Sources of Variance & ANOVA

- BG ANOVA
  - Partitioning Variation
  - “making” F
  - “making” effect sizes
- Things that “influence” F
  - Confounding
  - Inflated within-condition variability
- Integrating “stats” & “methods”

ANOVA → ANalysis Of VAriance

Variance means “variation”

- Sum of Squares (SS) is the most common variation index
- SS stands for, “Sum of squared deviations between each of a set of values and the mean of those values”
  \[ SS = \sum (\text{value} - \text{mean})^2 \]

So, Analysis Of Variance translates to “partitioning of SS”

In order to understand something about “how ANOVA works” we need to understand how BG and WG ANOVAs partition the SS differently and how F is constructed by each.

Variance partitioning for a BG design

\[ SS_{total} = SS_{Effect} + SS_{Error} \]
Constructing BG $F$ & $r$

$$SS_{Total} = SS_{Effect} + SS_{Error}$$

Mean Square is the SS converted to a “mean” → dividing it by “the number of things”

- $MS_{Effect} = \frac{SS_{Effect}}{df_{IV}}$  
  $df_{Effect} = k - 1$ represents design size
- $MS_{error} = \frac{SS_{error}}{df_{error}}$  
  $df_{error} = \sum n - k$ represents sample size

$F$ is the ratio of “effect variation” (mean difference) * “individual variation” (within condition differences)

$$F = \frac{MS_{IV}}{MS_{error}}$$

$$r^2 = \frac{Effect}{(Effect + error)} \quad \text{conceptual formula}$$

$$= \frac{SS_{Effect}}{(SS_{Effect} + SS_{error})} \quad \text{definitional formula}$$

$$= \frac{F}{(F + df_{error})} \quad \text{computational formula}$$

An Example …

**ANOVA** was designed to analyze data from studies with…

- Samples that represent the target populations
- True Experimental designs
  - proper RA
  - well-controlled IV manipulation
- Good procedural standardization
- No confounds

ANOVA is a very simple statistical model that assumes there are few sources of variability in the data

$$BG \quad SS_{Total} = SS_{Effect} + SS_{Error} \quad F = \frac{SS_{Effect}}{df_{Effect}} \frac{df_{Effect}}{df_{Error}}$$

However, as we’ve discussed, most data we’re asked to analyze are not from experimental designs.
2 other sources of variation we need to consider whenever we are working with quasi- or non-experiments are…

Between-condition procedural variation -- confounds
- any source of between-condition differences other than the IV
  - subject variable confounds (initial equiv)
  - procedural variable confounds (ongoing equiv.)
- influence the numerator of F

Within-condition procedural variation -- (sorry, no agreed-upon label)
- any source of within-condition differences other than "naturally occurring population individual differences"
  - subject variable diffs not representing the population
  - procedural variable influences on within cond variation
- influence the denominator of F

These considerations lead us to a somewhat more realistic model of the variation in the DV…

\[ SS_{Total} = SS_{IV} + SS_{confound} + SS_{indif} + SS_{wcvar} \]

Sources of variability…

- \( SS_{IV} \rightarrow IV \)
- \( SS_{confound} \rightarrow \) initial & ongoing equivalence problems
- \( SS_{indif} \rightarrow \) population individual differences
- \( SS_{wcvar} \rightarrow \) non-population individual differences

Imagine an educational study that compares the effects of two types of math instruction (IV) upon performance (% - DV)

Participants were randomly assigned to conditions, treated, then allowed to practice (Prac) as many problems as they wanted to before taking the DV-producing test

<table>
<thead>
<tr>
<th>Control Grp</th>
<th>Exper. Grp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prac DV</td>
<td>Prac DV</td>
</tr>
<tr>
<td>S1 5 75</td>
<td>S2 10 82</td>
</tr>
<tr>
<td>S3 5 74</td>
<td>S4 10 84</td>
</tr>
<tr>
<td>S5 10 78</td>
<td>S6 15 88</td>
</tr>
<tr>
<td>S7 10 79</td>
<td>S8 15 89</td>
</tr>
</tbody>
</table>

Confounding due to Practice
- mean prac dif between cond

WC variability due to Practice
- wg corrrelation or prac & DV

IV
- compare Ss 5&2 - 7&4

Individual differences
- compare Ss 1&3, 5&7, 2&4, or 6&8
The problem is that the F-formula will …

- Ignore the confounding caused by differential practice between the groups and attribute all BG variation to the type of instruction (IV) → overestimating the SSeffect
- Ignore the changes in within-condition variation caused by differential practice within the groups and attribute all WG variation to individual differences → overestimating SSerror
- As a result, the F & r values won't properly reflect the relationship between type of math instruction and performance → we will make statistical conclusion errors!

We have 2 strategies:
- Identify and procedurally control “inappropriate” sources
- Include those sources in our statistical analyses

Both strategies require that we have a “richer” model for variance sources – and that we can apply that model to our research!

The “better” SS model we’ll use …

\[
F = \frac{SS_{IV} + SS_{confound} / df_{IV}}{SS_{indif} + SS_{WCVAR} / df_{error}} \neq \frac{SS_{Effect} / df_{Effect}}{SS_{error} / df_{error}}
\]

Both strategies require that we have a “richer” model for SS sources – and that we can apply that model to our research!

The “better” SS model we’ll use …

\[
SS_{Total} = SS_{IV} + SS_{subcon} + SS_{procon} + SS_{indif} + SS_{wcsubinf} + SS_{wcprocinf}
\]

Sources of variability…
- \(SS_{IV} \rightarrow IV \smile\)
- \(SS_{subcon} \rightarrow\) subject variable confounds (initial eq problems)
- \(SS_{procon} \rightarrow\) procedural variable confounds (ongoing eq pbms)
- \(SS_{indif} \rightarrow\) population individual differences \(\smile\)
- \(SS_{wcsubinf} \rightarrow\) within-condition subject variable influences
- \(SS_{wcprocinf} \rightarrow\) within-condition procedural variable influences

In order to apply this model, we have to understand:
1. **How these variance sources relate to aspects of …**
   - IV manipulation & Population selection
   - Initial equivalence & ongoing equivalence
   - Sampling & procedural standardization

2. **How these variance sources befoul SS Effect, SS error & F of the simple ANOVA model**
   - \(SS_{IV}\) numerator
   - SS denominator
Sources of variability → their influence on $SS_{\text{Effect}}$, $SS_{\text{error}}$, & $F$ …

$SS_{\text{IV}} → IV$

“Bigger manipulations” produce larger mean differences
• larger $SS_{\text{effect}} →$ larger numerator → larger $F$
  • eg 0 v 50 practices instead of 0 v 25 practices
  • eg those receiving therapy condition get therapy twice a week instead of once a week

“Smaller manipulations” produce smaller mean differences
• smaller $SS_{\text{effect}} →$ smaller numerator → smaller $F$
  • eg 0 v 25mg antidepressant instead of 0 v 75mg
  • eg Big brother sessions monthly instead of each week

Sources of variability → their influence on $SS_{\text{Effect}}$, $SS_{\text{error}}$, & $F$ …

$SS_{\text{indir}} →$ Individual differences

More homogeneous populations have smaller within-condition differences
• smaller $SS_{\text{error}} →$ smaller denominator → larger $F$
  • eg studying one gender instead of both
  • eg studying “4th graders” instead of “grade schoolers”

More heterogeneous populations have larger within-condition differences
• larger $SS_{\text{error}} →$ larger denominator → smaller $F$
  • eg studying “young adults” instead of “college students”
  • eg studying “self-reported” instead of “vetted” groups

Sources of variability → their influence on $SS_{\text{Effect}}$, $SS_{\text{error}}$, & $F$ …

The influence a confound has on $SS_{\text{IV}}$ & $F$ depends upon the “direction of the confounding” relative to “the direction of the IV”
• if the confound “augments” the IV → $SS_{\text{IV}}$ & $F$ will be inflated
  • if the confound “offsets” the IV → $SS_{\text{IV}}$ & $F$ will be deflated

$SS_{\text{subcon}} →$ subject variable confounds (initial eq problems)

Augmenting confounds
• inflates $SS_{\text{Effect}} →$ inflates numerator → inflates $F$
  • eg 4th graders in Tx group & 2nd graders is Cx group
  • eg run Tx early in semester and Cx at end of semester

Offsetting confounds
• deflates $SS_{\text{Effect}} →$ deflates denominator → deflates $F$
  • eg 2nd graders in Tx group & 4th graders is Cx group
  • eg “better neighborhoods” in Cx than in Tx
Sources of variability \(\rightarrow\) their influence on \(S_{\text{Effect}}\), \(S_{\text{Error}}\) & \(F\)...

The influence a confound has on \(S_{\text{IV}}\) & \(F\) depends upon the “direction of the confounding” relative to “the direction of the IV”

- if the confound “augments” the IV \(\rightarrow S_{\text{IV}}\) & \(F\) will be inflated
- if the confound “offsets” the IV \(\rightarrow S_{\text{IV}}\) & \(F\) will be deflated

\(S_{\text{procon}} \rightarrow\) procedural variable confounds (ongoing eq problems)

Augmenting confounds

- inflates \(S_{\text{Effect}}\) \(\rightarrow\) inflates numerator \(\rightarrow\) inflates \(F\)
- eg Tx condition is more interesting/fun for assistants to run
- eg Run the Tx on the newer, nicer equipment

Offsetting confounds

- deflates \(S_{\text{Effect}}\) \(\rightarrow\) deflates denominator \(\rightarrow\) deflates \(F\)
- eg less effort put into instructions for Cx than for Tx
- eg extra practice for touch condition than for visual cond

\(S_{\text{wcsubinf}} \rightarrow\) within-condition subject variable confounds

More variation in the sample than in the population

- inflates \(S_{\text{Error}}\) \(\rightarrow\) inflates denominator \(\rightarrow\) deflates \(F\)
- eg letting participants practice as much as they want
- eg using multiple research stations

Less variation in the sample than in the population

- deflates \(S_{\text{Error}}\) \(\rightarrow\) deflates denominator \(\rightarrow\) inflates \(F\)
- eg DV measures that have floor or ceiling effects
- eg time allotments that produce floor or ceiling effects
Couple of important things to note !!!

\[ SS_{Total} = SS_{IV} + SS_{subcon} + SS_{proccon} + SS_{Indif} + SS_{wsubinf} + SS_{wprocinf} \]

Most variables that are confounds also inflate within condition variation!!!

• like in the simple example earlier
• “Participants were randomly assigned to conditions, treated, then allowed to practice as many problems as they wanted to before taking the DV-producing test”

There is an important difference between “picking a population that has more/less variation” and “sampling poorly so that your sample has more/less variation than the population”.

• is you intend to study 3rd, 4th & 5th graders instead of just 3rd graders, your SS_{error} will increase because your SS_{Indif} will be larger – but it is a choice, not a mistake!!!
• if intend to study 3rd stage cancer patients, but can’t find enough and use 2nd, 3rd & 4th stage patients instead, your SS_{error} will be inflated because of your SS_{wsubinf} – and that is a mistake not a choice!!!