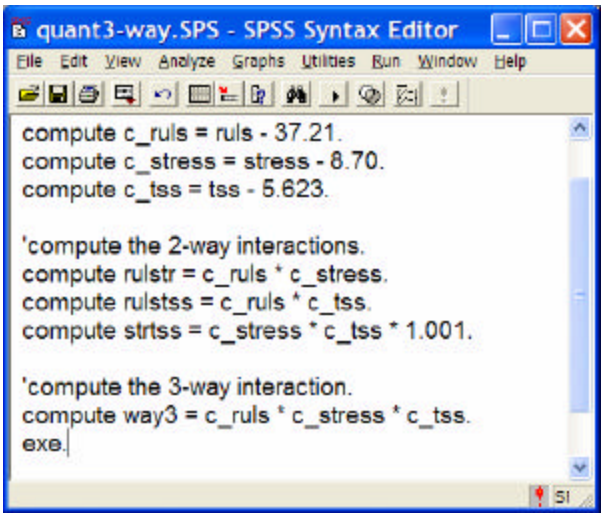
Example of a 3-way Interaction Among Quantitative Variables

An earlier analysis looked at how the relationship between stress and depress is moderated by social support. This analysis looks at an additional variable, loneliness, to see whether it and its interactions further explicate relationship between stress and depression.

We'll need to use centered variables, which requires knowing the mean of each predictor.

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
loneliness	405	20	73	37.21	11.377
total social support	405	1.00	7.00	5.6233	1.18204
STRESS	405	0	39	8.70	7.448
Valid N (listwise)	405				



Main effects are centered.

2-way interactions are computed as products of the centered main effect variables

The 3-way is also a product variable – using all three main effects.

A 3 stage model was used for this analysis – first the main effects alone, then the 2-ways were added, final the 3-way was included.

While the key information (except for R^2? values) can be obtained from the final model, this will allow us to watch how the ''story'' changes as the interactions and the 3-way are added to the model.

Model Summary

Model	R	R Square	Change Statistics				
			R Square Change	F Change	df1	df2	Sig. F Change
1	.641[a]	.411	.411	93.390	3	401	.000
2	.654[b]	.428	.016	3.759	3	398	.011
3	.668[c]	.446	.018	13.128	1	397	.000

a. Predictors: (Constant), C_TSS, C_STRESS, C_RULS

b. Predictors: (Constant), C_TSS, C_STRESS, C_RULS, STRTSS, RULSTSS, RULSTR

c. Predictors: (Constant), C_TSS, C_STRESS, C_RULS, STRTSS, RULSTSS, RULSTR, WAY3

ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	7115.756	3	2371.919	93.390	.000[a]
	Residual	10184.649	401	25.398		
	Total	17300.405	404			
2	Regression	7396.381	6	1232.730	49.538	.000[b]
	Residual	9904.024	398	24.884		
	Total	17300.405	404			
3	Regression	7713.393	7	1101.913	45.630	.000[c]
	Residual	9587.012	397	24.149		
	Total	17300.405	404			

a. Predictors: (Constant), C_TSS, C_STRESS, C_RULS

b. Predictors: (Constant), C_TSS, C_STRESS, C_RULS, STRTSS, RULSTSS, RULSTR

c. Predictors: (Constant), C_TSS, C_STRESS, C_RULS, STRTSS, RULSTSS, RULSTR, WAY3

d. Dependent Variable: depression (BDI)

Clearly the main effects account for the bulk of the variance accounted for, but with the large sample size the interactions increase the fit of the model to the data.

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations	
		B	Std. Error	Beta			Zero-order	Part
1	(Constant)	7.454	.250		29.764	.000		
	C_RULS	.232	.029	.403	7.974	.000	.537	.306
	C_STRESS	.319	.035	.364	9.094	.000	.487	.348
	C_TSS	-.272	.273	-.049	-.996	.320	-.369	-.038
2	(Constant)	7.047	.287		24.524	.000		
	C_RULS	.211	.030	.367	7.084	.000	.537	.269
	C_STRESS	.302	.036	.344	8.299	.000	.487	.315
	C_TSS	-.120	.275	-.022	-.436	.663	-.369	-.017
	RULSTR	1.297E-03	.003	.019	.376	.707	.271	.014
	RULSTSS	-3.81E-02	.017	-.103	-2.206	.028	-.352	-.084
	STRTSS	-3.33E-02	.035	-.051	-.964	.335	-.260	-.037
3	(Constant)	7.125	.284		25.097	.000		
	C_RULS	.219	.029	.380	7.426	.000	.537	.277
	C_STRESS	.260	.038	.296	6.879	.000	.487	.257
	C_TSS	.176	.283	.032	.622	.534	-.369	.023
	RULSTR	-2.48E-03	.004	-.037	-.698	.485	.271	-.026
	RULSTSS	-3.39E-02	.017	-.092	-1.989	.047	-.352	-.074
	STRTSS	2.167E-02	.037	.033	.582	.561	-.260	.022
	WAY3	-7.13E-03	.002	-.217	-3.623	.000	-.441	-.135

a. Dependent Variable: depression (BDI)

Plotting the Model

Start with the full model…

$y' = b_x X + b_z Z + b_v V + b_{xz} XZ + b_{xv} XV + b_{zv} ZV + b_{xzv} XZV + a$ ← full model

$y' = b_x X + b_{xz} XZ + b_{xv} XV + b_{xzv} XZV + b_z Z + b_v V + b_{zv} ZV + a$ ← gather the X-terms

$y' = (b_x + b_{xz} Z + b_{xv} V + b_{xzv} ZV) * X + (b_z Z + b_v V + b_{zv} ZV + a)$ ← factor out X for y' = bX + a form

Determine which variable will be X, Z & V (depending upon how you want to consider and portray the interactioni)

Then substitute values of Z & V to find the simple X-Y formula for different ZV combinations.

Often this is simplified by using just +1std and -1std values of for Z & V, which leads to plotting four simple regression lines to portray the interaction: 1) +1stdZ & +1stdV, 2) +1stdZ * -1stdV, 3) -1stdZ & +1stdV, 4) -1stdZ & -1stdV

Here are the results from using IntPlot to get the plotting coordinates. When using the program you have to decide which predictors will be X, Z & V.

- Since the major bivariate relationship is between stress and depression, stress will be used as X
- Since the next question was how social support moderated the stress-depression relationship, social will be Z
- And loneliness will be used V, as the 3rd main effect

So, we'll get the simple regression lines for the stress-depression relationship, for four combinations of social support (+1 std & -1 std) and loneliness (+1 std & -1 std).

Quant 3-way

"X" is the x-axis variable
"Z" and "V" are the other main effect variables

Plotting points for the X-Y relationship at each combination of +/- 1 std of Z and V will be given

Enter values to at least 3 decimals

0.26	* X	std of X	7.448
0.176	* Z	std of Z	1.182
0.219	* V	std of V	11.377
0.02167	* XZ		
-0.00248	* XV		
-0.00339	* ZV		
0.00713	* XZV		
7.125	constant		

Simple effect regression models

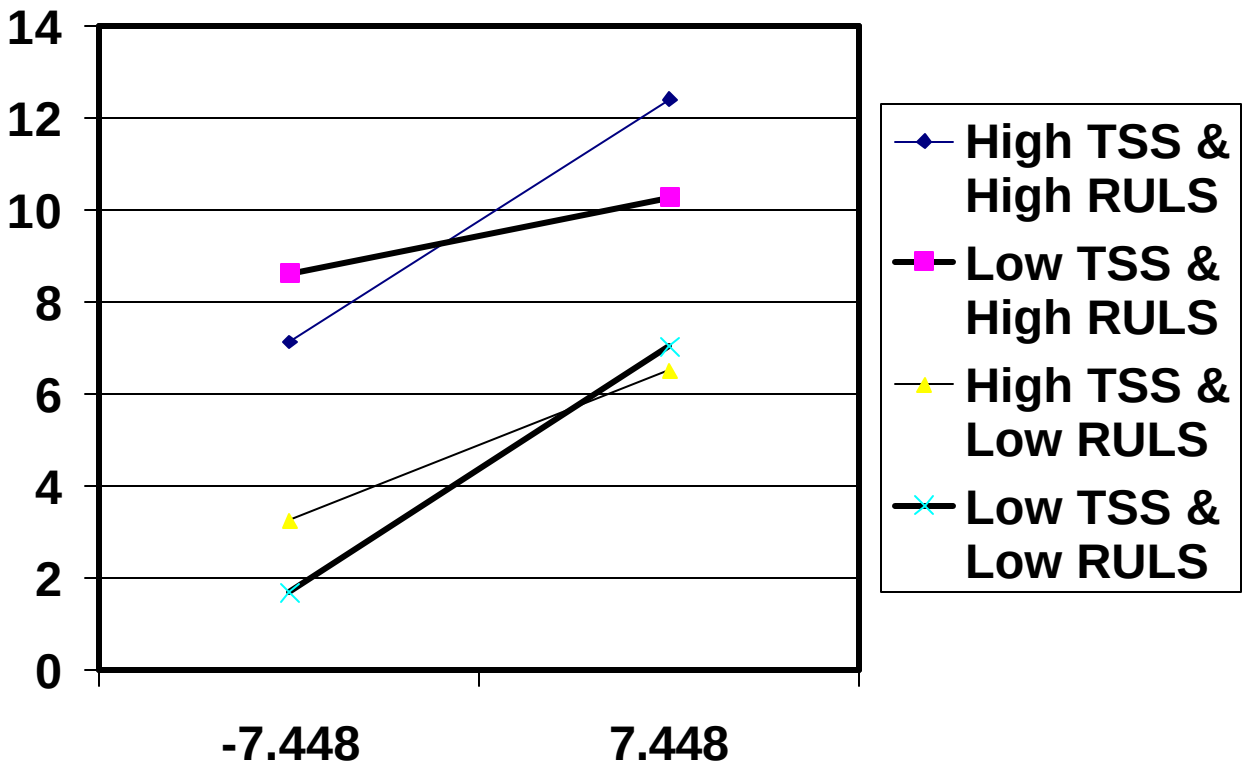
Compute Back

+1 std of Z and +1 std of V	b= 0.353	a= 9.779
+1 std of Z and -1 std of V	b= 0.217	a= 4.887
-1 std of Z and +1 std of V	b= 0.11	a= 9.454
-1 std of Z and -1 std of V	b= 0.358	a= 4.379

Plotting points

For +1 std of Z and +1 std of V,	plot	-7.448 ,	7.14	and	7.448 ,	12.41
For +1 std of Z and -1 std of V,	plot	-7.448 ,	3.26	and	7.448 ,	6.51
For -1 std of Z and +1 std of V,	plot	-7.448 ,	8.63	and	7.448 ,	10.27
For -1 std of Z and -1 std of V,	plot	-7.448 ,	1.7	and	7.448 ,	7.04

This is a really awful plot I made with an embedded chart



Interpreting the Model Using the Plot and the Regression Weights

There is a general positive relationship between stress and depression, after accounting for the other effects in the model – higher stress → higher depression.

In this model, there is no relationship between TSS and depression, after accounting for the other effects in the model -- the average of the two High TSS lines is not significantly higher than the average of the two Low TSS lines

There is a general positive relationship between RULS and depression, after accounting for the other effects in the model – higher RULS → higher depression

Seeing 2-way interactions within a 3-way plot can be difficult, fortunately there is only the one…

The RULS * TSS interaction shows that the relationship between RULS and Depression is more positive for lower values of TSS and less positive for higher values of TSS. You would have to change the X variable to be one of these and Z to be the other, get new plotting coordinates and make a new plot to see this 2-way clearly.

To describe the 3-way pattern we'll look at the simple 2-way of stress and TSS separately for High and and Los Ruls.

- For high RULS (diamond & square lines) and low stress, those with low TSS have higher depression scores than those with high TSS, whereas for those with high RULS and high stress, those with high TSS have higher depression scores than those with low TSS. (While this doesn't seem to make much sense, remember that these are not experimental data, so it is not appropriate to interpret this to mean that those with high RULS and high stress that more social support *causes* higher depression, only that more social support is associated with higher depression – maybe because those who are depressed "draw" more social support?

However,

- For low RULS (diamons and X lines) and low stress, those with high TSS have higher depression scores than those with low TSS, whereas for those with high stress, there is no difference in depression for those with low and high TSS (with the same cautions as above)

Some words of warning about 3-way interactions – Especially about their *Post-Hoc* interpretation…

- Remember that *a priori* hypotheses that are supported by higher-order effects are much more believable than are "discovered" (i.e., *post hoc*) effects
- This is especially true with the *post hoc* effect …
    - Is weak → consider that p=.047 with N=405, with a ß that isn't very large
    - Has pattern that requires a convoluted interpretation

The result pattern shown here – a weak, barely significant effect that "takes some explaining" from a large sample might turn out to be either a Type I or a Type III error upon replication & convergent research.