Spearman's Rank-order Correlation -- Analysis of the Relationship Between Two Quantitative Variables

Application: To test for a rank order relationship between two quantitative variables when concerned that one or both variables is ordinal (rather than interval) and/or not normally distributed or when the sample size is small. Thus, it is used in the same data situation as a Pearson's correlation, except that it is used when the data are either importantly non-normally distributed, the measurement scale of the dependent variable is ordinal (not interval or ratio), or from a too-small sample. It is important to remember the null hypothesis, and to differentiate it from the null for Pearson's correlation.

H0: The variables do not have a rank-order relationship in the population represented by the sample.

To reject H0: is to say that there is a rank-order relationship between the variables in the population.

The data: The variables for this analysis are **fishnum** (number of fish displayed) and **fishgood** (rating of fish quality on a 1-10 scale).

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32,6   41,5   31,3   38,3   21,7   13,9   17,9   22,8   24,6   11,9   17,7   20,8
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Research Hypothesis: Knowing that store owners are often over-worked, the researcher hypothesized that stores with fewer fish would have healthier fish (thus predicting a negative or inverse relationship between these variables in this population).

H0: for this analysis: There is no rank order relationship between the number of fish displayed in pet stores and the quality rating of the fish.

Step 1 Rearrange the data so that scores from each subject are in the appropriate columns, one for each variable.

fishnum	fishgood
32	6
41	5
31	3
38	3
21	7
13	9
17	9
22	8
24	6
11	9
17	7
20	8

Step 2 Rank order the scores SEPARATELY FOR EACH VARABLE with the smallest score getting a value of 1. Cases with the same score each receive the average rank they would have received. For example, there are two scores with values of 17. We wouldn't want to rank them 3 and 4, because it makes no sense to give different ranks to to values that are the same! Instead, we will assign the average rank ([3+4] / 2 = 3.5) to both.

fishnum	rank fishnum	fishgood	rank fishgood
32	10	6	4.5
41	12	5	3
31	9	3	1.5
38	11	3	1.5
21	6	7	6.5
13	2	9	11
17	3.5	9	11
22	7	8	8.5
24	8	6	4.5
11	1	9	11
17	3.5	7	6.5
20	5	8	8.5

Step 3 For each pair of scores, compute the difference (d) between the ranks, compute the square of this difference (d²) and then find the sum of these squared differences (Σd^2)

rank fishnum	rank fishgood	d	d²
10	4.5	5.5	30.25
12	3	9	81
9	1.5	7.5	56.25
11	1.5	9.5	90.25
6	6.5	-0.5	.25
2	11	-9	81
3.5	11	-7.5	56.25
7	8.5	-1.5	2.25
8	4.5	3.5	12.25
1	11	-10	100
3.5	6.5	-3	9
5	8.5	-3.5	12.25

$$\Sigma d^2 \ = \ 531$$

Step 4 Determine the sample size (n) for the analysis

N = 12

Step 5 Compute Spearman's correlation (ρ) using the following formula.

$$\rho = 1 - \frac{6 * \Sigma d^2}{N * (N^2-1)} = 1 - \frac{6 * 531}{12 * (12^2-1)} = 1 - \frac{3186}{1716} = 1 - 1.86 = -.86$$

Step 6 Look up the critical value of ρ for the appropriate sample size.

critical ρ (N = 12, α = .05) = .588

Step 7 Compare the obtained ρ and critical ρ values and determine whether to retain or reject the null hypothesis (that there in no rank order relationship between the variables in the population represented by the sample). Remember that correlation values can be positive or negative, and so we will compare the **absolute value** of the obtained ρ to the critical ρ.

-- if the **absolute value** of the obtained ρ is less than the critical ρ, then retain the null hypothesis and conclude that there is no rank order relationship between the two variables, in the population represented by the sample.

-- if the **absolute value** of the obtained ρ is greater than the critical ρ, then reject the null hypothesis and conclude that there is a rank order relationship between the variables in the population represented in the sample.

For the example data, we would decide to reject the null hypothesis, because the absolute value of the obtained ρ (.86) is larger than the critical ρ (.588).

Step 8 IF you reject the null hypothesis, determine whether the data support or do not support the research hypothesis.

  -- IF you reject the null hypothesis AND the direction of the rank order relationship (+ or - value of ρ) agrees with the direction of the research hypothesis, then the research hypothesis is supported

  -- IF you retain the null hypothesis OR if you reject the null BUT the direction of the rank order relationship (+ or - value of ρ) disagrees with the direction of the research hypothesis, then the research hypothesis is not supported.

By the way: Usually the researcher hypothesizes that there is a rank order relationship between the variables. Sometimes, however, the research hypothesis is that there is no rank order relationship between the variables. If so, the research hypothesis and H0: are the same! When this is the case, retaining H0: provides support for the research hypothesis, whereas rejecting H0: provides evidence that the research hypothesis is incorrect.

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        For the example data, we would decide that the research hypothesis is
supported, because we rejected the null hypothesis, and the negative obtained ρ value
agrees with the negative rank-order relationship hypothesized by the researcher.
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Step 9 Reporting the results

  You will want to compute medians and IQR values for both variables to help describe the data before reporting the results of the significance test. As for the other statistical tests, the report includes the "wordy" part and the statistical values upon which you made your statistical decision. Be sure to describe the pattern of the data that led to the positive, no, or negative relationship between the variables.

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        Spearman's correlation between the number of fish displayed in these stores (Mdn
= 21.5, IQR = 17-31.5)and the quality rating for the fish (Mdn = 7, IQR = 5.25-8.75)was
r = -.886 (p<.05). This result supports the research hypothesis that those stores with
fewer fish tended to have healthier fish, whereas those stores with more fish would
tend to have fish with lower health quality.
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Critical values for Spearman's correlation (**r**)

N	.10	.05	.01
5	.900	1.000	
6	.829	.886	1.000
7	.715	.786	.929
8	.620	.715	.881
9	.600	.700	.834
10	.564	.649	.794
11	.537	.619	.764
12	.504	.588	.735
13	.484	.561	.704
14	.464	.539	.680
15	.447	.522.	.658
16	.430	.503	.636
17	.415	.488	.618
18	.402	.474	.600
19	.392	.460	.585
20	.381	.447	.s70
21	.371	.437	.556
22	.361	.426	.544
23	.353	.417	.532
24	.345	.407	.521
25	.337	.399	.511
26	.331	.391	.501
27	.325	.383	.493
28	.319	.376	.484
29	.312	.369	.475
30	.307	.363	.467

For N > 30, use the critical value from Pearson's r

Selected critical values for Pearson's r

Be sure to use df -- df = n - 2*

df	.05	.01
28	.361	.463
29	.355	.456
30	.349	.449
35	.325	.418
40	.304	.393
45	.288	.372
50	.273	.354
60	.250	.325
70	.232	.302
80	.217	.283
90	.205	.267
∞	.195	.254

Notice that Pearson's (df=28, p = .05) = .361 and
      Spearman's (N=30, p = .05) = .363
                 "close enough"