## Comparing a Multiple Regression Model Across Criterion Variables

Sometimes we have multiple behaviors or responses that might be used as criterion variables. When this happens we might want to determine whether the models to predict the different criterion differ. This can be a useful way of determining whether or not behaviors or responses represent "the same thing".

For example we might have two different criteria by which we assess the "performance" of 1st year graduate students: 1) their grades and 2) the number of presentations and/or publications they obtain or collaborate on during their first year. Are the multiple regression models for predicting these two from GRE scores "the same"?

First we get the multiple regression model for each criterion, in turn. Here's the abbreviated output …

**For GPA as the criterion**

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .756[a] | .572 | .563 | .39752 |

a. Predictors: (Constant), Verbal subscore of GRE, Quantitative subscore of GRE, Analytic subscore of GRE

**Coefficients[a]**

| Model | | Unstandardized Coefficients B | Standardized Coefficients Beta | Sig. |
|---|---|---|---|---|
| 1 | (Constant) | -1.215 | | .003 |
| | Analytic subscore of GRE | 6.78E-03 | .551 | .000 |
| | Quantitative subscore of GRE | 3.42E-03 | .462 | .000 |
| | Verbal subscore of GRE | -2.3E-03 | -.235 | .001 |

a. Dependent Variable: 1st year graduate gpa -- criterion variable

**For #pubs & presentations as the criterion**

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .917[a] | .842 | .838 | .24176 |

a. Predictors: (Constant), Verbal subscore of GRE, Quantitative subscore of GRE, Analytic subscore of GRE

**Coefficients[a]**

| Model | | Unstandardized Coefficients B | Standardized Coefficients Beta | Sig. |
|---|---|---|---|---|
| 1 | (Constant) | 7.706 | | .000 |
| | Analytic subscore of GR | 1.1E-04 | .009 | .827 |
| | Quantitative subscore of GRE | 4.0E-04 | .058 | .152 |
| | Verbal subscore of GRE | 8.6E-03 | .893 | .000 |

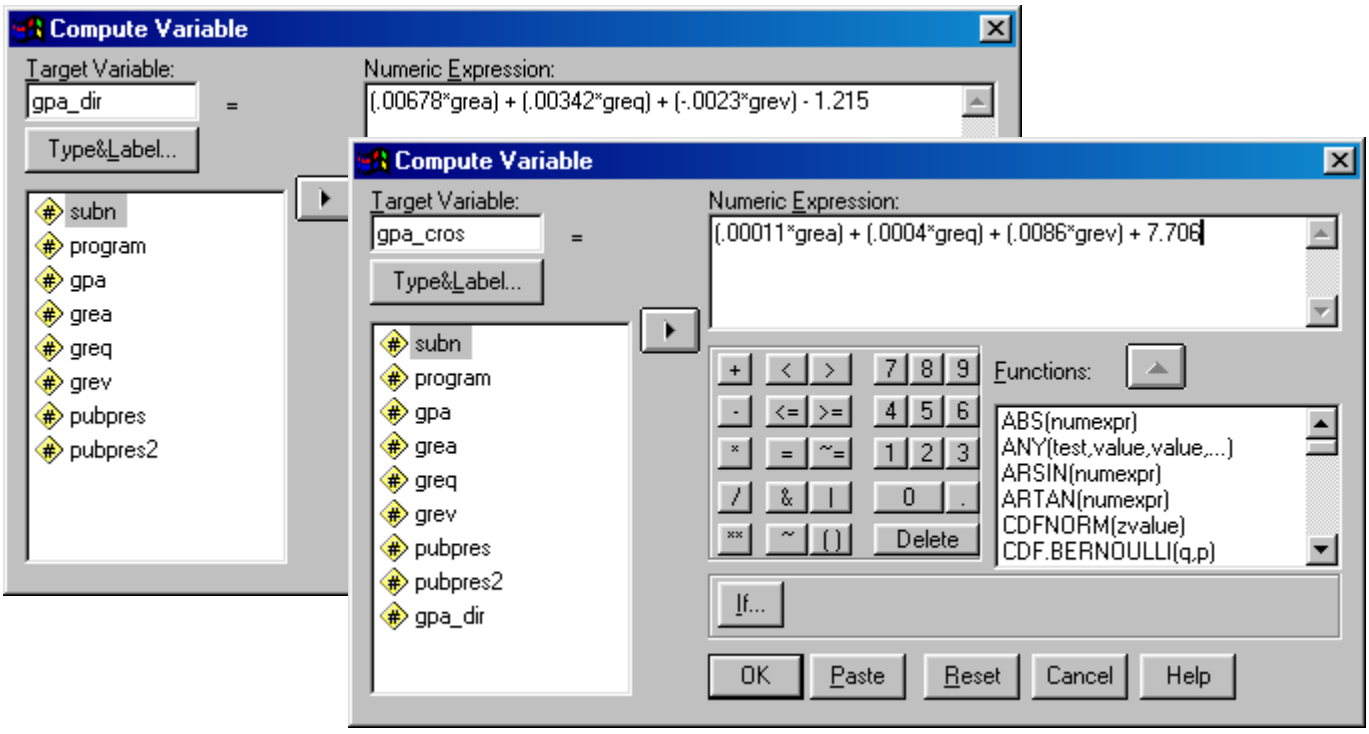
a. Dependent Variable: PUBPRES

Inspection of the β weights of the two models suggests that:

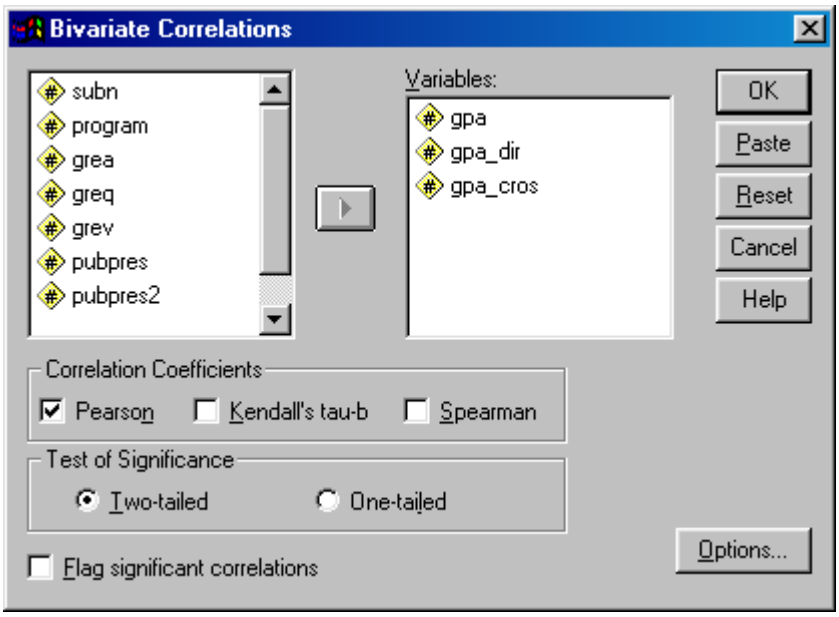
- The fit of the two models to the criteria appears to be different -- $R^2$ = .572 for GPA and .842 for publications/presentations.
- There is no direct test of these correlations -- can't use Fisher's Z-test because the models are from the same sample & can't use Hotelling's/Meng, etc. tests because both the criterion and the predictors producing the R values are different.

- The models appear to have very different "structures"
- Prediction of 1st year GPA involves all three GRE subscales (with somewhat less contribution from the suppressor variable GREV)
- Prediction of 1st year #publications/presentations involves only GREV, in this sample.
- To formally compare these models we will need to …
- Apply both models to the prediction of a single criterion (we'll use GPA)
- Compare the R obtained from these applications, taking the correlation between the models into account

We'll need to compute a predicted GPA score based on each model, in turn …




Finally, we get the correlation of each model with the selected criterion variable, as well as the correlation between the models.




**SPSS Syntax**

```
COMPUTE gpadir = (.00678*grea) + (.00342*greq) + (-.0023*grev) - 1.215.   ← compute the direct model y'
COMPUTE gpacros = (.00011*grea) + (.0004*greq) + (.0086*grev) + 7.706.    ← compute the crossed model y'

CORR VARIABLES = gpa  gpa_dir  gpa_cros.                                  ← compute the correlations
```

Here's the output…

**Correlations**

| | | 1st year graduate gpa -- criterion variable | GPA_DIR | GPA_CROS |
|---|---|---|---|---|
| 1st year graduate gpa -- criterion variable | Pearson Correlation | 1 | .756 | .310 |
| | Sig. (2-tailed) | . | .000 | .000 |
| | N | 140 | 140 | 140 |
| GPA_DIR | Pearson Correlation | .756 | 1 | .406 |
| | Sig. (2-tailed) | .000 | . | .000 |
| | N | 140 | 140 | 140 |
| GPA_CROS | Pearson Correlation | .310 | .406 | 1 |
| | Sig. (2-tailed) | .000 | .000 | . |
| | N | 140 | 140 | 140 |

Direct R -- the same as the R from the original multiple regression analysis of the GPA criterion

Crossed R -- when you apply the weights from the #pubs/presentations multiple regression model onto GPA

Model Correlation -- remember that the correlation between two models is represented by the correlation between their y' values

We use the Steiger's Z portion of the xls Computator to test if the Cirect and Crossed models fit significantly differently.

**Steiger's Z-test - Comparing Correlated Correlations**

| | |
|---|---|
| r(1,2) => | 0.756 |
| r(1,3) => | 0.31 |
| r(2,3) => | 0.406 |
| N => | 140 |
| Z = | 6.330 |
| p = | 2.46E-10 |

**Remember the "Steiger's Z" test uses R (r) values!**

Our intent it to compare the $R^2$ values! **But the formula uses R values!!**

So, the apparent differences between the structures of the multiple regression models for predicting these two criterion variables were revealed by the formal statistical comparison. The substantive implication of this finding is that the graduate faculty need to decide which criterion they want to predict, because the model for the two criterion isn't "interchangeable" -- that is predicting 1st year GPA is not the same as predicting 1st year #pubs/presentations (by the way, these two criterion variables are correlated -.285 in this sample).

**Example write-up of these analyses** (which used some univariate and correlation analyses not shown above)**:**

Separate regression analyses were run using the Verbal (GREV), Quantitative (GREQ) and Analytic (GREA) GRE subscales to predict the two criterion variables graduate grade point average (GGPA) and the number of graduate publications and presentations (NGPP). Table 1 shows the univariate statistics and Table 2 shows the correlations of each variable with respective criterion variables and the multiple regression weights for the models of the two criterion variables.

The graduate grade point average model had an $R^2 = .572$, $F(3,138) = 35.65$, $p < .001$, with all three GRE scores having significant regression weights and GREA and GREQ seeming to have the major contributions (based on inspection of the β weights). The number of publications and presentations model had an $R^2 = .842$, $F(3,138) = 72.53$, $p < .001$, with only GREV having significant a regression weight.

A comparison of the structure of the models for the two criterion variables was also conducted by applying the model derived from number of publications and presentations criterion to graduate grade point average and comparing the resulting "crossed" $R^2$ with the "direct" $R^2$ originally obtained for this criterion. The direct $R^2$=.572 and crossed $R^2$=..096 were significantly different, $Z = 6.33$, $p < .01$, which indicates that the apparent differential structure of the regression weights derived for the two criteria described above warrants further interpretation and investigation.

Table 1 Summary statistics for criterion and predictor variables.

| Variable | mean | std |
|---|---|---|
| NGPP | 11.21 | 3.32 |
| GGPA | 3.23 | .61 |
| GREV | 567.88 | 40.99 |
| GREQ | 589.62 | 82.01 |
| GREA | 576.03 | 66.86 |

Table 2 Correlations and multiple regression weights from models of graduate grade point average and number of publications and presentations.

| | Graduate Grade Point Average | | | Number of Publications & Presentations | | |
|---|---|---|---|---|---|---|
| Variable | r with GGPA | b | β | r with NPP | b | β |
| GREV | .270 | -.0023** | -.235 | .839** | .0086** | .893 |
| GREQ | .479** | .0034** | .462 | .320* | .0044 | .058 |
| GREA | .632** | .0068** | .551 | .194 | .0011 | .009 |
| constant | | -1.215 | | | 7.706 | |

* $p < .05$ ** $p < .01$