Multiple Regression Results You Should Remember

"**Multivariate Power"**

Example of a significant multivariate model built from predictors none of which are significantly correlated with the criterion.

The multiple regression model accounts for more variance than any single predictor, so the error term used to test the contribution of each predictor to the multivariate model ($1-R^2$) is smaller and the test is more powerful than for the bivariate test (same schtick as how adding a covariate that isn't correlated to the IV can increase the statistical power of the group comparison). Some consider this a suppressor effect, some don't.

Correlations

		Y	X1	X2	X3	X4	X5
Y	Pearson Correlation	1	.191	.192	.237	.174	.110
	Sig. (2-tailed)	.	.119	.117	.081	.155	.371
	N	68	68	68	68	68	68
X1	Pearson Correlation	.191	1	-.250*	-.077	-.079	-.110
	Sig. (2-tailed)	.119	.	.039	.535	.521	.371
	N	68	68	68	68	68	68
X2	Pearson Correlation	.192	-.250*	1	-.077	.361**	.013
	Sig. (2-tailed)	.117	.039	.	.532	.003	.917
		68	68	68	68	68	68
X3	Pearson Correlation	.237	-.077	-.077	1	.203	.219
	Sig. (2-tailed)	.081	.535	.532	.	.098	.073
	N	68	68	68	68	68	68
X4	Pearson Correlation	.174	-.079	.361**	.203	1	.162
	Sig. (2-tailed)	.155	.521	.003	.098	.	.187
	N	68	68	68	68	68	68
X5	Pearson Correlation	.110	-.110	.013	.219	.162	1
	Sig. (2-tailed)	.371	.371	.917	.073	.187	.
	N	68	68	68	68	68	68

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

The effect is most likely when there is little colinearity among the predictors – that way most of the variance each predictor shares with the criterion will be a unique contribution to the multivariate model.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.428[a]	.183	.117	522.14133

a. Predictors: (Constant), X5, X2, X3, X1, X4

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3781331	5	756266.188	2.774	.025[a]
	Residual	16903157	62	272631.569		
	Total	20684488	67			

a. Predictors: (Constant), X5, X2, X3, X1, X4

b. Dependent Variable: Y

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-350.742	195.472		-1.794	.078
	X1	3.327	1.376	.290	2.418	.019
	X2	2.485	1.185	.271	2.098	.040
	X3	3.125	1.479	.257	2.112	.039
	X4	.366	1.342	.035	.273	.786
	X5	.844	1.309	.077	.644	.522

a. Dependent Variable: Y

"Null Wash-out"

Example of mixing a significantly correlated predictor with several nulls. Also an example of having a significantly contributing predictor in a non-significant multivariate model.

Remember that the F-test of R^2 for a model really tests the "average contribution of the predictors to the model", so …. Be careful interpreting the results of a model which has mostly predictors that aren't correlated with the criterion!!!

Correlations

		P1	P2	P3	P4	P5	P6	P7	P8	P9
Y	Pearson Correlation	.230	.059	.004	.079	-.100	-.028	-.040	-.007	.013
	Sig. (2-tailed)	.002	.432	.953	.294	.186	.709	.595	.927	.863
	N	177	177	177	177	177	177	177	177	177

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.273[a]	.074	.024	9.68313

a. Predictors: (Constant), P9, P1, P4, P7, P5, P3, P2, P6, P8

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1257.906	9	139.767	1.491	.155[a]
	Residual	15658.410	167	93.763		
	Total	16916.316	176			

a. Predictors: (Constant), P9, P1, P4, P7, P5, P3, P2, P6, P8

b. Dependent Variable: Y

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	100.454	17.866		5.623	.000
	P1	.115	.038	.233	3.047	.003
	P2	4.511E-02	.077	.044	.583	.561
	P3	-1.93E-02	.076	-.019	-.254	.800
	P4	7.511E-02	.076	.075	.988	.325
	P5	-9.22E-02	.070	-.099	-1.320	.189
	P6	6.555E-04	.077	.001	.009	.993
	P7	-4.86E-02	.076	-.048	-.640	.523
	P8	-4.13E-02	.073	-.044	-.568	.571
	P9	6.592E-03	.076	.007	.087	.931

a. Dependent Variable: Y

Here we have a non-significant model – that "doesn't work", but which has a significantly contributing predictor!!

How, you might ask, can we have a significant contribution to a non-significant model?

Because most of the predictors aren't contributing, and the F-test of the model R^2 looks at the average contribution of the set of predictors!

Extreme Collinearity – When all the predictors are highly inter-correlated…

Remember that the b and the t-test of b reflect the independent contribution of that predictor to that model. So, a set of highly collinear predictors might form a "working model" which has "no contributors".

Correlations

		Y	P1	P2	P3	P4	P5
Y	Pearson Correlation	1	.298**	.198**	.221**	.221**	.251**
	Sig. (2-tailed)	.	.000	.008	.003	.003	.001
	N	177	177	177	177	177	177
P1	Pearson Correlation	.298**	1	.689**	.712**	.742**	.728**
	Sig. (2-tailed)	.000	.	.000	.000	.000	.000
	N	177	177	177	177	177	177
P2	Pearson Correlation	.198**	.689**	1	.499**	.500**	.520**
	Sig. (2-tailed)	.008	.000	.	.000	.000	.000
	N	177	177	177	177	177	177
P3	Pearson Correlation	.221**	.712**	.499**	1	.471**	.494**
	Sig. (2-tailed)	.003	.000	.000	.	.000	.000
	N	177	177	177	177	177	177
P4	Pearson Correlation	.221**	.742**	.500**	.471**	1	.593**
	Sig. (2-tailed)	.003	.000	.000	.000	.	.000
	N	177	177	177	177	177	177
P5	Pearson Correlation	.251**	.728**	.520**	.494**	.593**	1
	Sig. (2-tailed)	.001	.000	.000	.000	.000	.
	N	177	177	177	177	177	177

**. Correlation is significant at the 0.01 level (2-tailed).

Model

Mode	R	R	Adjuste R	Std. Error the
1	..43 [a]	.19	.06	9.4789

a. Predictors: (Constant), P5, P3,

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1551.944	5	310.389	3.455	.005[a]
	Residual	15364.372	171	89.850		
	Total	16916.316	176			

a. Predictors: (Constant), P5, P3, P2, P4, P1

b. Dependent Variable: Y

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	93.378	1.899		49.184	.000
	P1	.115	.080	.244	1.441	.151
	P2	-1.23E-02	.073	-.017	-.169	.866
	P3	1.555E-02	.076	.022	.206	.837
	P4	-4.41E-03	.077	-.006	-.057	.954
	P5	5.211E-02	.074	.076	.707	.481

a. Dependent Variable: Y

Patterns of Collinearity – When some of the predictors are highly inter-correlated…

Remember b and its t-test reflect the independent contribution of that predictor to that model. So, within a set of predictors with a ange of collinearities the "contribution of a predictor" may depend upon what variables are in the model.

Each of the four predictors has a substantial significant correlation with the criterion.

Most of the collinearities are around the same size as the correlations with the criterion, except the collinearity of P3 & P4, which is much larger!

When this happens, how much these two variables contribute to the model can depend on whether or not the other one is also I the model!

Correlations

		Y	P1	P2	P3	P4
Y	Pearson Correlation	1	.411	.479	.344	.397
	Sig. (2-tailed)		.000	.000	.000	.000
	N	200	200	200	200	200
P1	Pearson Correlation	.411	1	.429	.360	.265
	Sig. (2-tailed)	.000		.000	.000	.000
	N	200	200	200	200	200
P2	Pearson Correlation	.479	.429	1	.239	.169
	Sig. (2-tailed)	.000	.000		.001	.017
	N	200	200	200	200	200
P3	Pearson Correlation	.344	.360	.239	1	.869
	Sig. (2-tailed)	.000	.000	.001		.000
	N	200	200	200	200	200
P4	Pearson Correlation	.397	.265	.169	.869	1
	Sig. (2-tailed)	.000	.000	.017	.000	
	N	200	200	200	200	200

With only one or the other in the model, that predictor has a significant contribution!

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.559[a]	.313	.302	2871.69927

a. Predictors: (Constant), P3, P2, P1

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.558[a]	.312	.301	2874.01991

a. Predictors: (Constant), P4, P2, P1

Coefficients[a]

Model		B	Beta	t	Sig.
1	(Constant)	178		.168	.867
	P1	64	.192	2.796	.006
	P2	62	.351	5.332	.000
	P3	46	.191	2.993	.003

a. Dependent Variable: Y

Coefficients[a]

Model		B	Beta	t	Sig.
1	(Constant)	160.46		.150	.881
	P1	70.135	.209	3.116	.002
	P2	63.801	.359	5.457	.000
	P4	38.416	.181	2.937	.004

a. Dependent Variable: Y

However, with both of these predictors included, neither contributes! It might be tempting to conclude from this third analysis that neither P3 nor P4 should be included in the model – but that would be a poor conclusion!

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.561[a]	.315	.300	2875.26775

a. Predictors: (Constant), P4, P2, P1, P3

Coefficients[a]

Model		B	Beta	t	Sig.
1	(Constant)	97.36		.091	.928
	P1	65.55	.196	2.84	.005
	P2	62.85	.353	5.35	.000
	P3	27.57	.114	.911	.363
	P4	18.36	.086	.717	.474

a. Dependent Variable: Y

Entry Order Does Not Influence Full Model Fit (R^2) or Regression weights (b)

Regression models have no "memory" of predictor entry order – once the predictors are "all in there" the R^2 and b weights are "always the same".

Here's the results from a simultaneous entry of the four predictors

Model Summary

Model	R	R Square
1	.757[a]	.573

a. Predictors: (Constant), self seteem scale, STRESS, total social support, trait anxiety

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	15.838	3.130		5.060	.000
	total social support	-.527	.197	-.095	-2.678	.008
	trait anxiety	.193	.033	.292	5.928	.000
	STRESS	.191	.032	.217	5.994	.000
	self seteem scale	-.438	.061	-.350	-7.214	.000

a. Dependent Variable: depression (BDI)

Here's the results from adding each predictor one at a time

- Notice the last step includes all 4 predictors with the same R^2 and b values as above

Model Summary

Model	R	R Square
1	.369[a]	.136
2	.685[b]	.470
3	.720[c]	.518
4	.757[d]	.573

a. Predictors: (Constant), total social support

b. Predictors: (Constant), total social support, trait anxiety

c. Predictors: (Constant), total social support, trait anxiety, STRESS

d. Predictors: (Constant), total social support, trait anxiety, STRESS, self seteem scale

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	18.946	1.473		12.865	.000
	total social support	-2.044	.256	-.369	-7.973	.000
2	(Constant)	-3.717	1.836		-2.025	.044
	total social support	-.830	.215	-.150	-3.857	.000
	trait anxiety	.408	.026	.618	15.891	.000
3	(Constant)	-3.343	1.753		-1.907	.057
	total social support	-.776	.206	-.140	-3.774	.000
	trait anxiety	.343	.027	.519	12.894	.000
	STRESS	.213	.034	.243	6.341	.000
4	(Constant)	15.838	3.130		5.060	.000
	total social support	-.527	.197	-.095	-2.678	.008
	trait anxiety	.193	.033	.292	5.928	.000
	STRESS	.191	.032	.217	5.994	.000
	self seteem scale	-.438	.061	-.350	-7.214	.000

a. Dependent Variable: depression (BDI)

Here's the results from adding the same predictors in a different grouping and order

- Notice the last step includes all 4 predictors with the same R^2 and b values as above

Model Summary

Model	R	R Square
1	.369[a]	.136
2	.731[b]	.535
3	.757[c]	.573

a. Predictors: (Constant), total social support

b. Predictors: (Constant), total social support, trait anxiety, self seteem scale

c. Predictors: (Constant), total social support, trait anxiety, self seteem scale, STRESS

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	18.946	1.473		12.865	.000
	total social support	-2.044	.256	-.369	-7.973	.000
2	(Constant)	17.068	3.256		5.241	.000
	total social support	-.554	.205	-.100	-2.705	.007
	trait anxiety	.238	.033	.361	7.231	.000
	self seteem scale	-.473	.063	-.378	-7.518	.000
3	(Constant)	15.838	3.130		5.060	.000
	total social support	-.527	.197	-.095	-2.678	.008
	trait anxiety	.193	.033	.292	5.928	.000
	self seteem scale	-.438	.061	-.350	-7.214	.000
	STRESS	.191	.032	.217	5.994	.000

a. Dependent Variable: depression (BDI)

Here's the results from adding the same predictors in another different grouping and order

- Notice the last step includes all 4 predictors with the same R^2 and b values as above

Model Summary

Model	R	R Square
1	.723[a]	.523
2	.757[b]	.573

a. Predictors: (Constant), STRESS, self seteem scale

b. Predictors: (Constant), STRESS, self seteem scale, total social support, trait anxiety

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	29.353	1.686		17.411	.000
	self seteem scale	-.721	.046	-.576	-15.529	.000
	STRESS	.241	.033	.274	7.388	.000
2	(Constant)	15.838	3.130		5.060	.000
	self seteem scale	-.438	.061	-.350	-7.214	.000
	STRESS	.191	.032	.217	5.994	.000
	trait anxiety	.193	.033	.292	5.928	.000
	total social support	-.527	.197	-.095	-2.678	.008

a. Dependent Variable: depression (BDI)

Here's the results from adding the same predictors in yet another grouping and order

- Verily, the last step includes all 4 predictors with the same R^2 and b values as above

Model Summary

Model	R	R Square
1	.487[a]	.237
2	.564[b]	.318
3	.720[c]	.518
4	.757[d]	.573

a. Predictors: (Constant), STRESS

b. Predictors: (Constant), STRESS, total social support

c. Predictors: (Constant), STRESS, total social support, trait anxiety

d. Predictors: (Constant), STRESS, total social support, trait anxiety, self seteem scale

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	3.731	.437		8.529	.000
	STRESS	.428	.038	.487	11.198	.000
2	(Constant)	13.135	1.426		9.214	.000
	STRESS	.381	.037	.434	10.349	.000
	total social support	-1.600	.232	-.289	-6.894	.000
3	(Constant)	-3.343	1.753		-1.907	.057
	STRESS	.213	.034	.243	6.341	.000
	total social support	-.776	.206	-.140	-3.774	.000
	trait anxiety	.343	.027	.519	12.894	.000
4	(Constant)	15.838	3.130		5.060	.000
	STRESS	.191	.032	.217	5.994	.000
	total social support	-.527	.197	-.095	-2.678	.008
	trait anxiety	.193	.033	.292	5.928	.000
	self seteem scale	-.438	.061	-.350	-7.214	.000

a. Dependent Variable: depression (BDI)

Here's the results from a forward stepwise procedure – different entry order, same final model!

Variables Entered/Removed[a]

Model	Variables Entered	Method
1	self seteem scale	Stepwise (Criteria: Probability-of-F-to-enter <= .050
2	trait anxiety	Stepwise (Criteria: Probability-of-F-to-enter <= .050
3	STRESS	Stepwise (Criteria: Probability-of-F-to-enter <= .050
4	total social support	Stepwise (Criteria: Probability-of-F-to-enter <= .050

a. Dependent Variable: depression (BDI)

Model Summary

Model	R	R Square
1	.677[a]	.459
2	.726[b]	.527
3	.752[c]	.566
4	.757[d]	.573

a. Predictors: (Constant), self seteem scale

b. Predictors: (Constant), self seteem scale, trait anx

c. Predictors: (Constant), self seteem scale, trait anxiety, STRESS

d. Predictors: (Constant), self seteem scale, trait anxiety, STRESS, total social support

Coefficients [a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	35.674	1.546		23.073	.000
	self seteem scale	-.848	.046	-.677	-18.475	.000
2	(Constant)	14.504	3.140		4.619	.000
	self seteem scale	-.504	.062	-.402	-8.069	.000
	trait anxiety	.250	.033	.379	7.598	.000
3	(Constant)	13.391	3.017		4.439	.000
	self seteem scale	-.466	.060	-.372	-7.746	.000
	trait anxiety	.204	.033	.308	6.258	.000
	STRESS	.193	.032	.219	6.013	.000
4	(Constant)	15.838	3.130		5.060	.000
	self seteem scale	-.438	.061	-.350	-7.214	.000
	trait anxiety	.193	.033	.292	5.928	.000
	STRESS	.191	.032	.217	5.994	.000
	total social support	-.527	.197	-.095	-2.678	.008

a. Dependent Variable: depression (BDI)