

## Bivariate Correlation Comparisons

Our study involved two criterion variables

→ loneliness & depression

and two predictors

→ family social support & stress

		Correlations			
		stress	depression (BDI)	family social support	loneliness
stress	Pearson Correlation	1	.487	-.205	.285
	Sig. (2-tailed)	.	.000	.000	.000
	N	405	405	405	405
depression (BDI)	Pearson Correlation	.487	1	-.322	.537
	Sig. (2-tailed)	.000	.	.000	.000
	N	405	405	405	405
family social support	Pearson Correlation	-.205	-.322	1	-.494
	Sig. (2-tailed)	.000	.000	.	.000
	N	405	405	405	405
loneliness	Pearson Correlation	.285	.537	-.494	1
	Sig. (2-tailed)	.000	.000	.000	.
	N	405	405	405	405

### Comparing bivariate correlations within a single population

Our first question was whether there is a difference between the correlations that the two predictors family social support (FASS) and loneliness (RULS) have with the criterion variable stress.

This is called a question about "correlated correlations." The two correlations being compared are not independent, because they share a variable, in this case, the criterion variable.

### Hotelling's t-test for "correlated correlations" within a population

The long-time standard test of correlated correlations (originally designed for use with experimental data) is Hotelling's t-test, shown below:

$$t = [r_{12} - r_{13}] * \frac{\sqrt{[N-3] * [1 + r_{23}]}}{\sqrt{2[1 - r_{23}^2 - r_{12}^2 - r_{13}^2 + [2 * r_{23} * r_{12} * r_{13}]]}}$$

$$\begin{aligned} r_{12} &= r_{\text{Stress, FASS}} = -.205 \\ r_{13} &= r_{\text{Stress, RULS}} = .285 \\ r_{23} &= r_{\text{FASS, RULS}} = -.494 \\ N &= 405 \end{aligned}$$

df = N - 3    if |t| > t-critical, then Reject H0: at that p-value

### Steiger's Z-test for "correlated correlations" within a population

Under some circumstances Hotelling's t will overestimate the t-value, resulting in a Type I error. One of the difficulties is that the formula uses the actual correlation values, even though r-values are not normally distributed. Fortunately, this can be overcome using Fisher's transformation, changing r to a Z-score, and using these Zs in the significance testing formula. In addition, the Z-critical values do not depend on df, and so are consistent for all analyses.

$$Z = [Z_{12} - Z_{13}] * \frac{\sqrt{[N - 3]}}{\sqrt{2 * [1 - r_{23}] * h}}$$

Z<sub>12</sub> & Z<sub>13</sub> are the Fisher's Z transformations of r<sub>12</sub> & r<sub>13</sub>, respectively.

But then it gets ugly !!

And still worse!

Until finally !!

$$\text{where } h = \frac{1 - [f * r_{m^2}]}{1 - r_{m^2}} \quad \text{where } f = \frac{1 - r_{23}}{2 * [1 - r_{m^2}]} \quad \text{where } r_{m^2} = \frac{r_{12}^2 + r_{13}^2}{2}$$

If Z > 1.96. p < .05    Z > 2.58. p < .01

Both of these tests can be performed using the FZT Computator, shown below with the values entered for this problem. By the way, though I call this Steiger's t-test, it was originally proposed by Williams in '50. Steiger, who did a lot of work tracking down and creating all sorts of ways to compare correlations, regression weights, etc., preferred this test to Hotellings, and so, it often gets called "Steiger's". It is also sometimes referred to at "Meng's t-test" or the "Meng, Rosenthal, & Rubin's t-test" because they did a Psychological Bulletin paper that included it.

The screenshot shows the FZT Computator interface with the following data entered:

Test	Input	Output
R <sup>2</sup> change F-test	R <sup>2</sup> larger	
	k larger	
	R <sup>2</sup> smaller	
	k smaller	
	N	
Hotelling's t / Steiger's Z	r(y,1)	-.205
	r(y,2)	.285
	r(1,2)	-.494
	N	405
	Computed Results	t = -5.94712, df = 402, Z = -5.77331
Fisher's Z-test	r(y,1) group 1	
	n group 1	
	r(y,1) group 2	
	n group 2	
	Computed Results	Z = [blank]

Based on these results, we would conclude that there is a significant difference between the correlations each of these predictors has with the criterion variable. We might write up the results like this:

Greater stress is associated with lower family social support,  $r(403) = -.205$ ,  $p < .001$ , and greater loneliness,  $r(403) = .285$ ,  $p < .001$ . These correlations with stress are significantly different,  $Z = 5.773$ ,  $p < .01$ .

### An Alternative RH: Tested by these Models

Above we asked if two predictors have "different correlations" with a given criterion. We might, alternatively be interested in whether one of the predictors is a significantly "better predictor" of the criterion. When we used words like this we are asking if one predictor "accounts for more variance" or "has a larger  $r^2$ " than the other predictor.

While both Hotelling's t and Steiger's Z test r (not  $r^2$ ), we can compare the "utility" of two predictors by testing their absolute r-values ( $|r|$ ), rather than their signed r-values. Obviously these tests of "different correlations" and "better predictor" are equivalent if the two correlations have the same sign, the two tests can lead to different conclusions if the correlations have different signs. Specifically, two r-values with opposite signs are more likely to be different than their corresponding two  $r^2$  of two  $|r|$  values!

For example, if we use .205 as the  $r(y,1)$  value in the above computation, the resulting Z value is  $-.981$ , with  $p > .05$ , a non-significant difference in the "predictive utility" of the two predictors.

### A Variation on the Use of These Models

Most uses of these models involve comparing two correlations that share a common criterion, and are used to ask if one predictor is more correlated or a better predictor than the other. An important variation of this is to use these models to compare to correlations that have the same predictor variable, but different criterion variables. In this case, we would be asking if a given predictor was "differently correlated" with two criteria (if we compared the two signed r-values) or if a given predictor was a "better predictor" of two criteria (if we compared the two  $|r|$  values). Of course, we don't always think of the variables we are correlating and comparing the correlations as predictors and criteria, but when we do, it is useful to remember both issues of multicausality (using and comparing multiple predictors) and construct validity (using and comparing multiple criteria).

For example, we might be interested to explore whether loneliness was differently correlated with the two criterion variables included in this analysis, stress (.285) and depression (.537). Applying Steiger's Z-test (with the correlation of .487 between stress & depression) gives us  $Z = 5.650$ ,  $p < .01$  – indicating that loneliness is more strongly correlated with depression than it is with stress. Notice, that because both of these correlations are positive, comparing the r-values and the  $|r|$ -values leads to the same result and conclusion.

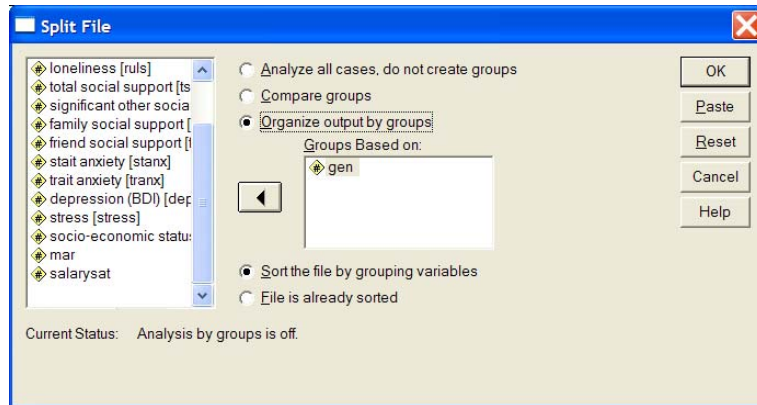
## Comparing bivariate correlations across populations

Another common question is whether two variables are equally correlated in two different populations. In this example we will ask if the correlation between depression (BDI) and family social support (FASS) is the same for males and females. To do this in SPSS we must first split the file into two subfiles (males and females) and obtain the desired correlation from each subfile.

### Data → Split File

Move the variable or variables into the "Groups Based on:" window and click "OK".

All subsequent analyses we request will be performed and presented separately for each of the resulting groups.



### Gen = Males

Correlations<sup>a</sup>

		depression (BDI)	family social support
depression (BDI)	Pearson Correlation	1	.111
	Sig. (2-tailed)	.	.373
	N	66	66
family social support	Pearson Correlation	.111	1
	Sig. (2-tailed)	.373	.
	N	66	66

a. gen = males

### Gen = Females

Correlations<sup>a</sup>

		depression (BDI)	family social support
depression (BDI)	Pearson Correlation	1	-.289
	Sig. (2-tailed)	.	.000
	N	170	170
family social support	Pearson Correlation	-.289	1
	Sig. (2-tailed)	.000	.
	N	170	170

a. gen = females

A significance test will require that we find the difference between these two correlations, relative to the expected variability in correlations for this sample size. The common Z-test is useful for this, but assumes that the values being compared are normally distributed, and we know that  $r$  is not normally distributed. Fisher, however, determined a way to transform  $r$ -values so that they will be normally distributed -- called Fisher's Z-transformation.

$$\text{The Z-test is computed as } Z = \frac{Z_1 - Z_2}{SE_{ZD}} \quad \text{Z-critical is } \begin{matrix} 1.96 \text{ for } p < .05 \\ 2.58 \text{ for } p < .01 \end{matrix}$$

$$SE_{ZD} = \sqrt{1/(n1-3) + 1/(n2-3)}$$

On the right is the portion of the FZT program used for Fisher's Z-test, with the values for this group comparison shown.

As with other correlation comparisons, you must decide if you want to test for "correlation differences" (including the sign of the correlations) or the "predictive utility differences" (using  $|r|$  for both correlations). In this case, the results from comparing the "utility" of the predictor for this criterion in the two groups was  $Z=1.258$ ,  $p > .05$ . Remember, the tests are equivalent if the signs of the two correlations are the same.

Family social support was correlated with depression for females,  $r(168) = -.289$ ,  $p < .001$ , for women, but not for men,  $r(64) = .111$ . The difference between these correlations was statistically significant,  $Z = 2.776$ ,  $p < .01$ .

### Fisher's Z-test

<b>r(y,1) group 1</b>	.111
<b>n group1</b>	66
<b>r(y,1) group 2</b>	-.289
<b>n group 2</b>	170

<b>Z</b>	2.766
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