Factorial ANCOVA with Regression Heterogeneity

The focus of the study was gender differences in depression, and whether gender differences are moderated by marital status. So, the starting point of the analysis was that 2x2 factorial design.

Considering Covariates

Neither Gender nor Marital Status were (could be) randomly assigned and manipulated and there is no hint of ongoing equivalence. So, the causal attribution of the mean differences among these groups is not on firm grounds! In a situation like this, there are several reasons we might want to include additional variables in the model.

Good candidates for covariates have two properties. 1) they are related to the DV – increasing the model fit and our understanding of how what variables contribute to model, 2) they are related to one or more of the IVs/interactions – controlling for the covariates may or may not change our impression of how the IVs (and their interaction) relate to the DVs, either of which increases our understanding of how what variables contribute to the model.

One variable that has both of these properties when considering depression differences between males and females is → stress. Stress has a well-known relationship with depression, gender and marital status.

Whenever including covariates, we have to decide whether or not to include the interactions between the IVs & the covariate. Excluding the interaction – making the homogeneity of regression slope assumption – assumes that each IV main effect and their interaction effect has exactly the same pattern for all values of the covariate. For this model, it would mean assuming that males and females have the same mean depression difference for all values of stress, that married and unmarried folks have the same mean depression difference for all values of stress, and that the pattern for the gender*marital status interaction is the same for all values of stress. Including the covariate interactions does increase the complexity of the model (adding two 2-ways and a 3-way to the mix), but if the interactions don't contribute (the homogeneity of regression slope assumption makes sense), we can always simplify the model.

Factorial ANOVA Model & Results

```
UNIANOVA depression BY marital gender                       ← dv BY ivs → sets order of Descriptive Stats table
  /METHOD = SSTYPE(3)                                       ← gets F-tests correcting each term for all others
  /EMMEANS = TABLES(marital*gender) COMPARE (gender)        ← gets simple effect tests of gender for each marital
  /EMMEAN S= TABLES(gender)          COMPARE (gender)       ← gets corrected marginal mean tests for gender
  /EMMEANS = TABLES(marital)         COMPARE (marital)      ← gets corrected marginal mean tests for marital
  /EMMEANS = TABLES(gender*marital) COMPARE (marital)       ← gets simple effect tests of marital for each gender
  /PRINT = DESCRIPTIVE                                      ← gets cell means & uncorrected/raw marginal means
  /DESIGN = gender   marital  gender*marital.               ← specifies the factorial model with interaction
```

Descriptive Statistics

Dependent Variable: depression

marital	gender	Mean	Std. Deviation	N
single	male	5.0492	5.58459	122
	female	9.0417	8.10995	120
	Total	7.0289	7.22053	242
married	male	6.1064	6.69923	47
	female	5.8919	4.57081	74
	Total	5.9752	5.47032	121
Total	male	5.3432	5.91411	169
	female	7.8402	7.12801	194
	Total	6.6777	6.69899	363

Tests of Between-Subjects Effects

Dependent Variable: depression

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1055.189[a]	3	351.730	8.313	.000
Intercept	13262.668	1	13262.668	313.447	.000
gender	278.122	1	278.122	6.573	.011
marital	85.324	1	85.324	2.017	.156
marital * gender	344.868	1	344.868	8.151	.005
Error	15190.100	359	42.312		
Total	32432.000	363			
Corrected Total	16245.289	362			

a. R Squared = .065 (Adjusted R Squared = .057)

There is a gender effect, no marital status effect and a significant interaction.

Remember that the main effects F-tests are not testing for differents beetween the marginal means shown in the Descriptive Statistics! Because the design is nonorthogonal (has unequal-n), the F-tests look at each of the effects correcting for the other effects in the model.

Describing the Factorial ANOVA Results

ANOVA Interaction of Gender * Marital Status

Estimates

Dependent Variable: depression

marital	gender	Mean	Std. Error
single	male	5.049	.589
	female	9.042	.594
married	male	6.106	.949
	female	5.892	.756

The raw and estimated cell means of the highest-order effect will be the same (there is nothing to correct for),

Univariate Tests

Dependent Variable: depression

marital		Sum of Squares	df	Mean Square	F	Sig.
single	Contrast	964.301	1	964.301	22.790	.000
	Error	15190.100	359	42.312		
married	Contrast	1.322	1	1.322	.031	.860
	Error	15190.100	359	42.312		

Each F tests the simple effects of gender within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

The simple effects F-tests and pairwise comparisons will be redundant, because there are only 2 gender conditions.

Pairwise Comparisons

Dependent Variable: depression

marital	(I) gender	(J) gender	Mean Difference (I-J)	Std. Error	Sig.[b]
single	male	female	-3.992*	.836	.000
	female	male	3.992*	.836	.000
married	male	female	.214	1.213	.860
	female	male	-.214	1.213	.860

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

The pattern of the interaction, looking at the simple effect of Gender for each Marital Status group is…

	Male		Female
Single		<	
Married		=	

ANOVA Main Effect of Gender

Estimates

Dependent Variable: depression

gender	Mean	Std. Error
male	5.578	.558
female	7.467	.481

Univariate Tests

Dependent Variable: depression

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	278.122	1	278.122	6.573	.011
Error	15190.100	359	42.312		

The F tests the effect of gender. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Pairwise Comparisons

Dependent Variable: depression

(I) gender	(J) gender	Mean Difference (I-J)	Std. Error	Sig.[b]
male	female	-1.889*	.737	.011
female	male	1.889*	.737	.011

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

The corrected Gender effect is a bit smaller than raw marginal mean difference. The F-test is the same value given in the ANOVA table above. The follow-ups are redundant with the F-test, because the effect has 2 conditions.

The main effect, females have higher average depression scores than males, is potentially misleading, because, as shown in the simple effects analysis above, while females have higher depression scores among singles, there is no difference among married individuals.

ANOVA Main Effect of Marital Status

Estimates

Dependent Variable: depression

marital	Mean	Std. Error
single	7.045	.418
married	5.999	.607

Overall, there is no Marital Status effect.

When there is a significant interaction, a null main effect is almost always misleading. The interaction tells us that the simple effects are different, and the simple effects can't (usually) be different from each other and both nulls.

The rare exception is when the simple effects are small and in opposite directions. It can happen that neither simple effect is strong enough to be significant, but, because they have different directions, they are different from each other!

Pairwise Comparisons

Dependent Variable: depression

(I) marital	(J) marital	Mean Difference (I-J)	Std. Error	Sig.[a]
single	married	1.046	.737	.156
married	single	-1.046	.737	.156

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: depression

	Sum of Squares	df	Mean Square	F	Sig.
Contrast	85.324	1	85.324	2.017	.156
Error	15190.100	359	42.312		

The F tests the effect of marital. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

The F-test is the same value given in the ANOVA table above. The follow-ups are redundant with the F-test, because the effect has 2 conditions.

Analysis of the simple effect of Marital Status for each Gender

Estimates

Dependent Variable: depression

gender	marital	Mean	Std. Error
male	single	5.049	.589
	married	6.106	.949
female	single	9.042	.594
	married	5.892	.756

Univariate Tests

Dependent Variable: depression

gender		Sum of Squares	df	Mean Square	F	Sig.
male	Contrast	37.922	1	37.922	.896	.344
	Error	15190.100	359	42.312		
female	Contrast	454.120	1	454.120	10.733	.001
	Error	15190.100	359	42.312		

Each F tests the simple effects of marital within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

Pairwise Comparisons

Dependent Variable: depression

gender	(I) marital	(J) marital	Mean Difference (I-J)	Std. Error	Sig.[b]
male	single	married	-1.057	1.117	.344
	married	single	1.057	1.117	.344
female	single	married	3.150*	.961	.001
	married	single	-3.150*	.961	.001

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Looking at the simple effect of Marital Status for each Gender shows…

	Single		Married
Male		=	
Female		>	

The main null main effect for Marital Status was descriptive only for males, single females have higher average depression scores than single males.

So, neither main effect is descriptive. The "full story" about the data pattern is captured in the simple effects. Especially when doing complex analyses like ANCOVAs, you usually don't want to describe misleading main effects and then tell how they are misleading! However, as always, a descriptive main effect can be an important "simplifying" result!

Factorial ANCOVA Model & Results

```
UNIANOVA depression BY marital  gender WITH stress          ← dvs  BY  ivs  WITH covariate(s)

  /METHOD = SSTYPE(3)                                       ← gets F-tests correcting each term for others

  /PRINT=DESCRIPTIVE

  /DESIGN=gender    marital    gender*marital               ← specifies the factorial model – IVs & interaction
          stress                                            ← covariate
          stress*gender  stress*marital   stress*gender*marital.   ← covariate interactions
```

Tests of Between-Subjects Effects

Dependent Variable: depression

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4108.543[a]	7	586.935	17.168	.000
Intercept	1807.389	1	1807.389	52.866	.000
gender	60.084	1	60.084	1.757	.186
marital	114.273	1	114.273	3.342	.068
marital * gender	24.140	1	24.140	.706	.401
stress	2521.175	1	2521.175	73.744	.000
gender * stress	.473	1	.473	.014	.906
marital * stress	207.654	1	207.654	6.074	.014
marital * gender * stress	469.408	1	469.408	13.730	.000
Error	12136.746	355	34.188		
Total	32432.000	363			
Corrected Total	16245.289	362			

a. R Squared = .253 (Adjusted R Squared = .238)

In the factorial ANOVA there was a significant Gender effect and a significant Gender * Marital Status interaction, neither of which are significant in this factorial ANCOVA with stress as the covariate.

However, both variables and Stress are involved in a 3-way interaction.

The significant Stress effect, along with the changes noted above, suggest the utility of this ANCOVA.

The significant Marital*Stress & significant 3-way both suggest the homogeneity of regression slope assumption isn't tenable.

Testing the Heterogeneity of Regression Slope Assumption (do we need the covariate interactions?)

It is possible to directly test of the regression slope homogeneity test. This involves getting the factorial ANCOVA model without the Stress*??? interactions and comparing the fit of the "full model" model with those interactions included and the "reduced model" without those interactions, using the $R^2\Delta$ F-test.

Code to get the ANCOVA without the stress*??? Interactions

```
UNIANOVA depression BY gender marital WITH stress
 /METHOD = SSTYPE(3)
 /DESIGN = gender   marital    gender*marital
           stress.
```

$$F = \frac{(R^2_L - R^2_S) / (k_L - k_S)}{(1 - R^2_L) / (N - k_L - 1)}$$

Tests of Between-Subjects Effects

Dependent Variable: depression

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3440.800[a]	4	860.200	24.050	.000
Intercept	2037.009	1	2037.009	56.953	.000
gender	232.844	1	232.844	6.510	.011
marital	1.355	1	1.355	.038	.846
gender * marital	270.217	1	270.217	7.555	.006
stress	2385.611	1	2385.611	66.699	.000
Error	12804.489	358	35.767		
Total	32432.000	363			
Corrected Total	16245.289	362			

a. R Squared = .212 (Adjusted R Squared = .203)

$F(3, 355) = 6.495$, $p = .0003$

Rejecting H0: $R^2_L = R^2_S$ we would conclude that the interactions contribute to the model and that the homogeneity of regression slope is untenable.

Using the Computator,,,

R-square Change F-test

Rsq Larger Model =>	**0.263**
# predictors =>	**7**
Rsq Smaller Model =>	**0.212**
# predictors =>	**4**
N =>	**363**
F =	**8.18860244**
df change =	**3**
df errror =	**355**
p =	**2.7681E-05**

$$F = \frac{(R^2_L - R^2_S) / (k_L - k_S)}{(1 - R^2_L) / (N - k_L - 1)}$$

$F(3, 355) = 8.1886$, $p = .000028$

Rejecting H0: $R^2_L = R^2_S$ we would conclude that the interactions contribute to the model and that the homogeneity of regression slope is untenable.

Describing the Factorial ANCOVA

Having decided that the Stress interactions should be part of the model, and finding a significant 3-way interaction in the Factorial ANCOVA, how should we describe the model?

The intent of the study was to examine Gender differences in depression, how those differences were moderated by Marital Status, and if that interaction was different for different values of Stress.

So, the analysis that gives the most direct examination of the research question, as it is phrased, would be to examine the Gender * Marital Status interaction effects at different levels of Stress. Based on an examination of the distribution of Stress Scores and an understanding of the clinical relevance of Stress score values, we chose to examine these effects at Stress values of 2 ("low"), the mean (8.62 "moderate") and 12 ("substantial").

The SPSS syntax below shows how to get the complete analyses to make these examinations. The EMMEANS statements provide the simple effects of Gender for each Marital Status (to describe the pattern of the Gender * Marital Status interaction & check the Gender main effect for a descriptive/misleading pattern), the main effect of Gender, the main effect of Marital Status, and the simple effects of Marital Status for each gender (to check the Marital Status main effect for a descriptive/misleading pattern).

Getting the main effects EMMEANS and both sets of interactions EMMEANs analyses is probably superfluous… The main effects were misleading within the ANCOVA, so it is unlikely they will be descriptive within this more complex 3-way.

```
UNIANOVA depression BY marital  gender WITH stress                  ← dvs  BY  ivs  WITH covariate(s)

 /METHOD = SSTYPE(3)                                                ← gets F-tests correcting each term for others

 /EMMEANS=TABLES(marital*gender) WITH (stress=2)    COMPARE (gender)   ← gets corrected simple 2-way interactions and
 /EMMEANS=TABLES(gender)         WITH (stress=2)    COMPARE (gender)     corrected simple effects among IVs with
 /EMMEANS=TABLES(marital)        WITH (stress=2)    COMPARE (marital)    Stress held constant at "2"
 /EMMEANS=TABLES(gender*marital) WITH (stress=2)    COMPARE (marital)

 /EMMEANS=TABLES(marital*gender) WITH (stress=mean) COMPARE (gender)   ← gets corrected simple 2-way interactions and
 /EMMEANS=TABLES(gender)         WITH (stress=mean) COMPARE (gender)     corrected simple effects among IVs with
 /EMMEANS=TABLES(marital)        WITH (stress=mean) COMPARE (marital)    Stress held constant at its mean
 /EMMEANS=TABLES(gender*marital) WITH (stress=mean) COMPARE (marital)

 /EMMEANS=TABLES(marital*gender) WITH (stress=12)   COMPARE (gender)   ← gets corrected simple 2-way interactions and
 /EMMEANS=TABLES(gender)         WITH (stress=12)   COMPARE (gender)     corrected simple effects among IVs with
 /EMMEANS=TABLES(marital)        WITH (stress=12)   COMPARE (marital)    Stress held constant at "12"
 /EMMEANS=TABLES(gender*marital) WITH (stress=12)   COMPARE (marital)

                                                                    ← gets cell means & uncorrected/raw marginal
 /PRINT=DESCRIPTIVE PARAMETER                                         means

                                                                    ← specifies the factorial model – IVs & interaction
 /DESIGN=gender   marital   gender*marital                          ← covariate
         stress                                                     ← covariate interactions
         stress*gender  stress*marital   stress*gender*marital.
```

Gender *Marital Status Interaction for Stress = 2, Mean(8.62) & 12

For brevity, I'm going to leave out the Univariate Tests – because these are all df=1 comparions, they are redundant with the Pairwise Comparisons

Stress = 2

Estimates

Dependent Variable: depression

marital	gender	Mean	Std. Error
single	male	4.344[a]	.751
	female	5.487[a]	.776
married	male	2.870[a]	1.014
	female	4.318[a]	.810

a. Covariates appearing in the model are evaluated at the following values: stress = 2.

Pairwise Comparisons

Dependent Variable: depression

marital	(I) gender	(J) gender	Mean Difference (I-J)	Std. Error	Sig.[a]
single	male	female	-1.143	1.079	.290
	female	male	1.143	1.079	.290
married	male	female	-1.448	1.298	.265
	female	male	1.448	1.298	.265

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Stress = mean (8.62)

Estimates

Dependent Variable: depression

marital	gender	Mean	Std. Error
single	male	5.017[a]	.530
	female	8.399[a]	.543
married	male	7.157[a]	.871
	female	6.503[a]	.701

a. Covariates appearing in the model are evaluated at the following values: stress = 8.62.

Pairwise Comparisons

Dependent Variable: depression

marital	(I) gender	(J) gender	Mean Difference (I-J)	Std. Error	Sig.[b]
single	male	female	-3.382*	.759	.000
	female	male	3.382*	.759	.000
married	male	female	.653	1.118	.559
	female	male	-.653	1.118	.559

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Stress = 12

Estimates

Dependent Variable: depression

marital	gender	Mean	Std. Error
single	male	5.361[a]	.579
	female	9.885[a]	.550
married	male	9.343[a]	1.014
	female	7.618[a]	.834

a. Covariates appearing in the model are evaluated at the following values: stress = 12.

Pairwise Comparisons

Dependent Variable: depression

marital	(I) gender	(J) gender	Mean Difference (I-J)	Std. Error	Sig.[b]
single	male	female	-4.524*	.799	.000
	female	male	4.524*	.799	.000
married	male	female	1.725	1.313	.190
	female	male	-1.725	1.313	.190

Based on estimated marginal means

*. The mean difference is significant at the .050 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Stress = 2

	Male		Female
Single		=	
Married		=	

Stress = mean (8.62)

	Male		Female
Single		<	
Married		=	

Stress = 12

	Male		Female
Single		<	
Married		=	

This reveals the 3-way interaction pattern. There were no Gender effects for either single or married people reporting low amounts of stress, however, for those reporting moderate and substantial amounts of stress, single females reported more depressive symptoms than single males, whereas married females and married males reported equivalent levels of depressive symptoms.