

# ANCOVA

- Workings of ANOVA & ANCOVA
- ANCOVA, Semi-Partial correlations, statistical control
- Using model plotting to think about ANCOVA & Statistical control

You know how ANOVA works

- the total variation among a set of scores on a quantitative variable is separated into between groups and within groups variation
- between groups variation reflects the extent of the bivariate relationship between the grouping variable and the quant variable -- systematic variance
- within groups variation reflects the extent that variability in the quant scores is attributable to something other than the bivariate relationship -- unsystematic variance
- F-ratio compares these two sources of variation, after taking into account the number of sources of variability
  - $df_{bg}$  -- # groups - 1
  - $df_{wg}$  -- # groups \* (number in each group - 1)
- the larger the F, the greater the systematic bivariate relationship

ANCOVA allows the inclusion of a 3rd source of variation into the F-formula (called the covariate) and changes the F-formula

Loosely speaking...

ANOVA Model  $F = \frac{\text{BG variation attributed to IV}}{\text{WG variation attributed to individual differences}}$

ANCOVA  $F = \frac{\text{BG variation attributed to IV} + \text{BG variation attributed to COV}}{\text{WG variation attributed to individual differences} + \text{WG variation attributed to COV}}$

Imagine an educational study that compares two types of spelling instruction. Students from 3rd, 4th and 5th graders are involved, leading to the following data.

Control Grp            Exper. Grp

S1 3rd 75	S2 4th 81
S3 3rd 74	S4 4th 84
S5 4th 78	S6 5th 88
S7 4th 79	S8 5th 89

Individual differences (compare those with same grade & grp)  
 • compare Ss 1-3, 5-7, 2-4, 6-8  
 Treatment (compare those with same grade & different grp)  
 compare 5,7 to 2,4

Grade (compare those with same group & different grade)  
 compare 1,3 to 5,7 or 2,4 to 6,8

Notice that Grade is:

- acting as a confound – will bias estimate of the treatment effect
- acting to increase within-group variability – will increase error

## ANOVA

- ignores the covariate
- attributes BG variation exclusively to the treatment
  - but BG variation actually combines Tx & covariate
- attributes WG variation exclusively to individual differences
  - but WG variation actually combines ind difs & covariate
- F-test of Tx effect “ain’t what it is supposed to be”

## ANCOVA

- considers the covariate (a multivariate analysis)
- separates BG variation into Tx and Cov
- separates WG variation into individual differences and Cov
- F-test of the TX effect while controlling for the Cov, using ind difs as the error term
- F-test of the Cov effect while controlling for the Tx, using ind difs as the error term

ANCOVA is the same thing as a semi-partial correlation between the IV and the DV, correcting the IV for the Covariate

Applying regression and residualization as we did before ...

- predict each person’s IV score from their Covariate score
- determine each person’s residual (IV - IV')
- use the residual in place of the IV in the ANOVA (drop 1 error df)
- The resulting ANOVA tells the relationship between the DV and IV that is unrelated to the Covariate

OR...

ANCOVA is the same thing as multiple regression using both the dummy coded IV and the quantitative covariate as predictors of the DV

- the “b” for each shows the relationship between that predictor and the DV, controlling the IV for the other predictor

Several things to remember when applying ANCOVA:

- $H_0$ : for ANOVA & ANCOVA are importantly different
  - ANOVA: No mean difference between the populations represented by the treatment groups.
  - ANCOVA: No mean difference between the populations represented by the treatment groups, assuming all the members of both populations have a covariate score equal to the overall covariate mean of the current sampled groups.
- Don't treat statistical control as if it were experimental control
- You don't have all the confounds/covariates in the model, so you have all the usual problems of "underspecified models"
- The underlying philosophy (or hope) of ANCOVA, like other multivariate models is, "Behavior is complicated, so more complicated models, will on average, be more accurate."
- Don't confuse this with, "Any given ANCOVA model is more accurate than the associated ANOVA model."



As you can see, there are different "applications" of ANCOVA

- "correcting" the assessment of the IV-DV relationship for within group variability attributable to the covariate
  - will usually increase F -- by decreasing the "error variation?"
- "correcting" the assessment of the IV-DV relationship for between group variability attributable to the covariate
  - will increase or decrease F by increasing or decreasing the "Tx effect" -- depending upon whether covariate and Tx effects are in "same" or "opposite" directions
- "correcting" for both influences of the covariate upon F
  - F will change as a joint influence of decreasing "error variation" and increasing/decreasing "systematic variation"

You should recognize the second as what was meant by "statistical control" when we discussed that topic in the last section of the course



How a corresponding ANOVA & ANCOVA differ...

$SS_{\text{error}}$  for ANCOVA will always be smaller than  $SS_{\text{error}}$  for ANOVA

- part of ANOVA error is partitioned into covariate of ANCOVA

$SS_{\text{IV}}$  for ANCOVA may be =, < or > than  $SS_{\text{IV}}$  for ANOVA

- depends on the "direction of effect" of IV & Covariate

Simplest situation first!

Case #1: If  $Tx = Cx$  for the covariate (i.e., there is no confounding)

- ANOVA  $SS_{\text{IV}} =$  ANCOVA  $SS_{\text{IV}}$  there's nothing to control for
- smaller  $SS_{\text{error}}$  – So F will be larger & more sensitive
- F-test for Tx may still be confounded by other variables

If  $Tx \neq Cx$  for the covariate (i.e., there is confounding)

- ANOVA  $SS_{IV} \neq$  ANCOVA  $SS_{IV}$
- we can anticipate the ANOVA-ANCOVA difference if we pay attention to the relative “direction” of the IV effect and the “direction” of confounding
- Case #2: if the Tx & Confounding are “in the opposite direction”
  - eg, the 3<sup>rd</sup> graders get the Tx (that improves performance) and 5<sup>th</sup> graders the Cx
  - ANOVA will underestimate the TX effect (combining Tx & the covariate into the  $SS_{IV}$ )
  - ANCOVA will correct for that underestimation (partitioning Tx & covariate into separate SS)
  - ANOVA  $SS_{IV} <$  ANCOVA  $SS_{IV}$  } ANCOVA  $F >$  ANOVA  $F$
  - smaller  $SS_{error}$
  - F-tests for Tx and for Grade will be “better” – but still only “control” for this one covariate (there are likely others)

Case #3: if the Tx & Confounding are “in the same direction”

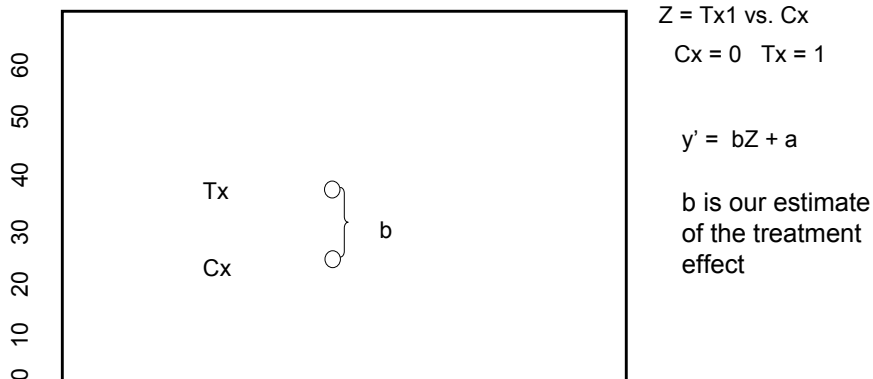
- eg, the 5<sup>th</sup> graders get the Tx (that improves performance) and 3<sup>rd</sup> graders the Cx
- ANOVA will overestimate the TX effect (combining Tx & the covariate into the  $SS_{IV}$ )
- ANCOVA will correct for that overestimation (partitioning Tx & covariate into separate SS)
- ANOVA  $SS_{IV} >$  ANCOVA  $SS_{IV}$
- smaller  $SS_{error}$
- Can’t anticipate whether F from ANCOVA or from ANOVA will be larger – ANCOVA has the smaller numerator & also the smaller denominator
- F-tests for Tx and for Grade will be “better” – but still only “control” for this one covariate (there are likely others)



Since we’ve recently learned about plotting ...

How do the plots of ANOVA & ANCOVA differ and what do we learn from each?

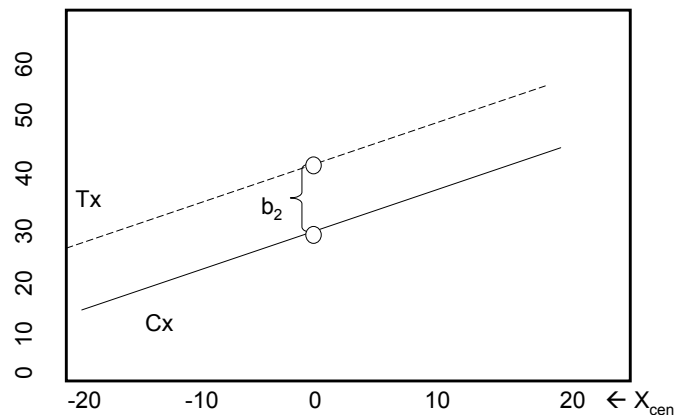
Here’s a plot of a 2-group ANOVA model



Here's a plot of the corresponding 2-group ANCOVA model ...

... with no confounding by "X" → for mean  $X_{cen}$   $Cx = Tx$

So, when we use ANCOVA to hold  $X_{cen}$  constant at 0 we're not changing anything, because there is no X confounding to control, "correct for" or "hold constant."



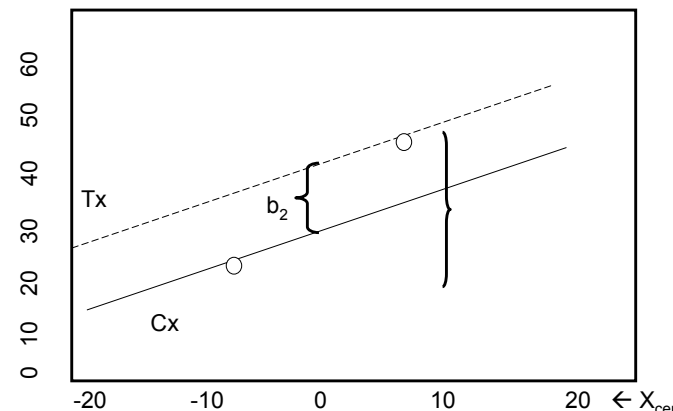
$Z = Tx1 \text{ vs. } Cx$   
 $Cx = 0 \quad Tx = 1$   
 $X_{cen} = X - X_{mean}$   
**b** is a good estimate of the treatment effect

Here's a plot of the corresponding 2-group ANCOVA model ...

... with confounding by "X" → for mean  $X_{cen}$   $Cx < Tx$

When we compare the mean Y of Cx & Tx using ANOVA, we ignore the group difference/confounding of X – and get a biased estimate of the treatment effect

When we use ANCOVA to compare the groups -- holding  $X_{cen}$  constant at 0 -- we're controlling for or correcting the confounding and get a better estimate of the treatment effect. Here the corrected treatment effect is smaller than the uncorrected treatment effect.



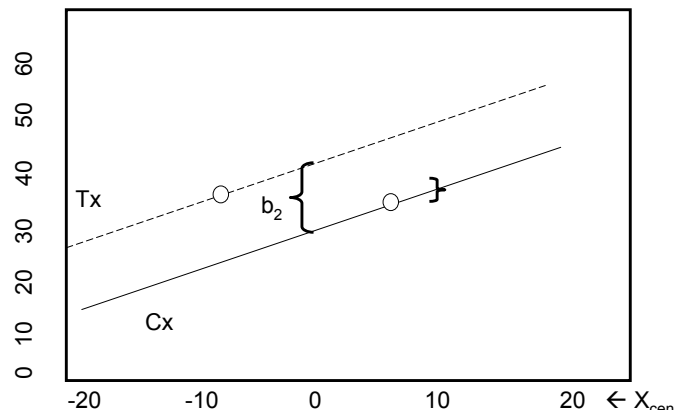
$Z = Tx1 \text{ vs. } Cx$   
 $Cx = 0 \quad Tx = 1$   
 $X_{cen} = X - X_{mean}$   
**b** is our estimate of the treatment effect

Here's a plot of the corresponding 2-group ANCOVA model ...

... with confounding by "X" → for mean  $X_{cen}$   $Cx > Tx$

When we compare the mean Y of Cx & Tx using ANOVA, we ignore the group difference/confounding of X – and get a biased estimate of the treatment effect

When we use ANCOVA to compare the groups -- holding  $X_{cen}$  constant at 0 -- we're controlling for or correcting the confounding and get a better estimate of the treatment effect. Here the corrected treatment effect is larger than the uncorrected treatment effect.



$Z = Tx1 \text{ vs. } Cx$   
 $Cx = 0 \quad Tx = 1$   
 $X_{cen} = X - X_{mean}$   
**b** is our estimate of the treatment effect

The “regression slope homogeneity assumption” in ANCOVA

You might have noticed that the 2 lines representing the Y-X relationship for each group in the ANCOVA plots were always parallel – had the same regression slope.

- these are main effects ANCOVA models that are based on...
- the homogeneity of regression slope assumption
- the reason it is called an “assumption” is that when constructing the main effects model we don’t check whether or not there is an interaction, we just build the model without an “interaction term” – so the lines are parallel (same slope)

There are two consequences of this assumption:

- Y & X have the same relationship/slope for both groups
- the group difference on Y is the same for every value of X
- REMEMBER → neither of these are “discoveries” they are both assumptions



But what if, you may ask, there are more than one “confound” you want to control for?

Just “expand” the model ...

$$SS_{\text{total}} = \frac{SS_{\text{IV}} + SS_{\text{cov1}} + \dots + SS_{\text{covk}}}{SS_{\text{Error}}}$$

- You get an F-test for each variable in model ...
- You get a b,  $\beta$  & t-test for each variable in model ...
- Each of which is a test of the unique contribution of that variable to the model after controlling for each of the other variables

**Remember: ANCOVA is just a multiple regression with one predictor called “the IV”**