Factor Rotation

Back to the adolescent data -- let's look at different rotations of the three factors with $\lambda > 1.00$.

Varimax rotation

- tends to produce multiple group factors
- maintaining orthogonality often results in increased multivocality (loadings of variables on "primary factors" is decreased a bit and loadings on "secondary factors" is raised a bit)

| Factor Analysis: Rol | ation | × |
|---|---|----------------------------|
| Method C <u>N</u> one C <u>V</u> arimax C Direct <u>D</u> blimin Delta: | C Quartimax C Equamax C Promax Kappa 4 | Continue Cancel Help |
| Display <u>R</u> otated solution Maximum Iterations for | | |

It is easy and reasonable to criticize orthogonal rotations for being "simplistic" or "artificial". After all, most things we study are at least somewhat related. For example, if I told you that I had several anxiety and several depression measures and that they factored into 2 factors, you'd not be surprised. But if I told you that the two factors were "depression" and "anxiety," you'd not be surprised. If I told you that those 2 factors were uncorrelated, you'd probably be surprised.

Oblique solution help to reduce this artificiality.

| | | Initial Eigenvalu | ies | Extractio | Extraction Sums of Squared Loadings | | | n Sums of Square | ed Loadings |
|-----------|-------|-------------------|--------------|-----------|-------------------------------------|--------------|-------|------------------|--------------|
| Component | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 3.048 | 38.097 | 38.097 | 3.048 | 38.097 | 38.097 | 2.304 | 28.798 | 28.798 |
| 2 | 1.709 | 21.363 | 59.459 | 1.709 | 21.363 | 59.459 | 2.093 | 26.163 | 54.961 |
| 3 | 1.340 | 16.746 | 76.205 | 1.340 | 16.746 | 76.205 | 1.699 | 21.244 | 76.205 |
| 4 | .636 | 7.953 | 84.158 | | | | | | |
| 5 | .483 | 6.036 | 90.194 | | | | | | |
| 6 | .340 | 4.244 | 94.438 | | | | | | |
| 7 | .240 | 3.000 | 97.438 | | | | | | |
| 8 | .205 | 2.562 | 100.000 | | | | | | |

Total Variance Explained

Extraction Method: Principal Component Analysis.

Notce the variance "spreads out" across the 3 factors with this rotation -- common with Varimax.

- "sad" is a classic example of increased simple structure \rightarrow goes from multivocal to univocal
- so do several others
- note however, "extreme verbal abuse" → goes from univocal to multivocal
 - seems it would be very important to properly interpret why this variable is multi-vocal!

| | Component | | | | | |
|----------------------|-----------|-----------|-----------|--|--|--|
| | 1 2 3 | | | | | |
| physical aggression | .758 | .413 | 1.164E-03 | | | |
| property damage | .693 | .489 | 199 | | | |
| theft | .362 | .656 | 204 | | | |
| extreme verbal abuse | .826 | 6.589E-02 | .235 | | | |
| sad | .540 | 510 | .441 | | | |
| anxious | .654 | 335 | .507 | | | |
| self-confidence | 349 | .539 | .669 | | | |
| compliance | 580 | .450 | .551 | | | |

Component Matrix^a

Rotated Component Matrix

| | Component | | | | |
|----------------------|-----------|-----------|-----------|--|--|
| | 1 | 2 | 3 | | |
| physical aggression | .807 | .301 | -6.51E-02 | | |
| property damage | .853 | 9.875E-02 | 148 | | |
| theft | .751 | 186 | 6.583E-02 | | |
| extreme verbal abuse | .562 | .645 | 105 | | |
| sad | -8.06E-02 | .846 | 155 | | |
| anxious | .110 | .884 | -5.44E-02 | | |
| self-confidence | 3.339E-02 | -2.60E-02 | .927 | | |
| compliance | 165 | 209 | .879 | | |

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

a. Rotation converged in 4 iterations.

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

Quartimax rotation

tends to produce a general factor and additional smaller multiple group factors

| Factor Analysis: Ro | tation | × |
|--|---|----------------------------|
| Method C <u>N</u> one C <u>V</u> arimax CDirect <u>O</u> blimin Delta: 0 | O Quartimax ○ Equamax ○ Promax ▲appa 4 | Continue Cancel Help |
| Display <u>R</u> otated solution Maximum Iterations for | | |

- Quartimax rotation won't make strong group factors "go away"
- but you know you have a weak factor structure if varimax & quqrtimax give importantly different solutions
- one source of a weak factor structure that gives different solutions from different rotations is over factoring

Total Variance Explained

| | | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | | Sums of Square | ed Loadings |
|-----------|-------|---------------------|--------------|-------|-------------------------------------|--------------|-------|----------------|--------------|
| Component | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 3.048 | 38.097 | 38.097 | 3.048 | 38.097 | 38.097 | 2.337 | 29.214 | 29.214 |
| 2 | 1.709 | 21.363 | 59.459 | 1.709 | 21.363 | 59.459 | 2.083 | 26.040 | 55.254 |
| 3 | 1.340 | 16.746 | 76.205 | 1.340 | 16.746 | 76.205 | 1.676 | 20.951 | 76.205 |
| 4 | .636 | 7.953 | 84.158 | | | | | | |
| 5 | .483 | 6.036 | 90.194 | | | | | | |
| 6 | .340 | 4.244 | 94.438 | | | | | | |
| 7 | .240 | 3.000 | 97.438 | | | | | | |
| 8 | .205 | 2.562 | 100.000 | | | | | | |

Extraction Method: Principal Component Analysis.

Still got some "spreading around" of the variance across the factors -- looks much the same as the varimax

- Pretty strong sign for "group factors" → got them from both varimax and quartimax
- Remember, "strong" solutions won't be "hidden" by the rotation you select

| | Component | | | | |
|----------------------|-----------|-----------|-----------|--|--|
| | 1 | 2 | 3 | | |
| physical aggression | .758 | .413 | 1.164E-03 | | |
| property damage | .693 | .489 | 199 | | |
| theft | .362 | .656 | 204 | | |
| extreme verbal abuse | .826 | 6.589E-02 | .235 | | |
| sad | .540 | 510 | .441 | | |
| anxious | .654 | 335 | .507 | | |
| self-confidence | 349 | .539 | .669 | | |
| compliance | 580 | .450 | .551 | | |

Component Matrix^a

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

Rotated Component Matrix

| | Component | | | | |
|----------------------|-----------|-----------|-----------|--|--|
| | 1 | 2 | 3 | | |
| physical aggression | .814 | .285 | -4.99E-02 | | |
| property damage | .856 | 8.321E-02 | 135 | | |
| theft | .746 | 203 | 7.244E-02 | | |
| extreme verbal abuse | .576 | .634 | -8.73E-02 | | |
| sad | -6.10E-02 | .850 | 142 | | |
| anxious | .129 | .882 | -3.86E-02 | | |
| self-confidence | 2.063E-02 | -4.15E-02 | .927 | | |
| compliance | 181 | 220 | .873 | | |

Extraction Method: Principal Component Analysis. Rotation Method: Quartimax with Kaiser Normalization.

a. Rotation converged in 4 iterations.

Direct Oblimin rotation

· Tends to produce varimax-looking factors, but which are oblique

| Factor Analysis: Rol | ation | × |
|---|---|----------------------------|
| Method C None C ⊻arimax C Direct <u>D</u> blimin Delta: | C Quartimax C Equamax C Promax Kappa 4 | Continue Cancel Help |
| Display <u>B</u> otated solution Maximum Iterations for | | |

Delta is a parameter that "controls" the extent of obliqueness amongst the factors.

- Negative values "decrease" factor correlations
- "0" is the default
- Positive values (don't go over .8) "permit" additional factor correlation

When you do an oblique rotation you get two different matrices that can be used for interpretation:

- the structure matrix holds the correlations between each variable and each factor (same as with orthogonal rotations)
- the pattern matrix holds the beta weights to reproduce variable scores from factor scores

There is considerable disagreement about which of these is the better basis for factor interpretation:

- Those who like using the structure matrix point out the long history of naming or interpreting factors (and other composite variables -- ldf, canonical correlation) in terms of the "variables with which they correlate."
- Those who like using the pattern matrix point out that there is often "simpler structure" in the pattern matrix
- Those who like using the structure matrix point out that the apparent "simpler structure" (i.e., fewer multivocal items) in the pattern matrix is an illusion, made possible because of the correction for collinearity by the beta weights.
- Typically, the interpretation based on the two matrices will be similar ...

| | Component | | | | |
|----------------------|-----------|-----------|-----------|--|--|
| | 1 2 3 | | | | |
| physical aggression | .241 | .787 | -1.36E-02 | | |
| property damage | 1.783E-02 | .848 | 119 | | |
| theft | 240 | .779 | 6.824E-02 | | |
| extreme verbal abuse | .608 | .507 | -2.52E-02 | | |
| sad | .858 | 163 | -7.26E-02 | | |
| anxious | .896 | 3.050E-02 | 3.954E-02 | | |
| self-confidence | 9.405E-02 | 7.397E-02 | .949 | | |
| compliance | -8.61E-02 | 113 | .875 | | |

Pattern Matrix^a

| Delta | = | 0 |
|-------|---|---|
| | | |

| | Component | | | |
|----------------------|-----------|------|------|--|
| | 1 | 2 | 3 | |
| physical aggression | .379 | .829 | 146 | |
| property damage | .191 | .862 | 203 | |
| theft | 123 | .731 | .051 | |
| extreme verbal abuse | .701 | .613 | 218 | |
| sad | .847 | 010 | 261 | |
| anxious | .891 | .180 | 176 | |
| self-confidence | 118 | .000 | .919 | |
| compliance | 313 | 211 | .906 | |

Structure Matrix

Extraction Method: Principal Component Analysis. Rotation Method: Oblimin with Kaiser Normalization.

Extraction Method: Principal Component Analysis. Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 9 iterations.

Promax Rotation

- An oblique rotation that tends to produce group factors that look like Direct Oblimin & Varimax
- Promax computations are more much quicker, so it is commonly used with very large factorings