Maximum Likelihood Factoring

Use the "Extraction" window to request a maximum likelihood factoring.

All other commands work as they did with PC & PAF analyses.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Initial Eigenvalues</th>
<th>Extraction Sums of Squared Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>% of Variance</td>
</tr>
<tr>
<td>1</td>
<td>3.048</td>
<td>38.097</td>
</tr>
<tr>
<td>2</td>
<td>1.709</td>
<td>21.363</td>
</tr>
<tr>
<td>3</td>
<td>1.340</td>
<td>16.746</td>
</tr>
<tr>
<td>4</td>
<td>.636</td>
<td>7.953</td>
</tr>
<tr>
<td>5</td>
<td>.483</td>
<td>6.036</td>
</tr>
<tr>
<td>6</td>
<td>.340</td>
<td>4.244</td>
</tr>
<tr>
<td>7</td>
<td>.240</td>
<td>3.000</td>
</tr>
<tr>
<td>8</td>
<td>.205</td>
<td>2.562</td>
</tr>
</tbody>
</table>

Extraction Method: Maximum Likelihood.

Above is the output from a ML factoring without rotation
- Notice that the "Initial Eigenvalues" are the same as for a PC and a PAF – because both a PAF and a ML start with a PC & use $\lambda > 1$ to determine the # factors (unless you specify a different criterion or select a specific number of factors in the Extraction window)
- Notice that the variance accounted for by the 3 retained PCs and 3 retained ML factors is different – again ML is accounting for common variance.
- Notice also that the PC & ML eigenvalues are different, and that the ML factors don’t necessarily account for decreasing amounts of common variance (though they are orthogonal)

<table>
<thead>
<tr>
<th>Goodness-of-fit Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
</tr>
<tr>
<td>7.693</td>
</tr>
</tbody>
</table>

Here is the significance test associated with this ML solution.

The X² test shows that there is no systematic variance in the reduced R (with communalities in the diagonal) after 3 factors have been extracted.
Communalities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial</th>
<th>Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>physical aggression</td>
<td>1.000</td>
<td>.746</td>
</tr>
<tr>
<td>property damage</td>
<td>1.000</td>
<td>.759</td>
</tr>
<tr>
<td>theft</td>
<td>1.000</td>
<td>.603</td>
</tr>
<tr>
<td>extreme verbal abuse</td>
<td>1.000</td>
<td>.742</td>
</tr>
<tr>
<td>sad</td>
<td>1.000</td>
<td>.747</td>
</tr>
<tr>
<td>anxious</td>
<td>1.000</td>
<td>.797</td>
</tr>
<tr>
<td>self-confidence</td>
<td>1.000</td>
<td>.861</td>
</tr>
<tr>
<td>compliance</td>
<td>1.000</td>
<td>.843</td>
</tr>
</tbody>
</table>

Extraction Method: Principal Component Analysis.

On the left are the communalities from the PC, PAF and ML factorings.

Notice that the PAF and the ML both start with the same communalities in SPSS – the $R^2$ predicting each variable from the other variables.

The PAF and ML solutions produce extraction communalities that are somewhat smaller (at least on average) than the PC, because they are reproducing common variance and the PCs are reproducing total variance.

PAF and ML communalities are usually very similar. Remember that ML will produce more accurate population parameter estimates than PAF if the assumptions of interval measurement and normal distribution are well met, while PAF is somewhat more robust to violations of these assumptions (but not really robust!).

Notice that we have encountered a “Heywood case” during the ML extraction.

When this happens SPSS will report the event, and display the communalities based on the solution from the previous iteration. (It tells you the iterations involved in the structure matrix, shown later).

While some accept the solution from the previous iteration, it is a good idea to explore the event, and see if the offending variable should be deleted or combined with another variable.

It is also possible to avoid the Heywood case by starting with a different/lower initial communality estimate for that variable.

Communalities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial</th>
<th>Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>physical aggression</td>
<td>.620</td>
<td>.680</td>
</tr>
<tr>
<td>property damage</td>
<td>.597</td>
<td>.727</td>
</tr>
<tr>
<td>theft</td>
<td>.265</td>
<td>.305</td>
</tr>
<tr>
<td>extreme verbal abuse</td>
<td>.587</td>
<td>.644</td>
</tr>
<tr>
<td>sad</td>
<td>.472</td>
<td>.498</td>
</tr>
<tr>
<td>anxious</td>
<td>.569</td>
<td>.790</td>
</tr>
<tr>
<td>self-confidence</td>
<td>.484</td>
<td>.604</td>
</tr>
<tr>
<td>compliance</td>
<td>.555</td>
<td>.805</td>
</tr>
</tbody>
</table>

Extraction Method: Principal Axis Factoring.

Communalities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial</th>
<th>Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>pa2</td>
<td>.620</td>
<td>.702</td>
</tr>
<tr>
<td>pd2</td>
<td>.597</td>
<td>.738</td>
</tr>
<tr>
<td>t2</td>
<td>.265</td>
<td>.272</td>
</tr>
<tr>
<td>va2</td>
<td>.587</td>
<td>.646</td>
</tr>
<tr>
<td>s2</td>
<td>.472</td>
<td>.433</td>
</tr>
<tr>
<td>ax2</td>
<td>.569</td>
<td>.864</td>
</tr>
<tr>
<td>sc2</td>
<td>.484</td>
<td>.476</td>
</tr>
<tr>
<td>co2</td>
<td>.555</td>
<td>.999</td>
</tr>
</tbody>
</table>

Extraction Method: Maximum Likelihood.

\(^a\) One or more communality estimates greater than 1 were encountered during iterations. The resulting solution should be interpreted with caution.
### Factor Matrix

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>pa2</td>
<td>-0.218</td>
<td>0.568</td>
<td>0.576</td>
</tr>
<tr>
<td>pd2</td>
<td>-0.298</td>
<td>0.449</td>
<td>0.669</td>
</tr>
<tr>
<td>t2</td>
<td>-0.033</td>
<td>0.226</td>
<td>0.469</td>
</tr>
<tr>
<td>va2</td>
<td>-0.302</td>
<td>0.736</td>
<td>0.118</td>
</tr>
<tr>
<td>s2</td>
<td>-0.279</td>
<td>0.501</td>
<td>-0.324</td>
</tr>
<tr>
<td>ax2</td>
<td>-0.271</td>
<td>0.791</td>
<td>-0.406</td>
</tr>
<tr>
<td>sc2</td>
<td>0.673</td>
<td>-0.138</td>
<td>0.064</td>
</tr>
<tr>
<td>co2</td>
<td>0.999</td>
<td>0.003</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Extraction Method: Maximum Likelihood.

a. 3 factors extracted. 12 iterations required.

### Structure Matrix

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>pa2</td>
<td>0.835</td>
<td>0.384</td>
<td>-0.165</td>
</tr>
<tr>
<td>pd2</td>
<td>0.853</td>
<td>0.279</td>
<td>-0.252</td>
</tr>
<tr>
<td>t2</td>
<td>0.492</td>
<td>0.054</td>
<td>-0.008</td>
</tr>
<tr>
<td>va2</td>
<td>0.628</td>
<td>0.711</td>
<td>-0.243</td>
</tr>
<tr>
<td>s2</td>
<td>0.153</td>
<td>0.644</td>
<td>-0.247</td>
</tr>
<tr>
<td>ax2</td>
<td>0.269</td>
<td>0.922</td>
<td>-0.218</td>
</tr>
<tr>
<td>sc2</td>
<td>-0.065</td>
<td>-0.146</td>
<td>0.682</td>
</tr>
<tr>
<td>co2</td>
<td>-0.289</td>
<td>-0.361</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Extraction Method: Maximum Likelihood.

Rotation Method: Promax with Kaiser Normalization.

### Pattern Matrix

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>pa2</td>
<td>0.817</td>
<td>0.070</td>
<td>0.046</td>
</tr>
<tr>
<td>pd2</td>
<td>0.873</td>
<td>-0.096</td>
<td>-0.078</td>
</tr>
<tr>
<td>t2</td>
<td>0.571</td>
<td>-0.152</td>
<td>0.078</td>
</tr>
<tr>
<td>va2</td>
<td>0.410</td>
<td>0.552</td>
<td>0.019</td>
</tr>
<tr>
<td>s2</td>
<td>-0.136</td>
<td>0.677</td>
<td>-0.073</td>
</tr>
<tr>
<td>ax2</td>
<td>-0.113</td>
<td>0.983</td>
<td>0.054</td>
</tr>
<tr>
<td>sc2</td>
<td>0.086</td>
<td>0.036</td>
<td>0.713</td>
</tr>
<tr>
<td>co2</td>
<td>-0.045</td>
<td>-0.048</td>
<td>0.972</td>
</tr>
</tbody>
</table>

Extraction Method: Maximum Likelihood.

Rotation Method: Promax with Kaiser Normalization.

a. Rotation converged in 4 iterations.

### Factor Correlation Matrix

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.402</td>
<td>-0.232</td>
</tr>
<tr>
<td>2</td>
<td>0.402</td>
<td>1.000</td>
<td>-0.303</td>
</tr>
<tr>
<td>3</td>
<td>-0.232</td>
<td>-0.303</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Extraction Method: Maximum Likelihood.

Rotation Method: Promax with Kaiser Normalization.

The extracted (unrotated) solution looks very different from the PC and PAF factorings of these same variables (which, you will remember, looked very similar to each other).

However, if it not clear that the difference is because of the extraction procedures. Remember that this analysis produced a Heywood case at the 13th iteration. That large communality for CO2 means that variable will dominate some factor (not always the first), changing the apparent structure of the factor solution.

The ML with Promax rotation structure matrix looks much like all the other combinations of extraction and rotation.

This highlights that the different extraction & rotation combinations will generally yield a similar solution, if there is a fairly strong structure to be found.

The corollary is that you should be very careful and explore your data further if you get apparently different solutions from different extraction & rotation combinations.

The Structure matrix shows the correlations between each factor and all variables.

A Pattern matrix is presented with any oblique rotation.

The Pattern matrix tells the unique contribution of each oblique factor to account for each variable.

Look at PA2. The Pattern matrix shows that only Factor 1 has an appreciable unique contribution to reproducing PA2. However, looking at the Structure matrix reminds us that Factor 2 has a non-zero correlation with PA2 (.384 is well above the minimum cutoff of .30).

A similar trend can be seen with CO2 for Factors 2 & 3.

The Factor Correlation matrix (also called Phi or \( \Phi \)) holds the intercorrelations among the oblique factors.

Very large values can suggest overfactoring (keeping more factors than there are kinds of variables). However, it is important to remember that while, for example an empirical \( r \) of .303 is a "medium" effect, it does not suggest that these factors are "too correlated".

An \( r = .303 \) means \( r^2 = .091 \), or these factors 2 & 3 share about 9% of their variance, and 91% of each factor’s variance is independent of the other.