Review of Factorial Designs

- 5 terms necessary to understand factorial designs
- 5 patterns of factorial results for a 2x2 factorial designs
- Descriptive & misleading main effects
- Research Hypotheses for Factorial Designs
- Causal Interpretation of Factorial Design Effects
- Statistical Analysis of 2x2 Designs
- "Sizes" and "Kinds" of Factorial Designs
- Statistical Analysis of kxk Designs

Introduction to factorial designs

Factorial designs have 2 (or more) Independent Variables

An Example…
Forty clients at a local clinic volunteered to participate in a research project designed to examine the individual and combined effects of the client's Initial Diagnosis (either general anxiety or social anxiety) and the Type of Therapy they receive (either group or individual). Twenty of the participants had been diagnosed with general anxiety and 20 had been diagnosed as having social anxiety. One-half of the clients with each diagnosis were assigned to receive group therapy and one-half received individual therapy. All clients underwent 6 months of the prescribed treatment, and then completed a battery of assessments which were combined into a DV score of "wellness from anxiety", for which larger scores indicate better outcome.
Here is a depiction of this design.

Type of Therapy

Initial Diagnosis	Group	Individual
General Anxiety	clients diagnosed w/ general anxiety who received group therapy	clients diagnosed w/ general anxiety who received individual therapy
Social Anxiety	clients diagnosed w/ social anxiety who received group therapy	clients diagnosed w/ social anxiety who received individual therapy

Participants in each "cell" of this design have a unique combination of IV conditions.

Effects examined by a factorial design

There are always THREE effects (IVs) examined ..

1 -- the interaction of the two IVs -- how they jointly relate to DV

2 -- the main effect of the one IV -- how it relates to the DV independently of the interaction and the other main effect

3 -- the main effect of the other IV -- how it relates to the DV independently of the interaction and the other main effect

For the example…

1 -- the "interaction" of Initial Diagnosis & Type of Therapy

2 -- the "main effect" of Initial Diagnosis

3 -- the "main effect" of Type of Therapy

The difficult part of learning about factorial designs is the large set of new terms that must be acquired. Here's a summary;

cell means -- the mean DV score of all the folks with a particular combination of IV treatments

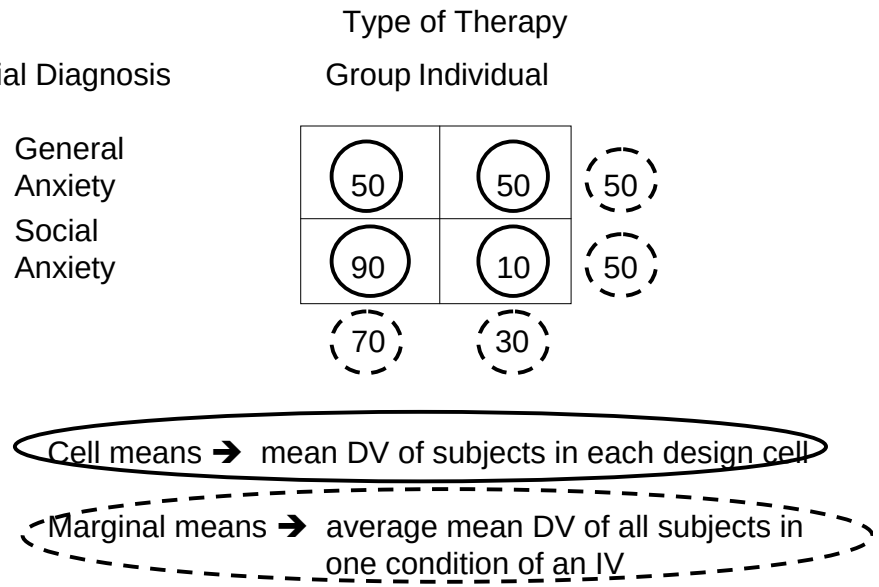
marginal means -- the mean DV score of all the folks in a particular condition of the specified IV (aggregated across conditions of the other IV)

Main effects involve the comparison of marginal means.

Simple effects involve the comparison of cell means.

Interactions involve the comparison of simple effects.

Identifying Cell Means and Marginal Means



Identifying Main Effects -- difference between the marginal means of that IV (ignoring the other IV)

Initial Diagnosis	Type of Therapy: Group	Individual	
General Anxiety	50	50	50
Social Anxiety	90	10	50
	70	30	

Main effect of Initial Diagnosis

Main effect of Type of Therapy

Identifying Simple Effects -- cell means differences between conditions of one IV for a specific level of the other IV

Initial Diagnosis	Type of Therapy: Group	Individual	
General Anxiety	50	50	1
Social Anxiety	90	10	2
	a	b	

Simple effects of Initial Diagnosis for each Type of Therapy

a Simple effect of Initial Diagnosis for group therapy

b Simple effect of Initial Diagnosis for individual therapy

Identifying Simple Effects -- cell means differences between conditions of one IV for a specific level of the other IV

Initial Diagnosis	Type of Therapy: Group	Individual	
General Anxiety	50	50	1
Social Anxiety	90	10	2
	a	b	

Simple effects of Type of Therapy for each Initial Diagnosis

1 Simple effect of Type of Therapy for general anxiety patients

2 Simple effect of Type of Therapy for social anxiety patients

Identifying Interactions

Patterns of data that include interactions can be identified and described using the "it depends" approach. This approach is referred to different ways, here are three commonly used expressions"

- the simple effect of one IV is different at different levels of the other IV
- "different differences"
- "different simple effects"

Here are the three basic patterns of interactions

#1

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer	
Easy	90	=	90	one simple effect "null"
Hard	40	<	70	one simple effect

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Easy tasks are performed equally well using paper and using the computer (90 vs. 90), however, hard tasks are performed better using the computer than using paper (70 vs. 40).

#2

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer	
Easy	80	<	90	simple effects in the same direction,
Hard	40	<	70	but of different sizes

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Performance was better using the computer than using paper, however this effect was larger for hard tasks (70 vs. 40) than for easy tasks (90 vs. 80).

#3

Task Presentation

Task Difficulty	Paper		Computer	
Easy	90	>	70	simple effects are
Hard	40	<	60	opposite directions

There is an interaction of Task Presentation and Task Difficulty as they relate to performance. Easy tasks are performed better using paper than using computer (90 vs. 70), whereas hard tasks are performed better using the computer than using paper (60 vs. 40).

#2 Task Presentation

Task Difficulty	Paper		Computer	
Easy	50	=	50	both simple effects
Hard	70	=	70	are nulls

There is no interaction of Task Presentation and Task Difficulty as they relate to performance. Performance is the same for computer and paper presentations (for both Easy and Hard tasks).

Notice the main effects will be descriptive.

Here are the two basic patterns of NON-interactions

#1 Task Presentation

Task Difficulty	Paper		Computer	
Easy	30	<	50	both simple effects are in the same direction and are the same size
Hard	50	<	70	

There is no interaction of Task Presentation and Task Difficulty as they relate to performance. Performance is better for computer than for paper presentations (for both Easy and Hard tasks).

Notice the main effects will be descriptive.

Identifying Main Effects

Patterns of data that include main effects can be identified by looking at the differences among the marginal means for a specific IV (the main effect of each IV must be examined and described separately !!!)

• When there is an interaction, each main effect (null or significant) must be carefully examined to determine if that main effect is

- •"descriptive" (unconditional, that is, descriptive for all levels of the other IV) or is
- • "potentially misleading (conditional, that is, descriptive for only some or none of the levels of the other IV)

• You must determine whether the pattern of each main effect (direction of any difference between the marginal means) is equivalent to each of the corresponding simple effects of that variable at the various levels of the other I

Identifying Main Effects

It is not uncommon to hear the advice to "ignore main effects if there is an interaction."

My best guess is that this is based on the correct idea that the pattern of some main effects can render the pattern of one or both main effects to be potentially or completely misleading.

However, it is also possible that there can be an interaction and that one or both of the main effects can be descriptive.

Discerning whether main effects are descriptive or misleading is a critical step in the examination of data from a factorial design! You must ensure that the reader has a thorough understanding of the pattern of your data!

You must give a complete accounting of each of the three effects involved in the factorial design, the interaction and each of the main effects!

Interpreting main effects … When there is an interaction, the pattern of the interaction may influence the interpretability (generality) of the description of the marginal means.

	Task Presentation: Paper		Task Presentation: Computer
Task Difficulty: Easy	80	<	90
Hard	40	<	70
	60	<	80

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation.

Notice: that the pattern of the main effect is consistent with both the simple effect of Task Presentation for easy tasks and the simple effect of Task Presentation for hard tasks.

Another example …

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	90	=	90
Hard	40	<	70
	65	<	80

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation. However, this pattern is descriptive for hard tasks, but not for easy tasks, for which there was no simple effect of Task Presentation.

Yet another example …

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	80	>	60
Hard	20	<	70
	50	<	65

There is a main effect for Task Presentation, overall performance was better using computer presentation than using paper presentation. However, this pattern is descriptive for hard tasks, but not for easy tasks, for which performance was better using paper presentations than using computer presentation.

"Null" main effects can also be misleading….

Task Difficulty	Task Presentation: Paper		Task Presentation: Computer
Easy	90	>	70
Hard	40	<	60
	65	=	65

There is no main effect for Task Presentation, overall performance was equivalent using computer presentation and using paper presentation. However, this pattern is descriptive for neither hard tasks, for which computer presentations worked better than paper, nor for easy tasks, for which performance was better using paper presentations than using computer presentation.

We can rearrange the **5** basic patterns of results from a 2x2 Factorial to help us think about interactions and descriptive/misleading main effects

Interaction -- simple effects of different size and/or direction	1. = vs. <	one null simple effect and one simple effect	Misleading main effects	
	2. < vs. >	simple effects in opposite directions		
	3. < vs. <	simple effects in same direction, but different sizes	Descriptive main effects	
No Interaction -- simple effects are null or same size	4. < vs. <	simple effects of the same size in the same direction		
	5. = vs. =	both null simple effects		

RH: for Factorial Designs

Research hypotheses for factorial designs may include

- RH: for main effects
  - involve the effects of one IV, while ignoring the other IV
  - tested by comparing the appropriate marginal means
- RH: for interactions
  - usually expressed as “different differences” -- differences between a set of simple effects
  - tested by comparing the results of the appropriate set of simple effects
  - That’s the hard part -- determining which set of simple effects gives the most direct test of the interaction RH:

Sometimes the Interaction RH: is explicitly stated

- when that happens, one set of SEs will provide a direct test of the RH: (the other won’t)

Here’s an example:

Easy tasks will be performed equally well using paper or computer presentation, however, hard tasks will be performed better using computer presentation than paper.

Task Diff.	Presentation: Comp	Presentation: Paper
Easy	=	
Hard	>	

This is most directly tested by inspecting the simple effect of paper vs. computer presentation for easy tasks, and comparing it to the simple effect of paper vs. computer for hard tasks.

Sometimes the set of SEs to examine use is “inferred” ...

Often one of the IVs in the study was used in previous research, and the other is “new”.

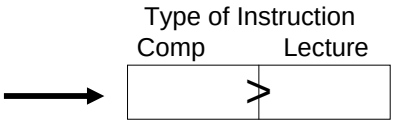
- In this case, we will usually examine the simple effect of the “old” variable, at each level of the “new” variable
- this approach gives us a clear picture of the replication and generalization of the “old” IV’s effect.

e.g., Previously I demonstrated that computer presentations lead to better learning of statistical designs than does using a conventional lecture. I would like to know if the same is true for teaching writing.

Let’s take this “apart” to determine which set of SEs to use to examine the pattern of the interaction...

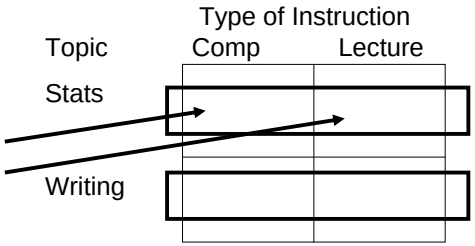
Previously I demonstrated that computer presentations lead to better learning of statistical designs than does using a conventional lecture. I would like to know if the same is true for teaching writing.

Here’s the design and result of the earlier study about learning stats.



Here’s the design of the study being planned.

What cells are a replication of the earlier study ?



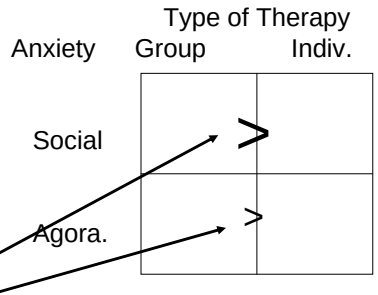
So, which set of SEs will allow us to check if we got the replication, and then go on to see of we get the same results with the new topic ?

Yep, SE of Type of Instruction, for each Topic ...

Sometimes the RH: about the interaction and one of the main effects are “combined”

- this is particularly likely when the expected interaction pattern is of the > vs. > type

Here’s an example…



Group therapy tends to work better than individual therapy, although this effect is larger for patients with social anxiety than with agoraphobia.

Int. RH: >

Main effect RH: >

So, we would examine the interaction by looking at the SEs of Type of Therapy for each type of Anxiety.

Describing Factorial Effects

Important things to remember:

- main effects and the interaction are 3 separate effects each must be separately interpreted -- three parts to the "story"
  - most common error -- "interaction is different main effects"
  - best thing -- be sure to carefully separate the three parts of the story and tell each completely
- Be careful of "causal words" when interpreting main effects and interactions (only use when really appropriate).
  - caused, effected influenced, produced, changed ….
- Consider more than the "significance"
  - consider effect sizes, confidence intervals, etc. when describing the results

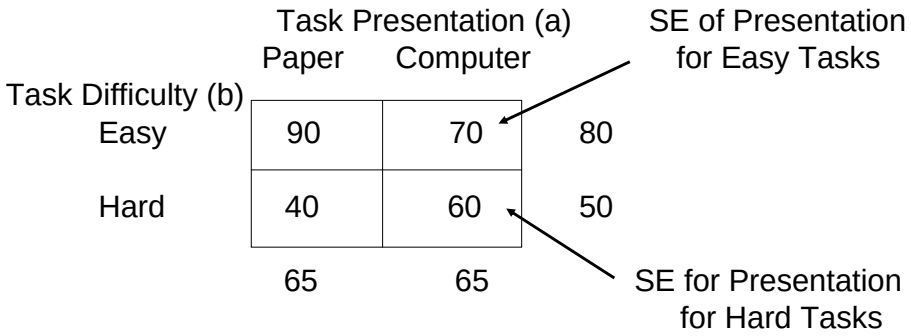
About the causal interpretation of effects of a factorial design…

In order to causally interpret an interaction, you must be able to casually interpret BOTH main effects …or…

In order to causally interpret an interaction you must be able to causally interpret the difference between casually interpretable simple effects.

Study of Age and Gender	no casually interpretable effects (main effects nor interaction)
Study of Age and Type of Toy (RA + Manip)	only casually interpretable effect would be the main effect of Type of Toy (not the main effect of Age, nor the interaction).
Study Type of Toy (RA + Manip) and Playing Situation (RA + manip)	all effects are causally interpreted (both main effects and the interaction).

Statistical Analysis of a 2x2 Design



Presentation Main Effect	Difficulty Main Effect	Interaction Effect
$F_{Presentation}$	$F_{Dificulty}$	$F_{Interaction}$
65 vs. 65	80 vs. 50	SE_{Easy} vs. SE_{Hard}

Statistical Analyses Necessary to Describe Main Effects of a 2x2 Design

In a 2x2 Design, the Main effects F-tests are sufficient to tell us about the relationship of each IV to the DV…

- since each main effect involves the comparison of two marginal means -- the corresponding significance test tells us what we need to know …
- whether or not those two marginal means are "significantly different"
- Don't forget to examine the means to see if a significant difference is in the hypothesized direction !!!

Statistical Analyses Necessary to Describe the Interaction of a 2x2 Design

However, the F-test of the interaction only tells us whether or not there is a "statistically significant" interaction…

- it does not tell use the pattern of that interaction
- to determine the pattern of the interaction we have to compare the simple effects
- to describe each simple effect, we must be able to compare the cell means

we need to know how much of a cell mean difference is "statistically significant"

Using LSD to Compare cell means to describe the simple effects of a 2x2 Factorial design

- LSD can be used to determine how large of a cell mean difference is required to treat it as a "statistically significant mean difference"
- Will need to know three values to use the computator
  - df_{error} -- look on the printout or use N – 4
  - MS_{error} – look on the printout
  - n = N / 4 -- use the decimal value – do not round to the nearest whole number!

Remember – only use the lsdmmd to compare cell means. Marginal means are compared using the man effect F-tests.

What statistic is used for which factorial effects????

Age	Male	Female	
5	30	30	30
10	20	30	25
	25	30	

There will be 4 statistics

1. F_{Age}
2. F_{Gender}
3. F_{Int}
4. LSD_{mmd}

This design as 7 "effects"

1. Main effect of age
2. Main effect of gender
3. Interaction of age & gender
4. SE of age for males
5. SE of age for females
6. SE of gender for 5 yr olds
7. SE of gender for 10 yr olds

Effect Sizes for 2x2 BG Factorial designs

For Main Effects & Interaction (each w/ df=1)

$$r = \sqrt{[F / (F + df_{error})]}$$

For Main Effects & Simple Effects

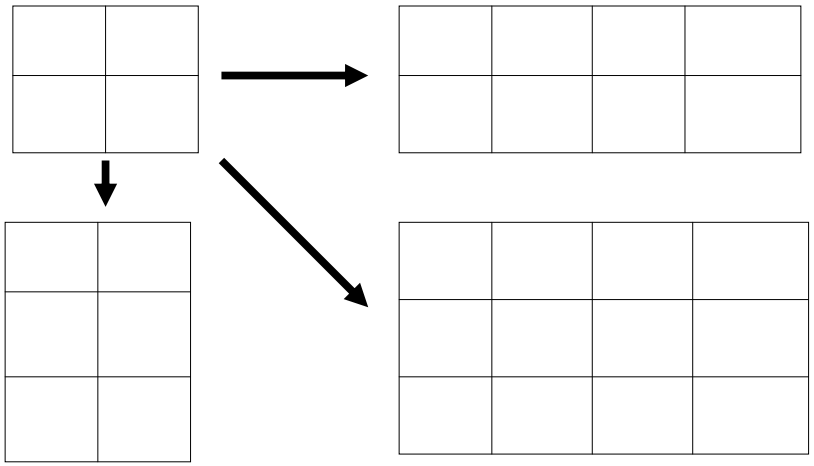
$$d = (M_1 - M_2) / \sqrt{Mserror}$$

$$r = \sqrt{\left[\frac{d^2}{d^2 + 4}\right]}$$ (This is an "approximation formula")

"Larger" Factorial Designs

The simplest factorial design is a 2x2, which can be expanded in two ways:

1) Adding conditions to one, the other, or both IVs



2) Add a 3rd IV (making a 3-way factorial design)

	Learning Psyc Methods		Learning Psyc Content	
	Ugrads	Grads	Ugrads	Grads
Computer Instruction				
Lecture Instruction				

Statistical Analyses Necessary to Describe Main Effects of a kxk Design

In a kxk Design, the Main effects F-tests are sufficient to tell us about the relationship of each IV to the DV only for 2-condition main effects…

- since a 2-condition main effect involves the comparison of two marginal means -- the corresponding F-test tells us what we need to know – the two marginal means are different
- however, for a k-condition main effect, the F-test only tells us that there is a pattern of significant differences among the marginal means, but doesn't tell us which means are significantly different
- for a k-condition main effect we need to use an LSDmmd to determine which pairs of marginal means are significantly different

Statistical Analyses Necessary to Describe the Interaction of a kxk Design

As with the 2x2 design, the interaction F-test for a kxk design only tells us whether or not there is a "statistically significant" interaction…

- it does not tell use the pattern of that interaction
- we need to use an LSDmmd to determine which pairs of cell means are significantly different

Be sure you are using the correct "n" when you compute LSDmmd

n = N / #conditions in that effect

Effect Sizes for kxk BG Factorial designs

For Main Effects & Interaction (each w/ df=1)

$$r = \sqrt{[F / (F + df_{error})]}$$

For specific comparisons among marginal or cell means

$$d = (M_1 - M_2) / \sqrt{Mserror}$$

$$r = \sqrt{\left[\frac{d^2}{d^2 + 4}\right]}$$ (This is an "approximation formula")

What statistic is used for which factorial effects????

There will be 5 statistics

1. F_{Gender}
2. F_{Age}
3. Age LSD_{mmd} (n=N/3)
4. F_{Int}
5. Int LSD_{mmd} (n = N/6)

15 "Effects" in this study

1. Main effect of gender
2. Main effect of age
3. 5 vs. 10 marginals
4. 5 vs. 15 marginals
5. 10 vs. 15 marginals
6. Interaction of age & gender
7. 5 vs. 10 yr old males
8. 5 vs. 15 yr old males
9. 10 vs. 15 yr old males
10. 5 vs. 10 yr old females
11. 5 vs. 15 yr old females
12. 10 vs. 15 yr old females
13. male vs. female 5 yr olds
14. male vs. female 10 yr olds
15. male vs. female 15 yr olds

Age	Male	Female	
	Gender		
5	30	30	30
10	20	30	25
15	25	30	27.5
	25	30	27.5

Back to → 100 males and 100 females completed the task, either under instructions to work quickly, work accurately, to work as quickly as possible without making unnecessary errors or no instructions.

Gender	Instruction: Quick	Accurate	Both	None
Male				
Female				

For the interaction p = .03

- will we need an LSD_{mmd} to compare cell means? why or why not? Yep! sig. Int & k = 8 ! 200 / 8 = 25
- what will "n" be?

For the main effect of instruction p = .02

- will we need an LSD_{mmd} to compare marginal means? why or why not? Yep! sig. ME & k = 4 ! 200 / 4 = 50
- what will "n" be?
- will we need an LSDmmd to compare cell means? why or why not? Yep! sig. Int ! 200 / 8 = 25
- what will "n" be?

For the main effect of gender p = .02

- will we need an LSD_{mmd} to compare marginal means? why or why not? Nope – k = 2 !
- what will "n" be?
- will we need an LSDmmd to compare cell means? why or why not? Yep! sig. Int ! 200 / 8 = 25
- what will "n" be?