

943 -- Determining the Number of Factors

Working again with the data from the adolescents receiving treatment for behavior disorders. But this time the question is about the number and kinds of information available from a series of variables that related to in-home and in-school behavior.

A PC analysis of these variables (without rotation) gives us the following (with some rearrangement)...

Factor Analysis: Descriptives

Statistics

Univariate descriptives

Initial solution

Continue Cancel Help

Correlation Matrix

Coefficients Inverse

Significance levels Reproduced

Determinant Anti-image

KMO and Bartlett's test of sphericity

Factor Analysis: Extraction

Method: Principal components

Continue Cancel Help

Analyze

Correlation matrix

Covariance matrix

Display

Unrotated factor solution

Screen plot

Extract

Eigenvalues over: 1

Number of factors:

Maximum Iterations for Convergence: 25

Communalities

| | Initial |
|-----------------------------------------------|---------|
| SEI | 1.000 |
| SEP | 1.000 |
| EI | 1.000 |
| EP | 1.000 |
| in-school disciplinary action | 1.000 |
| suspension from school | 1.000 |
| did the court remove the child from treatment | 1.000 |

Sutter-Eiberg Intensity -- frequency of specific behaviors exhibited by child -- teacher rating

Sutter-Eiberg Problem -- how problematic are the same behaviors -- teacher rating

Eiberg Intensity -- parent rating version

Eibher Problem -- parent rating version

Extraction Method: Principal Component Analysis.

Total Variance Explained

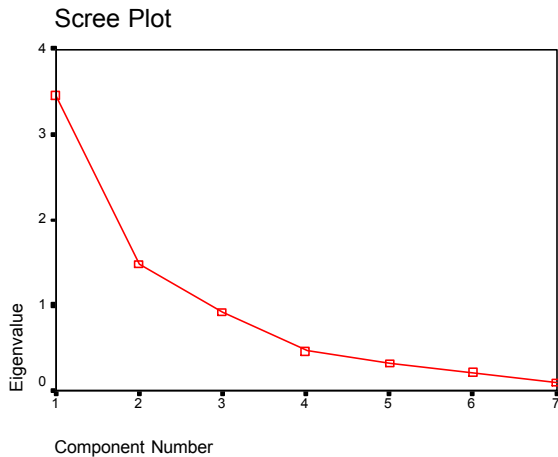
| Component | Initial Eigenvalues | | |
|-----------|---------------------|---------------|--------------|
| | Total | % of Variance | Cumulative % |
| 1 | 3.456 | 49.367 | 49.367 |
| 2 | 1.491 | 21.306 | 70.672 |
| 3 | .924 | 13.205 | 83.877 |
| 4 | .468 | 6.679 | 90.556 |
| 5 | .328 | 4.690 | 95.247 |
| 6 | .226 | 3.234 | 98.480 |
| 7 | .106 | 1.520 | 100.000 |

$\lambda > 1.00$ rule would lead to keeping 2 PCs, but notice the 3rd is "close" (13.2% of variance) and the 4th is "not even close" (6.7%).

Extraction Method: Principal Component Analysis.

Correlation Matrix^a

a. Determinant = 1.761E-02



The determinant represents the variances of the variables "minus" the covariances among them.

- So, the more of the variance of the variables that is related to the other variable the smaller the determinant will be.
- If the determinant = 0, it means that at least one of the variables is perfectly correlated with some combination of the other variables -- a singular matrix
- So, the smaller the determinant the more that the variable's variance can be reproduced using the other variables (and PC's formed from them, but ...)
- If the determinant is "too small" then the mathematics of the factoring formulas become unstable

The scree plot is sort of ugly! Looks like breaks at 2 and 4 -- suggesting keeping 1 or 3 factors ('67 rule).

KMO and Bartlett's Test

| | | |
|--------------------------------------------------|--------------------|---------|
| Kaiser-Meyer-Olkin Measure of Sampling Adequacy. | | .685 |
| Bartlett's Test of Sphericity | Approx. Chi-Square | 164.944 |
| | df | 21 |
| | Sig. | .000 |

The KMO -- indicates the proportion of variance in your variables which is common variance, i.e. which might be caused by underlying factors.

-- values < .50 suggest that the variables won't "factor well"

One of the "Bartlett's Sphericity Tests" is also provided by SPSS -- the one that tests if there's any systematic variance available to be factored.

I've provided a program that does the "Keep another factor" X^2 test, let's work with it a bit...

A minimum criterion for keeping a factor is that it is "significant" (probably not a Type I error). This is often referred to as a "minimum" criterion because the test is pretty sensitive, especially when N is large. So, a common result is analogous to having a significant effect, but one with a very small sample size -- the effect is "significant" but not "meaningful".

The $\lambda > 1$ rule says keep 2 factors, the 3rd is suggested by the scree, but we might check if the 3rd is "significant". To use the Bartlett's X^2 Computator, we need to know:

- the number of variables $\rightarrow 7$
- the sample size $\rightarrow 47$
- the determinant of the correlation matrix (R) $\rightarrow .01761$
- The number of factors we're sure we want to keep (so we can test the next one) $\rightarrow 2$

First we enter these values into the computer and click on the "Start" button

The program then asks for each of the eigenvalues for the factors we are certain we want to keep. Enter all these values and click the "Test" button.

Bartlett's X² Test for "the next PC"

Number of Variables: 7

Sample size: 47

Determinant of R: .01761

How many factors do you want to consider?: 2

Start

Test

| 1st 2 factors | Eigenvalue |
|---------------|------------|
| 1 | |
| 2 | |

Bartlett's X² Test for "the next PC"

Number of Variables: 7

Sample size: 47

Determinant of R: 0.01761

How many factors do you want to consider?: 2

Start

Test

Significant results indicate there is additional systematic variance – consider an

| 1st 2 factors | Eigenvalue | |
|---------------|------------|---------------------------------|
| 1 | 3.456 | $\chi^2(20) = 89.409$ $p < .01$ |
| 2 | 1.491 | $\chi^2(14) = 50.968$ $p < .01$ |

The results shows that there is systematic variance left among the correlations after extracting the second factor – suggesting that the third factor may be a "worthwhile" factor .

Here's Another Example

The researchers wanted to explore the relationship among a selection of dyadic behaviors. The extraction produced...

Total Variance Explained

| Component | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | |
|-----------|---------------------|---------------|--------------|-------------------------------------|---------------|--------------|
| | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 2.829 | 31.434 | 31.434 | 2.829 | 31.434 | 31.434 |
| 2 | 1.862 | 20.690 | 52.123 | 1.862 | 20.690 | 52.123 |
| 3 | 1.010 | 11.225 | 63.348 | 1.010 | 11.225 | 63.348 |
| 4 | .970 | 10.783 | 74.131 | | | |
| 5 | .670 | 7.448 | 81.579 | | | |
| 6 | .514 | 5.711 | 87.290 | | | |
| 7 | .440 | 4.893 | 92.183 | | | |
| 8 | .382 | 4.243 | 96.426 | | | |
| 9 | .322 | 3.574 | 100.000 | | | |

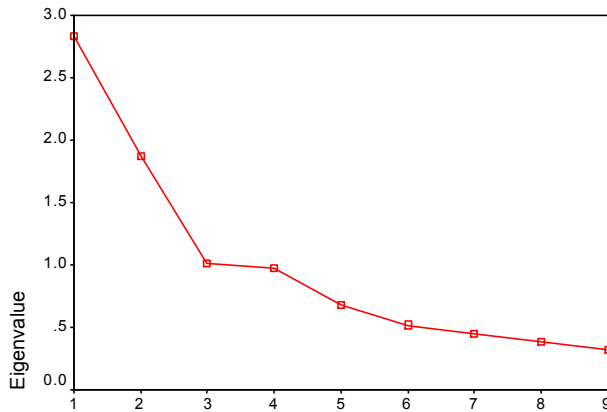
Extraction Method: Principal Component Analysis.

The $\lambda > 1.00$ rule leads to retaining 3 factors.

But notice that the λ of the 3rd factor is larger than, but very close to 1.00.

Also notice that the λ of the 4th factor is smaller than, but very close to 1.00

Scree Plot



Component Number

The screen plot looks like ...

Major elbow at 3 factors – suggesting 2 or 3 factors, depending upon which version of the scree rule you like.

Maybe another elbow at 5.

Bartlett's X² Computer

Bartlett's X² Test for "the next PC"

Number of Variables:

Sample size:

Determinant of R:

How many factors do you want to consider?

1st 4 factors

| Eigenvalue | Significant results indicate there is additional systematic variance – consider an |
|------------|------------------------------------------------------------------------------------|
| 2.829 | t(35)= 12.049 p < .05 |
| 1.862 | X ² (27)= 106.395 p < .01 |
| 1.010 | X ² (20)= 74.21 p < .01 |
| .970 | X ² (14)= 28.536 p < .05 |

Getting the significance tests shows that at least the first 5 factors are based systematic variation.