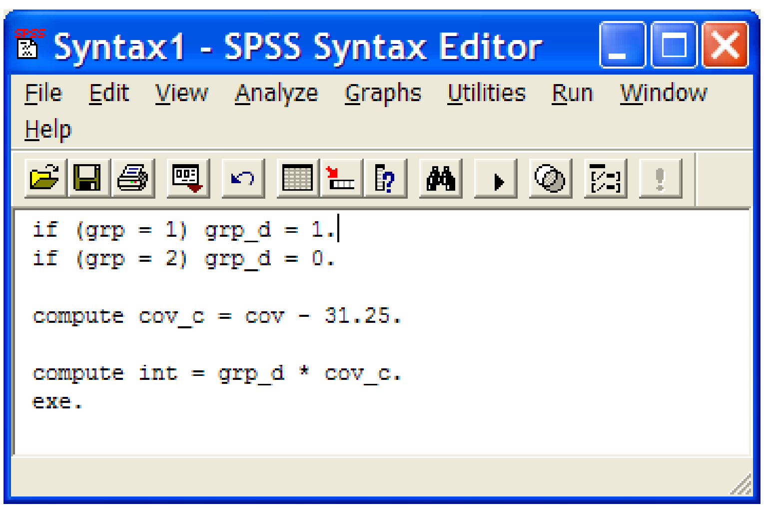
Report

treatment condition		rating of depression -- bigger scores are poore	time between diagnosis and rating of depression - in days
support group	Mean	4.4000	32.6000
	N	20	20
	Std. Deviation	1.81804	17.43680
cog-beh group	Mean	4.0000	29.9000
	N	20	20
	Std. Deviation	2.00000	17.95286
Total	Mean	4.2000	31.2500
	N	40	40
	Std. Deviation	1.89737	17.52178

Dummy-Coded Regression of the Data

Here's the SPSS syntax code to dummy code the grouping variable, to center the covariate (using mean overall mean for that variables from above) and to compute the interaction term.

- Dummy coding follows the GLM convention – the group with the highest original code as the control group (0)
- Centering of quantitative variables simplifies interpretation of the regression weights
- Interactions are "non-additive combinations" – represented as the product of the related main effect variables



IF statements to dummy-code the group variable: PEER is coded "1" as the target group and CBT is coded "0" as the comparison group

Centering the covariate requires subtracting the mean from each person's COV score

The product of the coded group variable and the centered covariate is the interaction term

Demonstration of Several Analyses with these Data

- Using Regression to obtain a "main effects" or "ANCOVA" model including both the Treatment & Delay
- A short tirade about the limitations of the "main effects model" and the importance of testing interactions
- Using Regression to obtain "interaction model" or "full model ANCOVA" with the grouping variable, the covariate and their interaction (and how to get all the information from the interaction test from a single model)
- Using nested regression models to test for an interaction
- Using GLM to obtain the "full model ANCOVA" – and how it is & isn't equivalent to the full model Regression

Why run the ANCOVA or Multiple Regression model? What's wrong with the ANOVA we ran?

When we looked at the means of the DV and delay for the two groups, we noticed that there was a mean difference between the groups on delay. This tells us that the relationship between group membership the DV is "confounded" by the group difference on delay.

So, following our general principle that when we don't have experimental data "on average, multivariate models are more accurate portrayals of complex behavior" we included both group and delay in this model.

If we find a different group effect with and without the delay included, we expect that the model with delay included is more likely to be accurate. If, as in this case, the group effect is the same with and without delay effect, we gain confidence in the generalizability of the group effect.

"Main Effects Model" or "ANCOVA" via Regression

Model Summary

Model	R	R Square
1	.126[a]	.016

a. Predictors: (Constant), COV_C, GRP_D

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2.219	2	1.109	.297	.745[a]
	Residual	138.181	37	3.735		
	Total	140.400	39			

a. Predictors: (Constant), COV_C, GRP_D

b. Dependent Variable: rating of depression -- bigger scores are poorer

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constan	3.990	.433		9.220	.000
	GRP_D	.419	.613	.112	.684	.498
	COV_C	-.007	.018	-.067	-.407	.686

a. Dependent Variable: rating of depression -- bigger scores are p

a is the expected value of dep when the value of group and the covariate are both 0. Which is the mean for members of the comparison group (CBT - coded 0) with exactly the mean time (which has been centered to 0)

COV_C is the slope of the DV-Delay regression line for each group (model assumes there is no interaction between GRP and COV) -- after correction for the IV

GRP_D is the direction and extent of the mean depression difference between the target group (PEER) and the comparison group (CBT), after correction for the covariate at its mean (0; the model assumes there is no interaction between GRP and COV). The target group (PEER) mean is .419 larger than that of the comparison group (CBT) for the mean delay (centered to 0)..

A Short Tirade About Main Effects Multiple Regression Models

Until now, all the multiple regression models we've looked at have been "main effects" models – that is, we've included any variables we want, without including any interactions.

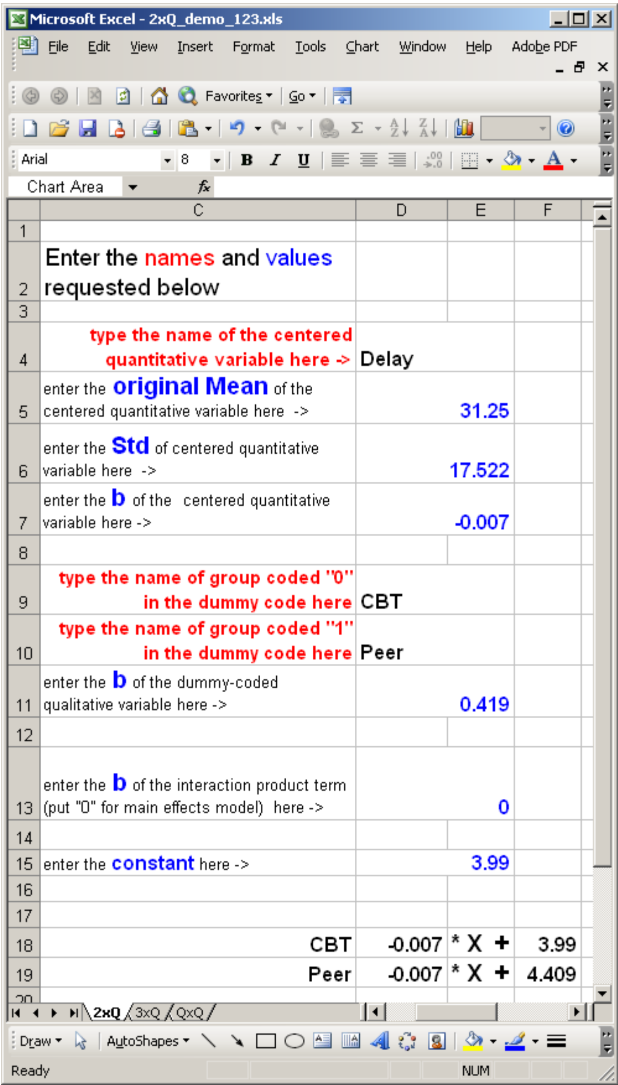
When we interpret the Group regression weight from this ANCOVA or Multiple Regression model we are invoking the "homogeneity of regression slope" assumption, which is another way of saying this is that we are assuming there is no interaction between the groping variable and the covariate.

- In ANCOVA language → the slope of the DV-Cov regression line is the same for all groups
- In ANOVA language → the Group difference on the DV is the same for all values of delay

This is the major historical difference between the uses of ANOVA and Multiple Regression. If we have multiple (qualitative) predictors we put them in an ANOVA and the very first thing we would look at is the interaction. If there is an interaction, we would very carefully examine the main effects, to see if they are descriptive or misleading. However, if we run a multiple regression, whether we have quantitative, coded categorical or a mixture of variables types, we put them all into the model and blithely interpret their regression weights (main effects) without considering that there might be interactions and that the interpretation of the regression weights might be conditional upon the values of the other variables.

Plotting this Regression / ANCOVA model

I've written a "computator" that computes plotting points to draw the regression lines from various models.



Select the one for a quant variable and 2 dummy-coded groups.

The computator was written for models including the interaction, but will work for the main effect & ANCOVA model also.

the interaction weight will be "0"

get the standard deviation of the covariate from above

The program gives you the regression models for each groups (notice they have the same slope) and the plotting points (based on +/- 1 std for the centered covariate)

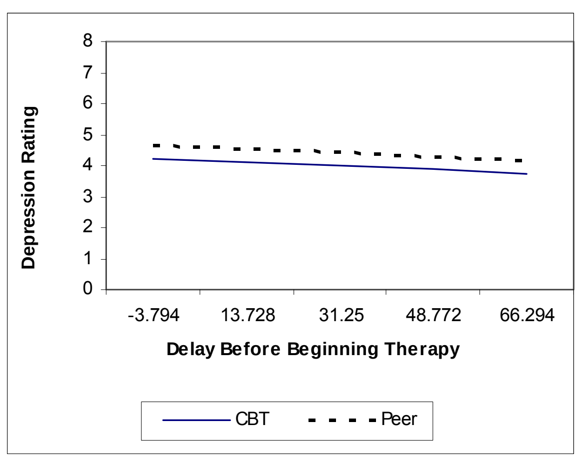
The program also plots the model using the original scale of the quantitative predictor.

You have to remember that the regression weight for the group variable refers to the group difference at the mean of the quant variable!!!

Notice that the regression slope for each group's regression line is the same as the slope of COV from the full model (because we assume there's no interaction, didn't put one in, and so have parallel regression lines for the two groups)

Notice that the difference between the teo group's constants is the regression weight for the grouping variable dummy code -- the corrected mean difference. Notice also that the corrected mean differences has the same for all values of the covariate – because there's no interaction.

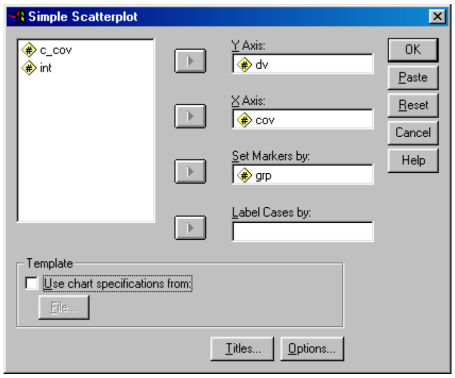
You should also remember that the lines are pretty flat, they are pretty close together, and that the R^2 is < .02 -- this model isn't doing very well.



This graph is the "model" of the data. However, we know that it fits the data poorly (R^2 < 2%). Maybe we should look at the data.

To graph the data…

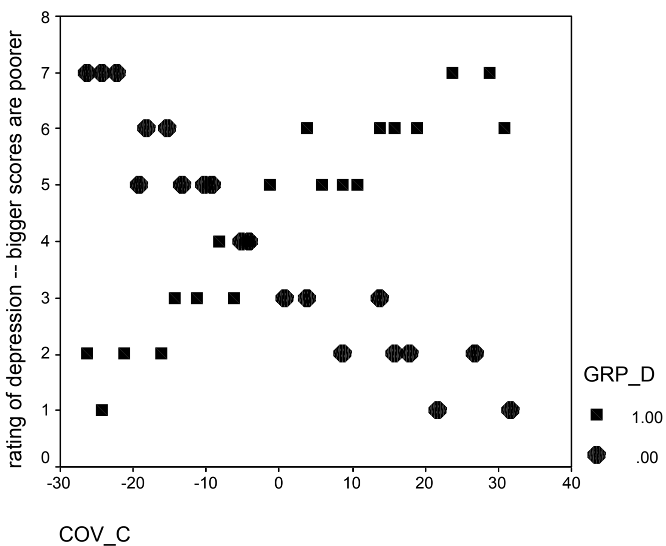
Graph → Scatter → highlight the "Simple" scatterplot and click on Define



By convention, the DV goes on the Y axis and the quantitative variable (the covariate) goes on the X axis.

Identify each plotted case with its group membership by setting the Marker to be "grp".

Add titles and footnotes by clicking the "Titles" button



This plot of the data looks very different from the plot of the model above, another suggestion that the above model isn't very good!!! This graph suggests the regression lines for the two groups have very different slopes -- suggesting an interaction.

Using Regression to Obtain an Interaction Model

There are a two common ways to perform the multiple regression with an interaction term:

- A full model with all the terms added
  - o Since each "effect" in the model is a single term, the t-test of the associated b tells whether or not that term contributes to the model after correction for the other terms in the model
  - o If you get the semi-partial correlations with the regression output you can compute the $R^2\Delta$ as the square of the semi-partial correlation of the interaction. The significance test of that regression weight is the significance test of the semi-partial correlation & is the significance test of the $R^2\Delta$.

- A 2-step hierarchical model
  - o Step 1 includes the main effects
    - ▪ the R^2 and F-test may or may not tell us much
  - o Step 2 adds the interaction effect – the full model
    - ▪ The R^2 and F-test tell us whether or not the "works"
    - ▪ The $R^2\Delta$ and F-test tell us whether or not the interaction contributes to the model
  - o Proponents of this often like having the $R^2\Delta$ to summarize the independent contribution of the interaction and the $R^2\Delta$F-test to test that contribution.

Full Regression Model Approach

Model Summary

Model	R	R Square
1	.952[a]	.907

a.

As expected, this looks just like the last step of the hierarchical model.

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	127.332	3	42.444	116.93	.000[a]
	Residual	13.068	36	.363		
	Total	140.400	39			

a. Predictors: (Constant), INT, GRP_D, COV_C

b. Dependent Variable: rating of depression -- bigger scores are poorer

Notice again that results of the full model are the same as the second step of the hierarchical approach below.

Also, the $R^2\Delta$ attributable to the interaction is computed as the square of the part correlation of the interaction

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations		
		B	Std. Error	Beta			Zero-order	Partial	Part
1	(Constant)	3.856	.135		28.536	.000			
	grp_d	.411	.191	.110	2.153	.038	.107	.338	.109
	cov_c	-.107	.008	-.159	-13.871	.000	-.058	-.318	-.066
	int	.205	.011	.804	18.565	.000	.636	.952	.944

a. Dependent Variable: rating of depression -- bigger scores are poorer

Looking at the regression weights from the full model:

- The significant interaction tells us that the direction and/or size of the group difference *depends upon* the specific value of the quantitative variable.
- When there is a significant interaction term in the model, be careful not to interpret the other regression weights as main effects – they are **simple effects**. We must examine the main effects very carefully, to determine if it is “descriptive” (unconditionally true for all values of the quantitative variable) or “potentially misleading” (conditionally true for only some values of the quantitative variable)
- The group difference is tested specifically at the mean of the quantitative variable (0 because of centering – it is a simple effect, not a main effect). We don’t directly test the group difference at other values of the quantitative variable from the regression weight.
- The covariate effect is tested specifically for the group coded 0 – it is a simple effect, not a main effect. We don’t directly test whether the regression slope for the other group is significantly different from zero

a -- the mean for the comparison group (CBT=0) when the covariate = 0 (mean after centering)

Simple effect for group when covariate = 0 (its mean after centering)
grp_d -- The regression weight for group dummy code (CBT = 0, PEER = 1) tells us that the peer support group has a mean depression score that is .411 larger than the CBT group, when holding time and the group*time interaction constant at zero (the mean of the centered variable). This mean difference is statistically significant.

Simple effect for the covariate for the comparison group (CBT coded 0)
cov_c -- The regression weight for the centered covariate tells that depression is expected to decrease .107 for each additional day before the participant starts therapy, holding the other variables constant. Specifically, this is the slope of the regression line for those in the comparison group (CBT) with a score of “0” on the interaction term. This slope is significantly different from zero.

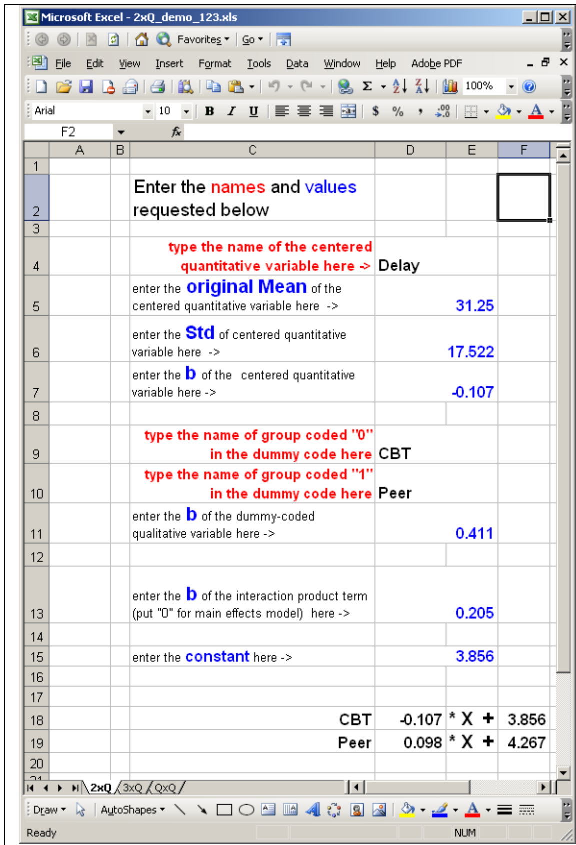
Interaction – simple effect differences for CBT & Peer groups
int -- The interaction regression weight tells us that the slope of the depression-time regression line for the Peer group (1) is .205 greater than the slope of the depression-time regression line for the CBT group (0). The interaction regression weight also tells us that each time the delay increases by 1, the mean difference between the groups increases by .205,

Things to Notice:

- The regression weight for group is actually a tiny bit smaller in the interaction model (.411) than in the main effects model, but is significant (p=.03) in the interaction model. Why? This is an example of “multivariate power.” Adding the interaction raised the R^2 from .016 to .907, and decreased the MSerror from 3.735 to .363. The R^2 & the regression weights in the interaction model are tested much more powerfully with this much smaller error term.
- The regression weight for delay is slightly larger (more negative) in the interaction model and is significant, again because adding the interaction term increased the fit of the model and the error term was much smaller.

Plotting the Interaction Model

It is important to get a sense for the statistical model represented by the regression weights. That is why it is important to plot the model, and be able to describe the "story" that it tells.



Describing the plot of the model:

Significant interaction regression weight

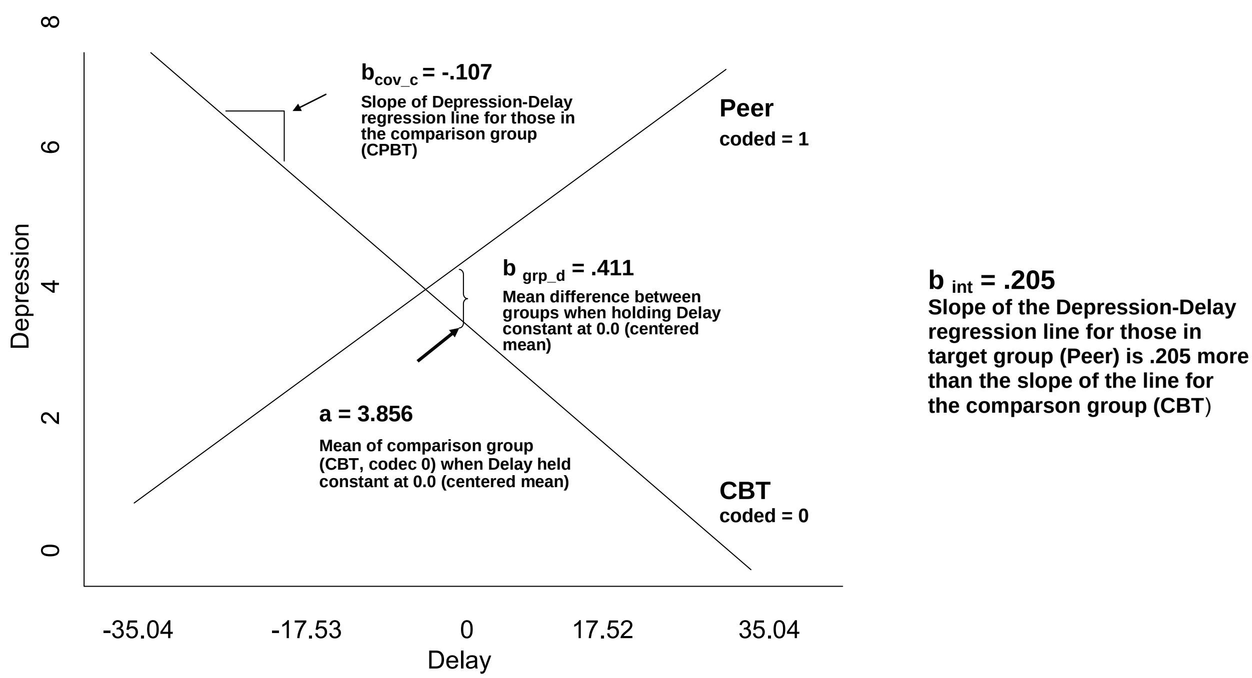
- there is a negative relationship between delay and depression for those in the CBT group
  - o those who delayed longer had higher depression scores than those who delayed a shorter time
- whereas there was a positive relationship between delay and depression for those in the PEER group
  - o those who delayed longer had lower depression scores than those who delayed a shorter time

Whatever the regression weight for delay (remember it is a simple effect) the main effect is potentially misleading

- the relationship between delay and the DV depends upon which treatment group you look at
- when you have an interaction, always check main effects for descriptive/misleading

Whatever the regression weight for group (remember it is a simple effect) the main effect is potentially misleading

- the direction and size of the treatment effects depends upon the value of delay you look at
- when you have an interaction, always check main effects for descriptive/misleading



Hierarchical Regression Approach

This is the "classic" approach and still preferred by many. SPSS Regression was used to obtain first the main effects model and then add the interaction term to obtain the full model. $R^2\Delta$ tests and semi-partial correlations were also requested.

Model Summary

Model	R	R Square	R Square Change	F Change	df1	df2	Sig. F Change
			Change Statistics				
1	.126[a]	.016	.016	.297	2	37	.745
2	.952[b]	.907	.891	344.677	1	36	.000

a. Predictors: (Constant), COV_C, GRP_D

b. Predictors: (Constant), COV_C, GRP_D, INT

As we saw above, the main effects ANCOVA model didn't account for much variance and neither the COV nor the grouping variable contributed to that model.

Adding the interaction term increased the R^2 considerably.

ANOVA[c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2.219	2	1.109	.297	.745[a]
	Residual	138.181	37	3.735		
	Total	140.400	39			
2	Regression	127.332	3	42.444	116.930	.000[b]
	Residual	13.068	36	.363		
	Total	140.400	39			

a. Predictors: (Constant), COV_C, GRP_D

b. Predictors: (Constant), COV_C, GRP_D, INT

c. Dependent Variable: rating of depression -- bigger scores are poorer

Also with this large increase in the fit of the model, the error term decreased dramatically, so that the main effects of the COV and GRP are both significant.

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	Correlations Zero-order	Correlations Partial	Correlations Part
1	(Constant)	3.990	.433		9.220	.000			
	grp_d	.419	.613	.112	.684	.498	.107	.112	.112
	cov_c	-.007	.018	-.067	-.407	.686	-.058	-.067	-.066
2	(Constant)	3.856	.135		28.536	.000			
	grp_d	.411	.191	.110	2.153	.038	.107	.338	.109
	cov_c	-.107	.008	-.159	-13.871	.000	-.058	-.318	-.066
	int	.205	.011	.804	18.565	.000	.636	.952	.944

a. Dependent Variable: rating of depression -- bigger scores are poorer

Things to notice:

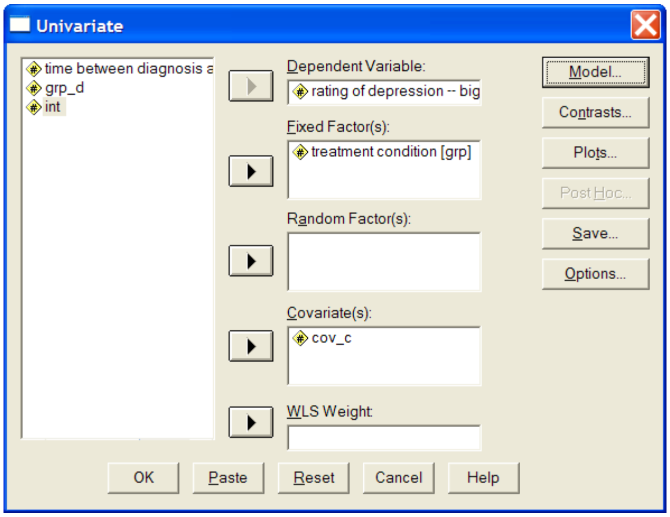
Notice that squaring the part correlation of the interaction in the full model ($.944^2 = .891$) gives the $R^2\Delta$ from adding the interaction to the main effect model.

The second step of the hierarchical model shows exactly the same results as the full model above. As always, "The full model is always the same, no matter what the intermediate steps!"

2xQ Interaction Models using SPSS GLM

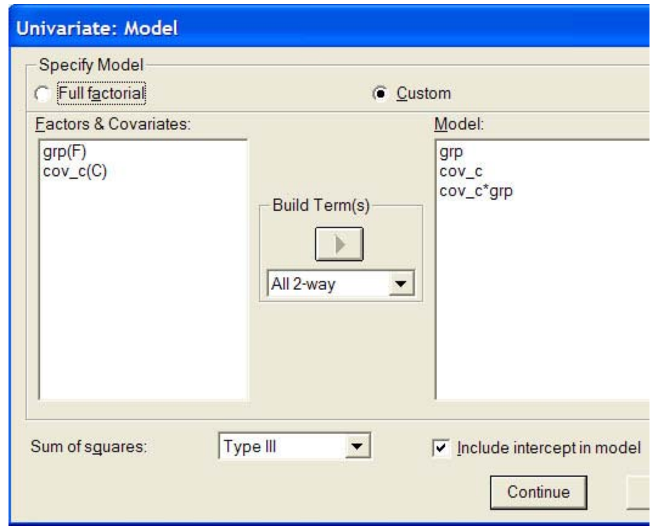
There are different ways to coerce GLM into computing this model. The following variation gives all the information provided by the multiple regression approach plus giving the group means after correction for the covariate and the interaction.

Analyze ➔ General Linear Model ➔ Univariate



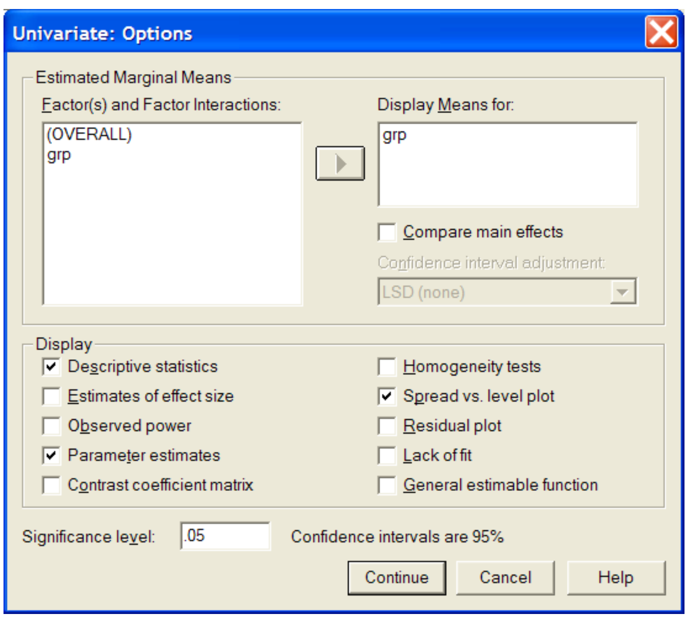
Use the original group variable as the Fixed Factor

Be sure to use the centered delay term as the “Covariate”



Put the grouping variable and the covariate in as main effects

Combine them into a 2-way interaction



Move the IV into the “Display Means for” window -- to obtain the corrected group means

Be sure to check Descriptive Statistics and Parameter estimates

Descriptive Statistics

Dependent Variable: rating of depression -- bigger scores are poorer

treatment condition	Mean	Std. Deviation	N
support group	4.4000	1.81804	20
cog-beh group	4.0000	2.00000	20
Total	4.2000	1.89737	40

These are the same group means that we've seen before.

These are the raw or uncorrected group means. The group means corrected for the covariate and the interaction that are tested in the ANCOVA model are shown down below.

Parameter Estimates

Dependent Variable: rating of depression -- bigger scores are poorer

Parameter	B	Std. Error	t	Sig.
Intercept	3.856	.135	28.536	.000
[grp=1.00]	.411	.191	2.153	.038
[grp=2.00]	0[a]	.	.	.
cov_c	-.107	.008	-13.871	.000
[grp=1.00] * cov_c	.205	.011	18.565	.000
[grp=2.00] * cov_c	0[a]	.	.	.

a. This parameter is set to zero because it is redundant.

Remember that SPSS dummy codes the grouping variable with the highest-valued group as the comparison group (CBT).

SPSS computes the interaction codes as the product of the dummy code of the grouping variable and of the centered continuous variable -- ***you must remember to use the centered version of the quant variable when you submit the analysis.***

These "b" values are the same as from the regression analysis. GLM does not give "beta" values. However they can be calculated.

$$\beta = (b * Std_{\mathbf{pred}}) / Std_{\mathbf{crit}}$$

Tests of Between-Subjects Effects

Dependent Variable: rating of depression -- bigger scores are poorer

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	127.332[a]	3	42.444	116.930	.000
Intercept	655.817	1	655.817	1806.717	.000
grp	1.682	1	1.682	4.634	.038
cov_c	.212	1	.212	.583	.450
grp * cov_c	125.114	1	125.114	344.677	.000
Error	13.068	36	.363		
Total	846.000	40			
Corrected Total	140.400	39			

a. R Squared = .907 (Adjusted R Squared = .899)

Using GLM in this way provides an F-test for each specific "effect" in the model.

The $F_{\mathbf{interaction}} = t^2$ from the interaction regression weight.

The $F_{\mathbf{group}} = t^2$ from the group regression weight

The $F_{\mathbf{cov\_c}} \neq t^2$ from the co_c regression weight !!!

Why? GLM uses dummy coding (0,1) to compute the regression weight for grp, but it uses effect coding (-.5, .5) to compute the SS for grp.

treatment condition

Dependent Variable: rating of depression -- bigger scores are poorer

treatment condition	Mean	Std. Error
support group	4.406[a]	.135
cog-beh group	3.994[a]	.135

a. Covariates appearing in the model are evaluated at the following values: COV_C = .0000, INT = .6750.

Above are the group means after correction for the covariate and the interaction. These are the same values as on the respective regression lines for each group at COV = 0.

The difference between these corrected means is the same as the regression weight for the dummy-coded group term in both the Parameter Estimates from GLM and the regression weights from the regression analysis up above.

So, when grp is dummy coded (in the parameter estimates) , the regression weight for cov_c tells the simple effect slope of the DV-cov_c regression line for those in the comparison group (coded 0).

But, when grp is effect coded, -.5 & .5 as in the ANOVA table, the cov_c effect is testing the slope of the DV-cov_c regression line for those with grp = 0 (which is no one because of effects coding). If the groups have the same n, then this F tests the "main effect" slope of the DV-cov_c regression line (i.e., "on average" for those coded -.5 & .5). Looking back at the plot of the model, we see that the "average" the two regression lines would have a slope very close to zero.

To summarize:

- ANOVA F-tests are of the main effects & interaction
- Regression t-tests are of simple effects & interaction