## Example of Multiple-group ldf – with Follow-up Analyses

In this example, three sections of a research methods class were conducted using three different formats for test preparation. Group 1 was a "control group" that received the lectures, and took the exams; Group 2 received a steady stream of homework assignments, which were similar to items which appeared on the exams; Group 3 received no homework assignments, but completed "exam preps" that was similar to items which appeared on the exam. There were four "DVs" for this analysie: total points from the quizzes, Midterm Exam #1, Midterm Exam 2, and the Final (cummulative) Exam.

				Valid N (li	stwise)
GROUP		Mean	Std. Deviation	Unweighted	Weighted
lecture	QUIZ	518.8628	108.77367	20	20.000
	EXAM1	43.6497	9.23430	20	20.000
	EXAM2	39.6246	10.00930	20	20.000
	FINAL	94.4252	8.10272	20	20.000
homework	QUIZ	594.3515	71.29752	20	20.000
	EXAM1	50.9138	8.81464	20	20.000
	EXAM2	49.5690	9.64761	20	20.000
	FINAL	99.4550	6.23406	20	20.000
examprep	QUIZ	472.3838	133.61485	20	20.000
	EXAM1	59.6089	7.27711	20	20.000
	EXAM2	52.9495	11.57586	20	20.000
	FINAL	130.6873	6.76939	20	20.000
Total	QUIZ	528.5327	117.32600	60	60.000
	EXAM1	51.3908	10.62168	60	60.000
	EXAM2	47.3810	11.74384	60	60.000
	FINAL	108.1892	17.60895	60	60.000

**Group Statistics** 

## **Tests of Equality of Group Means**

	Wilks' Lambda	F	df1	df2	Sig.
QUIZ	.813	6.539	2	57	.003
EXAM	.616	17.741	2	57	.000
EXAM	.764	8.796	2	57	.000
FINAL	.156	154.028	2	57	.000

#### Eigenvalues

				Canonical
Function	Eigenvalue	% of Variance	Cumulative %	Correlation
1	7.110 <sup>a</sup>	96.0	96.0	.936
2	.295 <sup>a</sup>	4.0	100.0	.477

a. First 2 canonical discriminant functions were used in the analysis.

## Structure Matrix

	Fund	Function	
	1	2	
FINAL	.869*	338	
QUIZ	116	.673*	
EXAM2	.173	.572*	
FXAM1	286	380*	

Pooled within-groups correlations between discriminating variables and standardized canonical discriminant function Variables ordered by absolute size of correlation within fun

\*. Largest absolute correlation between each variable an any discriminant function

#### Classification Results

			Predicte	d Group Mem	bership	
		GROUP	lecture	homework	examprep	Total
Original	Count	lecture	17	3	0	20
		homework	6	14	0	20
		examprep	0	0	20	20
	%	lecture	85.0	15.0	.0	100.0
		homework	30.0	70.0	.0	100.0
		examprep	.0	.0	100.0	100.0

a. 85.0% of original grouped cases correctly classified.

# .772

Test of Function(s)

Function

2

-.557

.712 <u>-.</u>154

1 through 2

**Functions at Group Centroids** 

1

-2.454

-1.143

3.597

Unstandardized canonical discriminant

functions evaluated at group means

GROUP

homework

examprep

lecture

Wilks'

Lambda

.095

Chi-square

130.514

14.348

Wilks' Lambda

## Functions at Group Centroids

	Fund	ction
GROUP	1	2
lecture	-2.454	557
homework	-1.143	.712
examprep	3.597	154

df

8

3

Sig.

.000

.002

Unstandardized canonical discriminant functions evaluated at group means

So, we have two ldfs, that seem to do a pretty good job of discriminating between the groups. However, some presentations of these results would benefit from more formal tests of the "contributions" of the two ldfs, to the discrimination among the groups. There are three common types of such "follow-up analyses" for multiple group discriminant analyses:

- 1) bivariate follow-ups emphasis returns to how the groups differ on each of the DVs
- 2) multivariate pairwise ldf analyses which groups are differentiable using which ldfs
- 3) multivariate pairwise group analyses building separate ldf models for each pair of groups

Here are Examples of each of the approaches:

## **Bivariate Follow-ups:**

Analyze  $\rightarrow$  Compare Means  $\rightarrow$  One-way ANOVA

One-Way ANOVA	One-Way ANOVA: Post Hoc Multiple Comp
Dependent List: <ul> <li></li></ul>	OK       Equal Variances Assumed         Paste       SD       S·N·K       Waller-Duncan         Bonferroni       Tukey       Type I/Type II Error Ratio:       Image: Control Category:         Beset       Sgheffe       Duncan       Control Category:       Last         Beset       R-E-G-W F       Hochberg's GT2       Test         Help       R-E-G-W Q       Gabriel       C < Control C         Equal Variances Not Assumed       Tambane's T2       Dunnett's T3       Games-Howell       Dunnett's
Click on "Post Hoc"	Significance level:  .05

QUIZ					
	Sum of				
	Squares	df	Mean Square	F	Sig.
Between Groups	151566.5	2	75783.262	6.539	.003
Within Groups	660591.5	57	11589.325		
Total	812158.1	59			

ANOVA

#### Multiple Comparisons

Dependent Variable: QUIZ

LSD

-						
		Mean Difference			95% Confide	ence Interval
(I) GROUP	(J) GROUP	(I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
lecture	homework	-75.4887*	34.04310	.031	-143.6589	-7.3186
	examprep	46.4790	34.04310	.178	-21.6911	114.6491
homework	lecture	75.4887*	34.04310	.031	7.3186	143.6589
	examprep	121.9677*	34.04310	.001	53.7976	190.1379
examprep	lecture	-46.4790	34.04310	.178	-114.6491	21.6911
	homework	-121.9677*	34.04310	.001	-190.1379	-53.7976

\*. The mean difference is significant at the .05 level.

Using LSD has the advantage that you get the exact probabilities for each pairwise comparison. Then you can apply whatever amount of a inflation control you think appropriate.

This approach emphasizes "specificity" but doesn't take advantage of any multivariate analyses.

## Pairwise ldf Follow-ups:

When getting the ldf analysis click the "Save" button and check "Discriminant scores"

Discriminant Analysis: Save	×
Eved oted group membership	Continue
Description of any superstantial	Cancel
1 Pyceaulias of group memorianip	Help
- Export model information to XML file	
	Browse

This approach is an obvious extension of the descriptive procedures we were using earlier. It emphasizes the ldfs that were identified and interpreted, and gives statistical information about which groups can be discriminated based on each ldf. Remember, larger F  $\sim$  less overlap  $\sim$  better classification.

Then use oneway to get pairwise comparisons using these ldf scores as the DVs.

One-Way ANC	VA	×
<ul> <li>Image: with a state of the sta</li></ul>	Dependent List: Discriminant Scores from Discriminant Scores from D	OK <u>P</u> aste <u>R</u> eset Cancel
	Factor:	Help

Equal Volation A IF 1503 IF Bonfenoni IF Sidak IF Schefe IF 8-EG-W F IF 8-EG-W £	Visuned SNK Likey Likey Likey'sb Duncen Hochbergie G Gabrel	☐ WelenDuncan           Tope (Cupe) Law from           ☐ Duringti           [12]           [14]           [16]
Equal Variances N	loi Assumed	E Games House E D months E

		Sum of Squares	df	Mean Square	F	Sig.
Discriminant Scores from	Between Groups	405.263	2	202.632	202.632	.000
Function 1 for Analysis 1	Within Groups	57.000	57	1.000		
	Total	462.263	59			
Discriminant Scores from	Between Groups	16.815	2	8.408	8.408	.001
Function 2 for Analysis 1	Within Groups	57.000	57	1.000		
	Total	73.815	59			

## **Multiple Comparisons**

LSD							
			Mean			95% Confide	ance Interval
Dependent Variable			Difference	Std Error	Sig	Lower Bound	Lipper Bound
Discriminant Scores from		(J) GROUP	(I-J) 1 2107522*	31622777		1 0/20002	6775190
Discriminant Scores norm	lecture	HOMEWORK	-1.3107552	.31022111	.000	-1.9439003	0775160
Function 1 for Analysis 1		examprep	-6.0503915*	.31622777	.000	-6.6836267	-5.4171563
	homework	lecture	1.3107532*	.31622777	.000	.6775180	1.9439883
		examprep	-4.7396383*	.31622777	.000	-5.3728735	-4.1064031
	examprep	lecture	6.0503915*	.31622777	.000	5.4171563	6.6836267
		homework	4.7396383*	.31622777	.000	4.1064031	5.3728735
Discriminant Scores from	lecture	homework	-1.2689528*	.31622777	.000	-1.9021880	6357176
Function 2 for Analysis 1		examprep	4032513	.31622777	.207	-1.0364865	.2299839
	homework	lecture	1.2689528*	.31622777	.000	.6357176	1.9021880
		examprep	.8657015*	.31622777	.008	.2324663	1.4989367
	examprep	lecture	.4032513	.31622777	.207	2299839	1.0364865
		homework	8657015*	.31622777	.008	-1.4989367	2324663

\*- The mean difference is significant at the .05 level.

## Pairwise group comparisons

This involves getting separate 2-group analyses for each pair of groups. It can be especially helpful when the overall ldf doesn't discriminate between one or more group pairs.

You have to recode the group variable to get comparison of nonadjacent groups, like this.

Recode group (1=4) (3=5). Then ask for a discriminant analysis of groups 4 & 5.

Eigenvalues						
				Canonical		
Function	Eigenvalue	% of Variance	Cumulative %	Correlation		
1	.987 <sup>a</sup>	100.0	100.0	.705		

a. First 1 canonical discriminant functions were used in the analysis.

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1	.503	24.711	4	.000

Wilks' Lambda

#### Structure Matrix

	Function		
	1		
EXAM2	.522		
QUIZ	.424		
EXAM1	.416		
FINAL	.359		

Pooled within-groups correlations between discriminating variables and standardized canonical discriminant functions Variables ordered by absolute size of correlation within function Standardized Canonical Discriminant Function Coefficients

	Function
	1
QUIZ	.838
EXAM1	.713
EXAM2	.270
FINAL	.577

**Functions at Group Centroids** 

	Function	
GROUP	1	
4.00	968	
5.00	.968	

Unstandardized canonical discriminant functions evaluated at group means

## Classification Results<sup>a</sup>

			Predicted Group Membership		
		GROUP	4.00	5.00	Total
Original	Count	4.00	16	4	20
		5.00	5	15	20
		Ungrouped cases	0	20	20
	%	4.00	80.0	20.0	100.0
		5.00	25.0	75.0	100.0
		Ungrouped cases	.0	100.0	100.0

a. 77.5% of original grouped cases correctly classified.

## How to choose the best follow-up ??

If the multivariate results "add nothing" to the bivariate analyses and you won't be doing classifications, then the bivariate follow-ups might be easier for your audience to understand. This approach is also "more specific" in that it uses the original DVs that you selected, rather than constructing new variates out of them.

If you have carefully identified and interpreted the ldfs, then the pairwise ldf analysis "completes" the story by telling giving a statistical description of which groups are different on which ldfs.

If you are interested in how pairs of groups differ from each other (especially if the full model ldf doesn't separate particular groups) the multivariate group comparison approach may give useful information.