# Example of Very Simple Path Analysis via Regression (with correlation matrix input)
Using data from Pedhazur (1997)

Certainly the most three important sets of decisions leading to a path analysis are:
1. Which causal variables to include in the model
2. How to order the causal chain of those variables
3. Which paths are not "important" to the model – the only part that is statistically tested

Here's the hypothesized causal ordering for how SES, IQ & Achievement Motivation cause GPA. Usually a path analysis involves the analysis and comparison of two models – a "full model" with all of the possible paths included and a "reduced model" which has some of the paths deleted, because they are hypothesized to not contribute to the model.
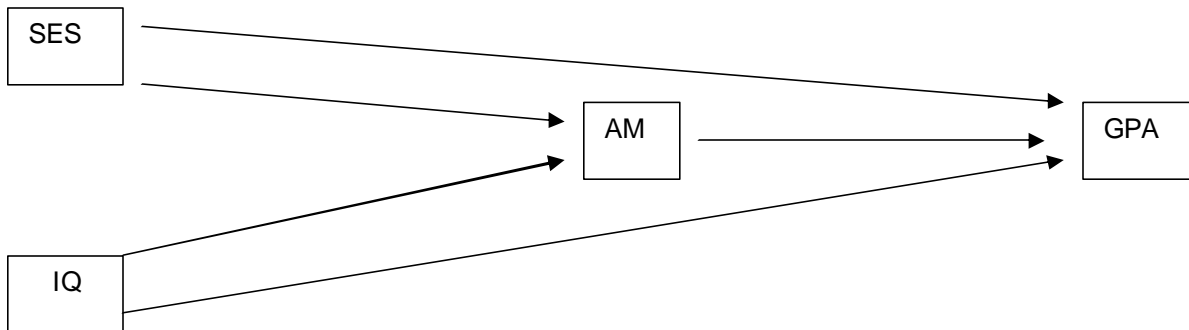
The path coefficients for the full model (with all the arrows) are derived from a series of "layered" multiple regression analyses. For each multiple regression, the criterion is the variable in the box (all boxes after the leftmost layer) and the predictors are all the variables that have arrows leading to that box.

For the full model above, we will need two "layers" of multiple regressions:
1. with AM as the criterion and SES & IQ as the predictors
2. with GPA as the criterion and SES, IQ & AM as the predictors

One of the nice things about SPSS is that it will allow you to start with a correlation matrix (you don't need the raw data – this is nice because more articles now include the correlation matrix of the variables, providing you an opportunity to reanalyze their variables using your model).

Entering a Correlation Matrix into SPSS

```
matrix data variables = rowtype_ ses iq am gpa   ← tells variable names
 / format = lower diagonal.
begin data.
mean 0.0 0.0 0.0 0.0                              ←  don't need means
stddev 2.10 15.00 3.25 1.25                       ← need std to get raw b weights
n 300 300 300 300                                 ← need N to get significance tests
corr 1.00
corr .30 1.00
corr .410 .160 1.00
corr .330 .570 .500 1.00
end data.
```

Getting the "First layer" multiple regression for the full model

```
regression matrix = in(*)/ dep am/ enter ses iq.
```

Model Summary

Model	R	R Square
1	.412a	.169

a. Predictors: (Constant), IQ, SES

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.000	.172		.000	1.000
	SES	.616	.086	.398	7.177	.000
	IQ	8.810E-03	.012	.041	.734	.464

a. Dependent Variable: AM

Getting the "Second Layer" multiple regression for the full model

```
regression matrix = in(*)/ dep gpa/ enter ses iq am.
```

Model Summary

Model	R	R Square
1	.705a	.496

a. Predictors: (Constant), AM, IQ, SES

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.000	.051		.000	1.000
	SES	5.470E-03	.028	.009	.196	.845
	IQ	4.172E-02	.004	.501	11.569	.000
	AM	.160	.017	.416	9.194	.000

a. Dependent Variable: GPA

Portraying the Full Path Model
- The path coefficients are the ß weights (and correlations) from the multiple regression analyses.
- The "e" values (roughly error variance) are computed as $\sqrt{(1-R^2)}$ (e.g., $e_{AM} = \sqrt{(1-.169)} = .9116$)
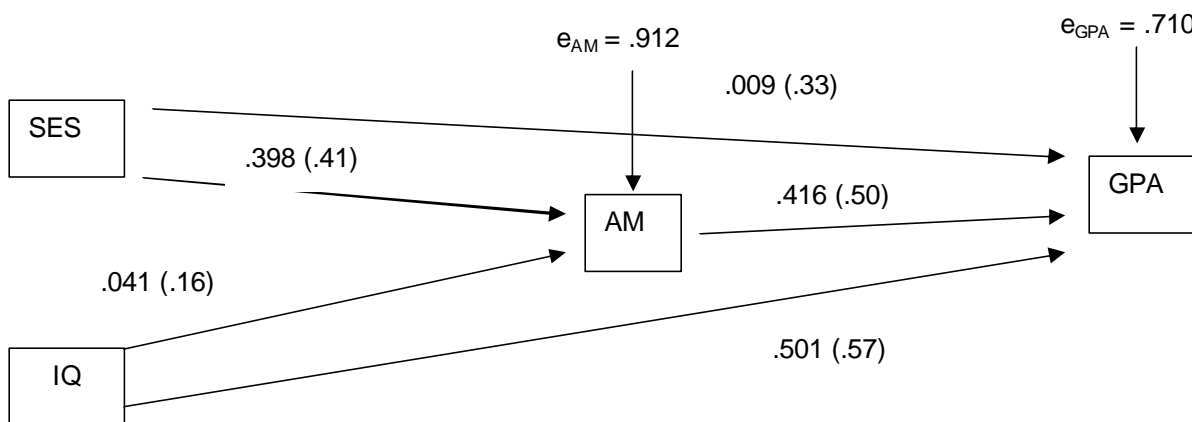
Examining this model we would note:
1. AM influences GPA
2. SES has no direct effect upon GPA, but has an indirect effect through AM
3. IQ has only a direct effect upon GPA

It is important to consider the e values as well. These tell us how much of the different variables are "not accounted for" – they can be an important wake up call !!!

For the final criterion, GPA, e reminds is that most of the variance in the criterion is not accounted for by the model – clearly there are important other predictors/causal variables important to a complete understanding of this variable!

For Academic Motivation, e reminds us that we have accounted for very little of the sources of variation for this variable that our model suggests has an important direct and indirect role in accounting for GPA! We need to know more about what predicts/causes Academic Motivation!

Path Analysis Hypotheses & Testing

While some path analyses are "descriptive" in that they compute and describe this soft of "full model" others test hypotheses about which model paths do not portray causal links among the variables. Below is such a reduced model.

Remember → you need to be very honest with yourself and with your audience about whether the reduced model is an *a priori* theory-driven model, or the result of inspecting a full path model (usually involves tossing the noncontributing paths, know as *theory trimming*). As in all the other analyses we've discussed, confirmed *a priori* hypotheses have a special place in our hearts!

This model posits that there is no direct effect of SES on GPA (that it's only effect is an indirect one channeled through AM) and IQ has only a direct effect (without any additional indirect effect channeled through AM).
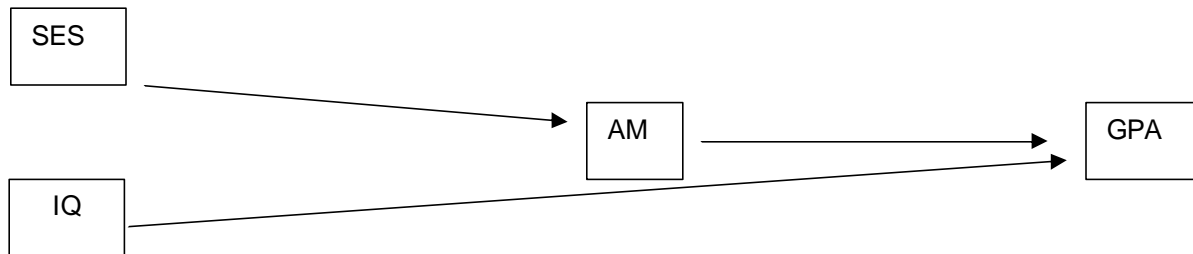
Once again, two multiple regression models would be used to obtain the path coefficients.

Getting the "First layer" multiple regression for the reduced model

The first layer doesn't require an actual multiple regression model, because there is only one predictor. So for AM as the criterion SES as the single predictor $R^2 = r^2 = .41^2 = .1681$, $\beta = r = .41$ and $e_{AM} = \sqrt{(1 - .1681)} = .9121$

Getting the "Second Layer" multiple regression for the reduced model

```
regression matrix = in(*)/ dep gpa/ enter iq am.
```
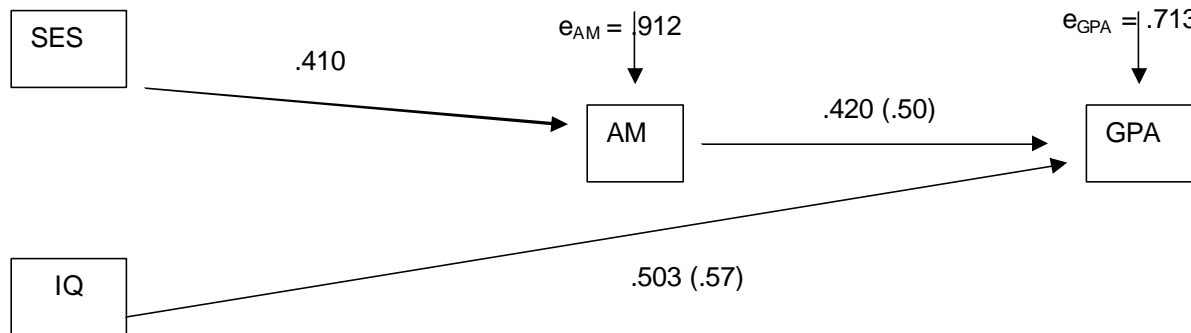
Model Summary

Model	R	R Square
1	.705 [a]	.491

a. Predictors: (Constant), AM, IQ

Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	.000	.051		.000	1.000
	IQ	4.191E-02	.003	.503	12.055	.000
	AM	.161	.016	.420	10.057	.000

a. Dependent Variable: GPA

Portraying the Reduced or Hypothesized Path Model

Testing the Reduced or Hypothesized Model

Testing the reduced model involves comparing how well it fits the data compared to how well the full model fits the data. This is much like the R²? test for comparing nested models. As with those analyses, the test of the models actually tests the average contribution of the predictors (paths) being deleted from the model, so results from dropping several predictors can be uninformative or misleading.

$$R^2_{Full} = 1 - p\ (e^2)\quad =\quad 1 - .9116^2 * .7099^2\quad =\quad .5812$$

$$R^2_{Reduced} = 1 - p\ (e^2)\quad =\quad 1 - .9121^2 * .7134^2\quad =\quad .5766$$

The summary statistic showing the relative fit of the reduced model to the full model is

$$Q = \frac{1 - R^2_{Full}}{1 - R^2_{Reduced}} = \frac{1 - .5812}{1 - .5766} = .9891$$

The significance test to compare the fit of the two models is (N = sample size d = number of dropped paths)

$$W = -(N - d) * log_e Q = -(100 - 2) * log_e .9890 = 1.074$$

W is distributed as X² with df = d. For this analysis X²(df=2, p = .05) = 5.991. We would retain the null and conclude that the reduced model fits the data as well as does the full model. That is, a model deleting the direct influence of SES and the indirect influence of IQ channeled through AM fit the data about as well as did the model including these paths.