Example of Testing Research Hypotheses by Comparing Multiple Regression Models

Three educational researcher disagreed about the best way to anticipate college performance. The first hypothesized that three variables were important: locus of control (those with an internal locus of control would "knuckle down" when the going got tough), reading (a basic skill for all academic performance), and science (since these are the courses that "drag down" and frustrate most young college students). The second researcher hypothesized that locus of control was an unimportant predictor (that silly psycho-babble stuff). The third researcher agreed with the first that locus of control was important, but felt that since many students managed to avoid "serious" science courses, locus of control and reading would work as well as the first researcher's model.

So, we have three models
1. 1st Researcher's model → locus, reading & science
2. 2nd Researcher's model → reading & science
3. 3rd Researcher's model → reading & locus

.. and three research questions:
1. Does the locus-reading-science model work better than the reading-science model → comparing nested models
2. Does the locus-reading-science model work better than the locus-reading model → comparing nested models
3. Does the reading-science model work better than the locus-reading model → comparing non-nested models

Comparing Nested Models using SPSS

There are two different ways to compare nested models using SPSS.
- Get the multiple regression results for each model and then make the nested model comparisons using the "R^2 change F-test" part of the FZT Computator.
- Use SPSS to change from one model to another and compute resulting the R^2-change F-test for us. (While convenient, some versions of SPSS don't use the correct dferror for this test under some circumstances.)

Here's an example using the "Enter" and "Remove" functions of SPSS Regression

Analyze → Regression → Linear

Getting the full model → locus, rdg & sci …

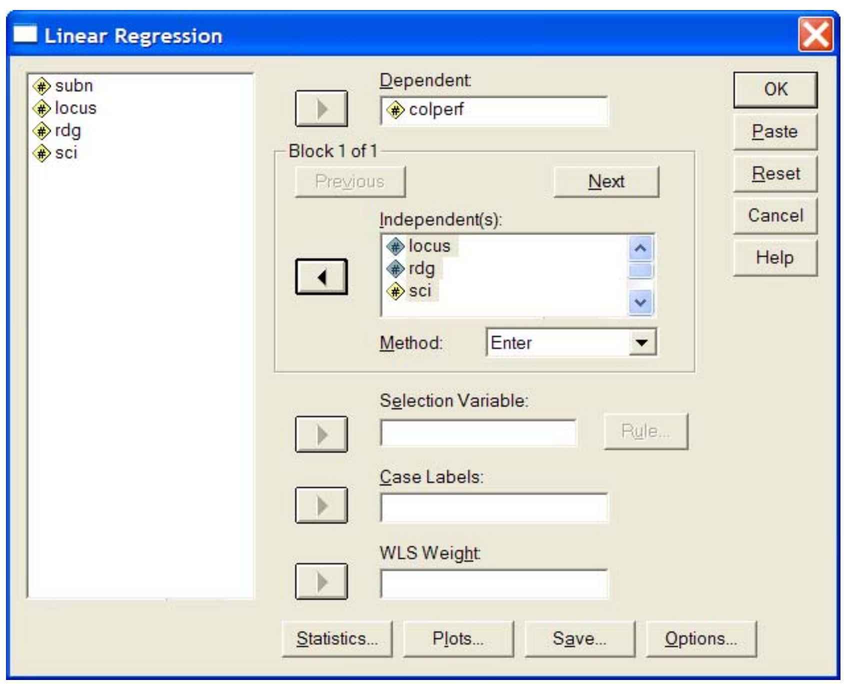
Move the criterion variable into the "Dependent" window

Move the predictors into the "Independent(s)" window

Be sure "Enter" is showing in the "Method" window

Click the "Next" button

A new window will appear that says "Block 2 of 2"



Getting the reading & science model and comparing it to the full model …

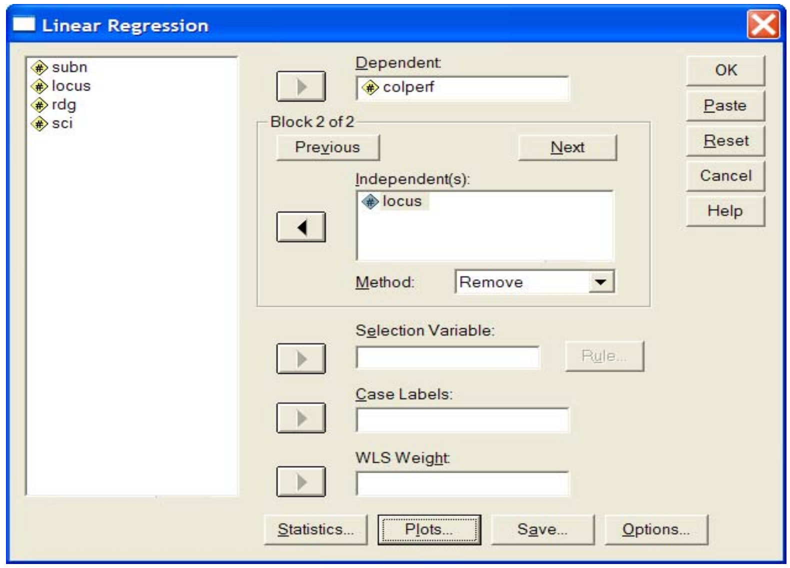
In "Block 2 of 2"

Move locus into the "Independent(s)" window .

Be sure "Remove" is in the "Method" window/

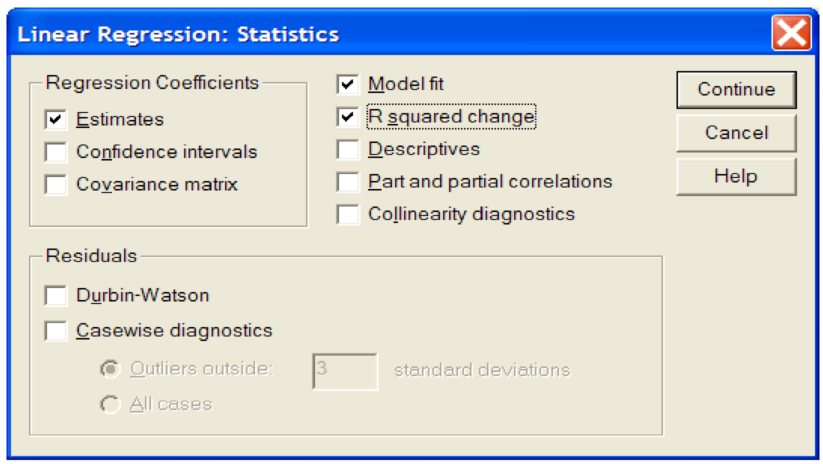
This tells SPSS to make a second model by removing locus from the previous model.

We could also have started with a 1st model Entering reading & science and then making a second model by Entering locus. Adding variables to a model and removing them from a model are equivalent – they both compare the same models,



Click the "Statistics" button.

Be sure that R squared change is checked – this will get you the R-square change F-test



SPSS Syntax

*Full model & removing predictor(s) to form reduced model.
REGRESSION
/STATISTICS COEFF OUTS R ANOVA CHANGE ← asks for usual stats & R^2-change F-test
/DEPENDENT colperf ← set criterion variable
/METHOD=ENTER locus rdg sci ← enter these predictors as first model with locus, rdg & sci

/METHOD=REMOVE locus. ← remove this predictor for form second model including rdg & sci

*alternative comparison of same two models.
*form reduced model and then add predictor(s) to form full model.
*Full model & removing predictor(s) to form reduced model.
REGRESSION
/STATISTICS COEFF OUTS R ANOVA CHANGE
/DEPENDENT colperf
/METHOD=ENTER rdg sci ← forms first model with red & sci
/METHOD=ENTER locus. ← adds locus to make the second model

SPSS Output:

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.658[a]	.432	.424	149.97822	.432	49.511	3	196	.000
2	.657[b]	.431	.426	149.71207	-.001	.305	1	196	.581

a. Predictors: (Constant), science score, reading score, locus of control

b. Predictors: (Constant), science score, reading score

Notice that for model 1 the R^2 and R^2 change are the same (as are the associated F-tests), since this model is "changing" from a 0-predictor model to this one.

ANOVA[c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3340992	3	1113664.060	49.511	.000[a]
	Residual	4386226	196	22493.468		
	Total	7727218	199			
2	Regression	3334132	2	1667066.090	74.377	.000[b]
	Residual	4393086	197	22413.705		
	Total	7727218	199			

a. Predictors: (Constant), science score, reading score, locus of control

b. Predictors: (Constant), science score, reading score

c. Dependent Variable: college gpa

Coefficients[a]

Model		Unstandardized Coefficients: B	Std. Error	Standardized Coefficients: Beta	t	Sig.
1	(Constant)	509.497	52.104		9.778	.000
	locus of control	8.895	16.108	.032	.552	.581
	reading score	5.083	.940	.376	5.405	.000
	science score	6.172	1.248	.342	4.946	.000
2	(Constant)	499.825	48.985		10.204	.000
	reading score	5.178	.923	.383	5.611	.000
	science score	6.279	1.231	.348	5.101	.000

a. Dependent Variable: college gpa

Science-Reading-Locus model

- R^2 is significant $p < .001$
- and substantial .432
- reading and science contribute
  - and about equally -- look at βs
- locus does not contribute

Science-Reading model

- R^2 is significant $p < .001$
- and substantial .431
- reading and science contribute
  - and about equally -- look at βs

Comparing the two nested models

- we can see this one coming -- we dropped the noncontributor from the larger model to form the smaller model
- R^2 change is small .001
- R^2 change is not significant p = .581
- Conclusion:
  - Locus does not add to a model including reading and science

The third researcher was working alone (and with an older version of SPSS), and did this analysis:

Comparing Nested Models using FZT

This is the older version of SPSS Regression output. Having seen examples of it in a couple recently published textbooks, I thought you should see what it looks like.

```
Equation Number 1    Dependent Variable..   COLPERF    COLLEGE PERFORMANCE
 Block Number  1.  Method:  Enter      LOCUS    RDG

 Multiple R           .60097
 R Square             .36117
 Adjusted R Square    .35465
 Standard Error    158.69973

 Analysis of Variance
                     DF      Sum of Squares      Mean Square
 Regression           2       2790840.01117    1395420.00558
 Residual           197       4936378.38079      25185.60398
 F =      55.40546       Signif F =  .0000

 ------------------ Variables in the Equation ------------------
 Variable              B         SE B       Beta          T  Sig T
 RDG            7.724519     .818899    .571216      9.433  .0000
 LOCUS         21.214956   16.839453    .076291      1.260  .2092
 (Constant)   666.380357   43.738513                15.236  .0000
```

We can use the $R^2\Delta$ F-test compare the R^2 from this model and the full model derived earlier.

$$F = \frac{(R^2(L) - R^2(S)) \; / \; (k(L) - k(S))}{(1 - R^2(L)) \; / \; (N - k(L) - 1)} = \frac{(.432 - .361) \; / \; (3 - 2)}{(1 - .432) \; / \; (200 - 3 - 1)} = 24.38$$

where:

$R^2(L)$	= R^2 from the larger model	= .432	$k(L)$	= number of predictors in larger model	= 3
$R^2(S)$	= R^2 from the smaller model	= .361	$k(S)$	= number of predictors in smaller model	= 2

N = number of subjects = df(regression) + df(residual) + 1 = (2 + 197 + 1) or (3 + 196 + 1) = 200

looking at an F-table ➔ $F(1,200, \alpha = .01) = 6.76$ so, this R^2-change is significant at the .01 level.

Remember that the "R^2-change" part of the FZT program uses R^2 values!

Using R^2 larger = .432, k larger = 3, R^2 smaller = .361, k smaller = 2 and N = 200 gives us F= 24.50

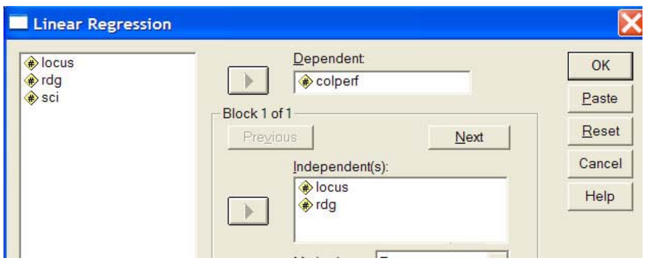
Comparing Non-Nested Models

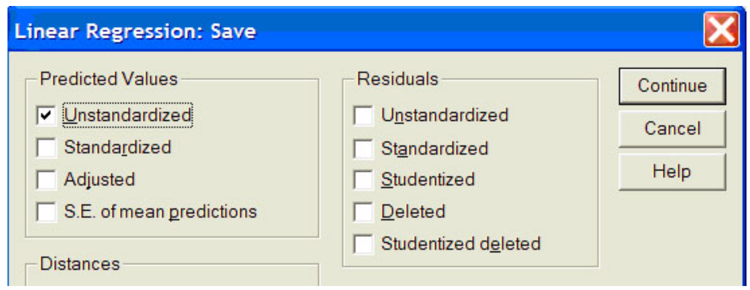
Having compared each of the reduced models to the full model, we might next want to compare the two reduced models to each other. Is there a difference in the variance accounted for by the Rdg & Sci model and the Rdg & Locus model? **Remember**, we can not be sure that the R^2 for the two reduced models are significantly different, just because one is equivalent to the full model and one is significantly smaller than the full model!!!

In order to compare these models we need to know the correlation between them. This is obtained as the correlation between the y' values computed from each model. This is easy to do in SPSS.

Analyze → Regression → Linear

- Enter the Dependent and Independent variables for the model (the science & reading model is below)
- Click the "Save" button (at the bottom of the Linear Regression window)
- Click "Unstandardized" under "Predicted Values)
- Run the regression analysis
- Repeat for the other model



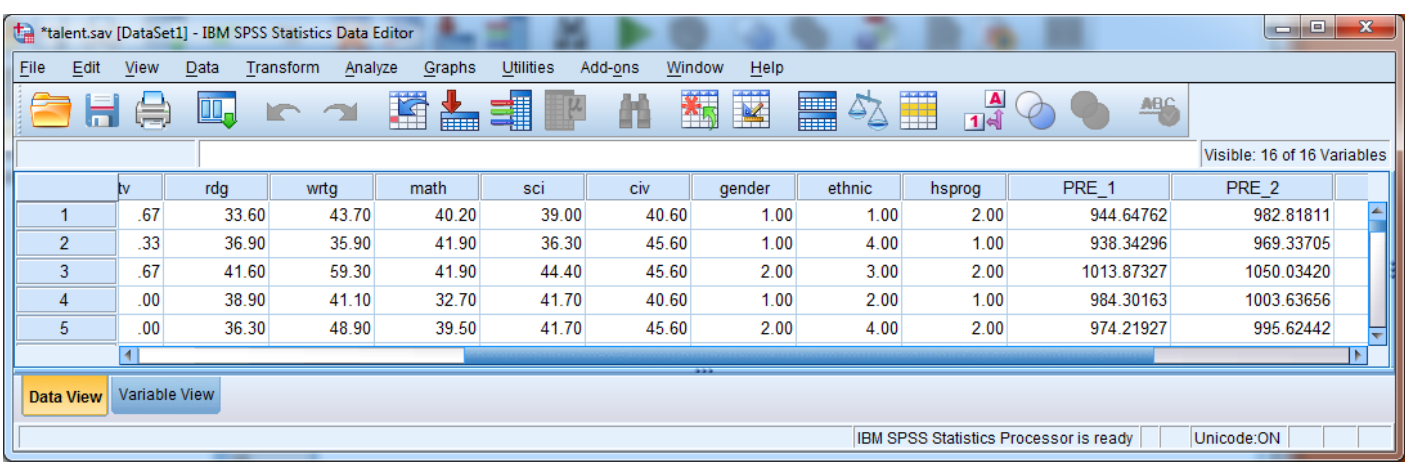


SPSS Syntax

```
REGRESSION
  /STATISTICS COEFF OUTS R ANOVA
  /DEPENDENT colperf
  /METHOD=ENTER rdg sci
  /SAVE PRED.

REGRESSION
  /STATISTICS COEFF OUTS R ANOVA
  /DEPENDENT colperf
  /METHOD=ENTER rdg locus
  /SAVE PRED.
```

SPSS will compute 2 new variables that are the y' values for the two models. These variables will be called PRE_1 and PRE_2. You must be careful to remember which is which – renaming them is a great idea!



	tv	rdg	wrtg	math	sci	civ	gender	ethnic	hsprog	PRE_1	PRE_2
1	.67	33.60	43.70	40.20	39.00	40.60	1.00	1.00	2.00	944.64762	982.81811
2	.33	36.90	35.90	41.90	36.30	45.60	1.00	4.00	1.00	938.34296	969.33705
3	.67	41.60	59.30	41.90	44.40	45.60	2.00	3.00	2.00	1013.87327	1050.03420
4	.00	38.90	41.10	32.70	41.70	40.60	1.00	2.00	1.00	984.30163	1003.63656
5	.00	36.30	48.90	39.50	41.70	45.60	2.00	4.00	2.00	974.21927	995.62442

Then we get the correlation between these two new variables (capturing the correlation between the two non-nested models) and the criterion (duplicating the Rs from the models – just to check!).

CORRELATION
  VARIABLES = pre_1 pre_2 colperf.

The correlation of each should equal the R from the multiple regression of that model.

The correlations between the models (r(1,2)

R-value for the science & reading model

R-value for the reading & locus model

Correlations

		Unstandardized Predicted Value	Unstandardized Predicted Value	COLLEGE PERFORMANCE -- DV
Unstandardized Predicted Value	Pearson Correlation	1	.910	.657
	Sig. (2-tailed)		.000	.000
	N	200	200	200
Unstandardized Predicted Value	Pearson Correlation	.910	1	.601
	Sig. (2-tailed)	.000		.000
	N	200	200	200
COLLEGE PERFORMANCE -- DV	Pearson Correlation	.657	.601	1
	Sig. (2-tailed)	.000	.000	
	N	200	200	200

Remember that the "Hotellings t / Steiger's Z" formulas & commutators use R (r) values!

Using r_{y1} = .657, r_{y2} = .601 and r_{12} = .910 and N = 200 gives t = 2.46 & Z = 2.42. p = .0155.

We would conclude that the science-reading model predicts college performance significantly better than does the reading-locus model.

Example write-up of these analyses (which used some univariate and correlation info not shown above)**:**

A series of regression analyses were run to examine the relationships between college performance (colperf) and locus of control (locus), reading skills (rdg) and science skills (sci). Table 1 shows the univariate statistics, correlations of each variable with college performance, and the regression weights for the various models. The full model had an $R^2 = .432$, $F(3,196) = 49.51$, $p < .0001$, with science and reading having significant regression weights with similar relative contribution to the model.

The first research hypothesis was that a model including just reading and science skills would perform as well as the full model. This reduced model has an $R^2 = .431$, $F(2,197) = 74.38$, $p < .0001$, with both predictors having a significant contribution to the model. As hypothesized, this model did perform as well as the full model, R^2-change = .00089, $F(1, 196) = .305$, $p = .58$.

The second hypothesis was that a model including just reading skill and locus of control would also perform as well as the full model. This reduced model had an $R^2 = .36$, $F(2,197) = 55.41$, $p < .0001$, with only reading skill having a significant contribution. However this hypothesis was not supported, as this reduced model had a significantly lower R^2, R^2-change = .071, $F(1,196) = 24.38$, $p < .01$.

Finally the predictive utility of the two reduced models was compared, using the Hotelling's t-test for non-independent correlations. The correlation between these two models was $r = .90$, $p = .001$. The model including science and reading accounted for significantly more variance among college grades than did the model including reading and locus of control, $t(197) = 2.45$, $p < .05$.

Table 1 Summary statistics, correlations and results from the various regression models

				Beta weights from various models		
Variable	mean	std	correlation with college performance	full model	reading & science	locus of control & reading
colperf	3.65	1.02				
locus	.10	.35	.213	.031		.076
rdg	48.34	9.86	.431**	.342**	.383**	.571**
sci	47.47	8.76	.447**	.376**	.348**	

* p < .05 ** p < .01